Fundamental of Control Engineering

Systems Engineering Department
Second Class **II semester**

Introduction:

Control Theory: It is that part of science which concern control problems.

Control Problem: If we want something to act or vary according to a certain performance specification, then we say that we have a control problem.

Ex. We want to keep the temperature in a room at certain level and as we order, then we say that we have temperature control problem.

***Plant*:** A piece of equipments the purpose of which is to perform a particular operation (we will call any object to be controlled a plant).

Ex. Heating furnace, chemical reactor or space craft.

Process: A progressively continuing operation (natural or artificial) that consist of a series of actions or changes in a certain way leading towards a particular result or end. We will call any operation to be controlled a process. Processes could be chemical, economic, or biological.

System: A combination of components that act together and perform a certain objective (could be physical, biological, or economic).

Disturbance: A signal which tends to conversely affect the value of the output of a system (of course it is undesired signal).

Command input i/p: The motivating input signal to the system which is independent of the output of the system.

Reference i/p elements: An element which modifies the command i/p into suitable signal (called reference i/p) for the controlled system.

Reference input: It is almost the desired output**.**

Actuating signal**:** The difference between the reference input and feed back (f/b) signals. It actuates the control unit (controller) to maintain the output at the desired value.

Control unit: The unit which receives the actuating signal and delivers the control signal.

Controlled variable (actual o/p)*:* The variable which we need actually to control it.
Ex. temperature, pressure, liquid level, flow rate, etc.

Feedback signal: A signal representing a measure of the actual o/p which is fed back into control system for purpose of comparison with the reference signal.

Feedback element: Usually it represents a transducer, the purpose of which is to convert the o/p of the system in to a signal of suitable physical nature for the next stage in the system (error detector).

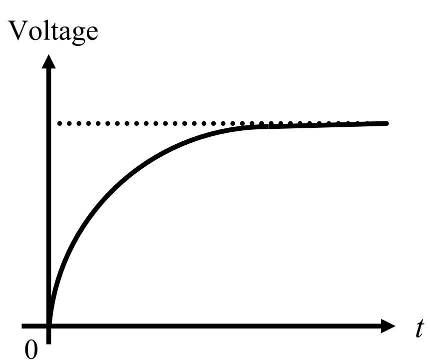
Feedback control: An operation which tends to reduce the difference between the o/p of the system and the reference i/p.

***Servomechanism control system*:** A feedback control system in which the o/p is mechanical variable (position, speed, acceleration).

Process control system: A feedback control system in which the o/p is a variable such as temperature, pressure liquid level.

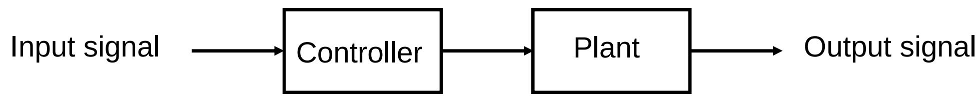
Automatic regulating system*:* A feedback control system in which the reference i/p (desired output) is either constant or slowly varying with time and the primary task is to maintain the o/p at the desired value in the presences of disturbance.

Close loop control system*:* A control system in which the o/p signal has a direct effect upon the control action.



Open loop control system: A control system in which the o/p signal has no effect upon the control action.

Ex. heater, light, washing machine



Open loop control system

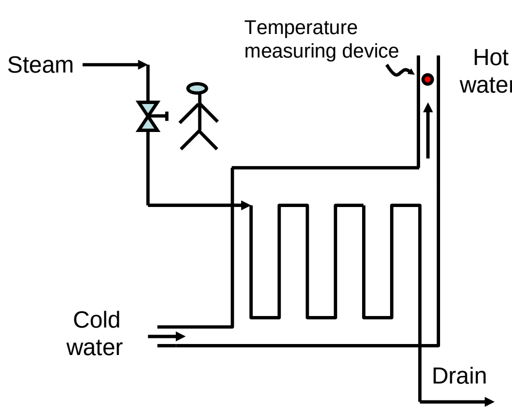
C/L control system versus O/L control system:

1) F/b control system is relatively insensitive to external disturbance and internal variation in system parameters. So we can use relatively inexpensive components with close loop control.

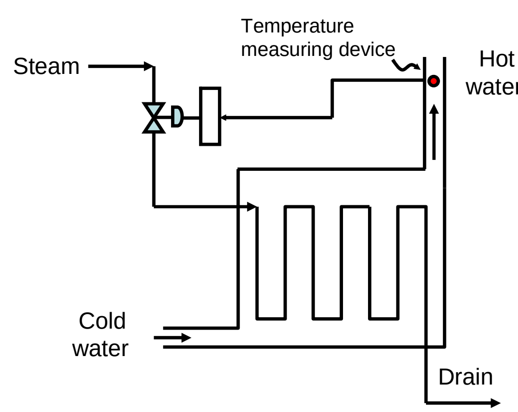
2) The required power of the system is less in O/L than in C/L control system.

Note: finally, which one to be used depends on the situation, sometimes we might use both of them in a certain way to get optimum case.

Manual and automatic feedback control:



Manual F/b control system
of a steam thermal system



Automatic F/b control system
of a steam thermal system

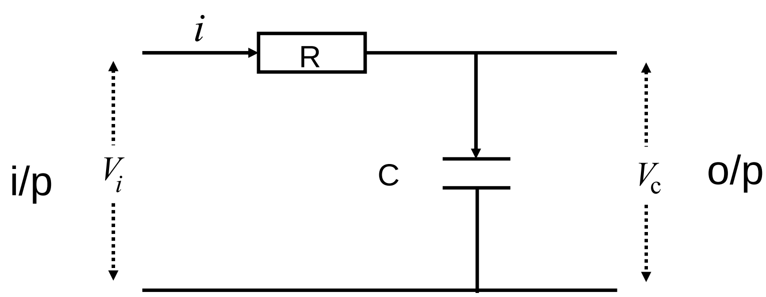
Classification of control system:

1. linear or nonlinear
2. C/L or O/L
3. Electrical. mechanical,…, etc
4. Continuous or discrete
5. Time variant or time invariant.

Mathematical Representation of Control System:

1. ***Electrical system:***

Ex(1).



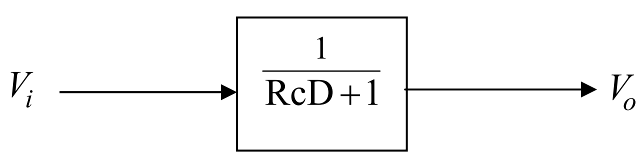
$$V_c = \frac{1}{c}\int i \; dt = \frac{1}{cD} i$$

$$i = \frac{V_i}{R + \frac{1}{cD}}$$

$$V_c = \frac{\frac{1}{cD}}{R + \frac{1}{cD}} V_i$$

$$\boxed{V_c = \frac{1}{RcD + 1} V_i}$$

