2.4.4 Time reversal (folding)

Folding operation is important for filtering and is given by:

$$y[n] = x[-n]. \tag{2.5}$$

The input sequence is flipped at $n=0$ to produce the output sequence.

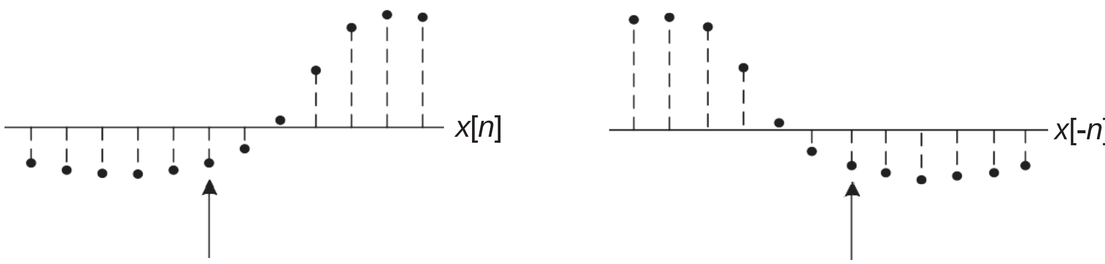

Figure 2.8: Folding operation (arrow points to n=0).

2.4.5 Branching

Branching is used to provide multiple copies of the input sequence:

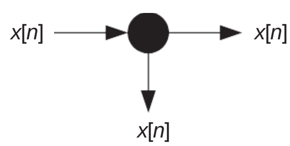

Figure 2.9: Branching operation.

2.4.6 Time shifting

Time shifting denotes delaying or advancing the input by N samples:

$$\begin{aligned} y[n] &= x[n-N] \quad \text{(delay)} \\ y[n] &= x[n+N] \quad \text{(advance)} \end{aligned} \tag{2.6}$$

Figure 2.8: Folding operation (arrow points to n=0).

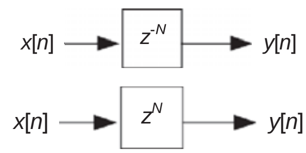

Figure 2.10: Block diagram for time shifting operation.

2.4.7 Time scaling

Time scaling can be categorised into down sampling or up sampling. In down sampling, every Mth sample of the input sequence is kept and M-1 in-between samples are removed. Hence, the number of samples is reduced or in other words, the sampling frequency is reduced. Down sampling can be denoted as

$$y[n] = x[Mn];$$
$$x[n] \rightarrow \downarrow M \rightarrow y[n].$$

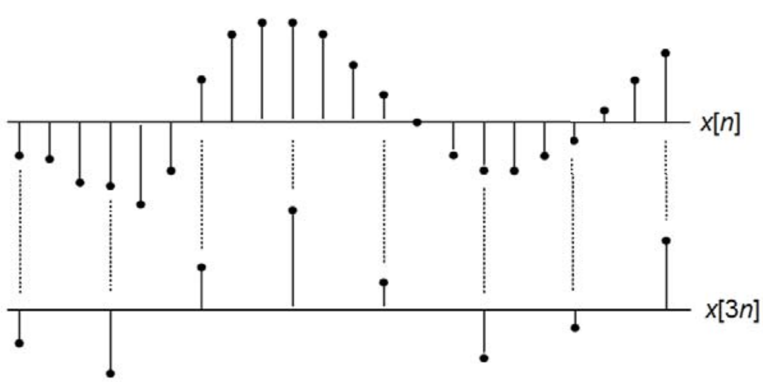

Figure 2.11: Example showing the down sampling process for M=3.

Up sampling is the opposite process of down sampling. In up sampling, N-1 equidistant zero-valued samples are inserted by the up sampler device between each consecutive samples of the input sequence. As a result, the number of samples is increased which effectively increases the sampling frequency.

$$y[n] = x[n/N];$$
$$x[n] \rightarrow \uparrow N \rightarrow y[n]. \tag{2.8}$$

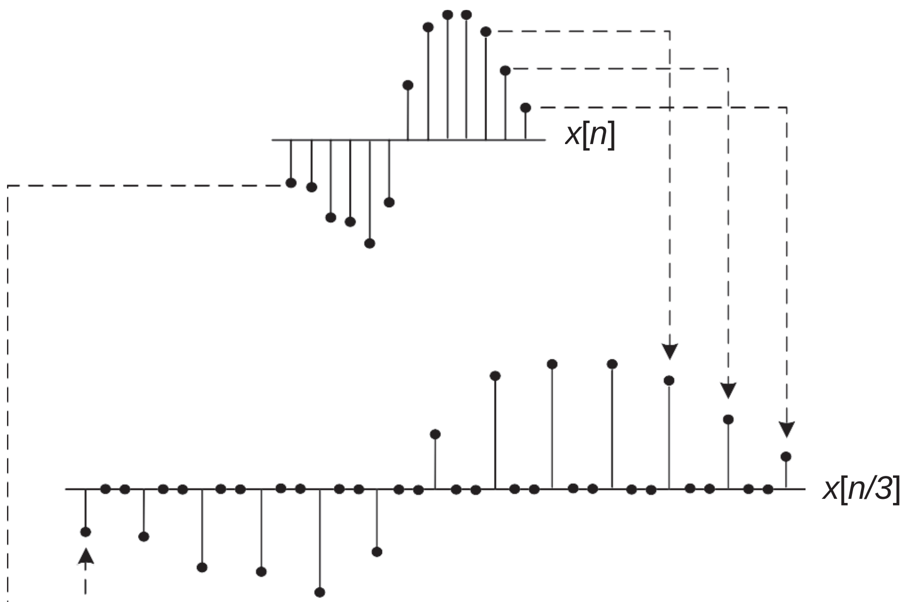

Figure 2.12: Example showing the up sampling process for *N*=3.

2.4.8 Combination of operations

Often, a system includes a combination of these operations. For example, Figure 2.13 shows a discrete-time system block diagram that includes 3 multipliers, 3 single sample delays and 1 adder operations for *y*[*n*]:

$$y[n] = \alpha x[n] + \beta x[n-1] + \lambda x[n-3]$$

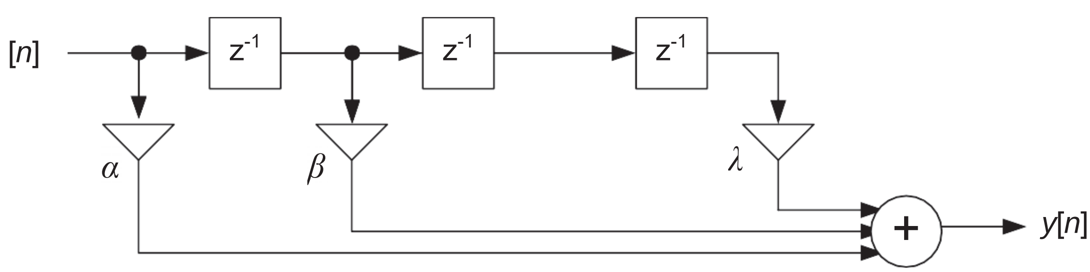

Figure 2.13: Block diagram of a discrete-time system *y*[*n*].

2.5 Examples on sequence operations

Several examples are given here to illustrate the concept of combining different sequence operations.

Example 1

For the following sequences, defined for $1 \le n \le 5$ (i.e. length=5),

- $a[n]$={1 2 4 -9 1};
- $b[n]$={2 -1 3 3 0}.

obtain the new sequences

- $c[n]=\{2.a[n].b[n]\}$;
- $d[n]=\{a[n]+b[n]\}$;
- $e[n]=0.5\{a[n]\}$.

Answer

- $c[n]$={4 -4 24 -54 0};
- $d[n]$={3 1 7 -6 1};
- $e[n]$={0.5 1 2 -4.5 0.5}.

These are easy as both $a[n]$ and $b[n]$ have same length. What if their lengths differ? If the lengths of sequences differ, then pad with zeros (in front or end) to obtain same length sequences and same defined ranges before applying the operations.

Example 2

For the following sequence $f[n]$={-5 2 -3} defined for $1 \le n \le 3$, what would be $g[n]=a[n]+f[n]$?

Answer

As $f[n]$ has length 3 and $a[n]$ has length 5, pad $f[n]$ with 2 zeros.

$a[n]$={1 2 4 -9 1} defined for $1 \le n \le 5$;

$f_{pad}[n]$={-5 2 -3 0 0} defined for $1 \le n \le 5$.

$f[n]$ is padded with 2 zeros at the end as to make the defined ranges of $a[n]$ and $f_{pad}[n]$ equal. So, $g[n]$={-4 4 1 -9 1}.

Example 3

Consider the following sequences:

- $a[n]$={-3 4 2 -6 -9}, $-3 \le n \le 1$;
- $b[n]$={-3 -1 0 8 7 -2}, $-2 \le n \le 3$;
- $c[n]$={1 8 -3 2 -6}, $3 \le n \le 7$.

The sample values of each of the above sequences outside ranges specified are all zeros. Generate the following sequences (put an arrow at $n=0$):

- $d[n]=a[-n+3]$;
- $e[n]=b[-n]$;
- $f[n]=a[n]+b[-n+2]+c[-n]$.

Answer

Making a table will make it easier to obtain the answer.

n	-3	-2	-1	0	1	2	3	4	5	6	7	8
a[*n*]	-3	4	2	-6	-9							
b[*n*]		-3	-1	0	8	7	-2					
c[*n*]							1	8	-3	2	-6	

For $d[n]=a[-n+3]$, the folding operation is performed first and then the shift to obtain

$$d[n] = \{\underset{\uparrow}{0} \quad 0 \quad -9 \quad -6 \quad 2 \quad 4 \quad 3\}.$$

n	-3	-2	-1	0	1	2	3	4	5	6
a[*n*]	-3	4	2	-6	-9					
a[-*n*]			-9	-6	2	4	-3			
a[-*n*+3]				**0**	**0**	-9	-6	2	4	-3

Similarly, for $e[n]=b[-n]$, we have

$$e[n] = \{-2 \quad 7 \quad 8 \quad \underset{\uparrow}{0} \quad -1 \quad 3\}.$$

n	-3	-2	-1	0	1	2	3	4	5	6	7	8
b[*n*]		-3	-1	0	8	7	-2					
b[-*n*]	-2	7	8	0	-1	-3						

For $f[n]=a[\mathrm{n}]+b[-n-2]+c[-n]$, we have

$$f[n] = \{-6 \quad 2 \quad -5 \quad 15 \quad 6 \quad 4 \quad 1 \quad \underset{\uparrow}{-9} \quad 9\}.$$

n	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8
a[*n*]					-3	4	2	-6	-9							
b[*n*]						-3	-1	0	8	7	-2					
b[-*n*]					-2	7	8	0	-1	-3						
b[-*n*-2]			-2	7	8	0	-1	-3								
c[*n*]								**0**	**0**	**0**	1	8	-3	2	-6	
c[-*n*]	-6	2	-3	8	1	0	0	0								
f[*n*]	-6	2	-5	15	6	4	1	-9	-9							

Often, combination of folding and shifting operations causes confusion on which direction to shift the sequence. The following hints will make it easier to remember the direction to make the shift operation:

- $x(n+k)$, then the signal moves $k \leftarrow$
- $x(n-k)$ signal moves $k \rightarrow$
- $x(-n+k)$ signal moves $k \rightarrow$
- $x(-n-k)$ signal moves $k \leftarrow$

From the examples above, we can verify that $a(-3-n)= a(-n-3)$ but it is always easier to if we rewrite the expression with the folding operation first.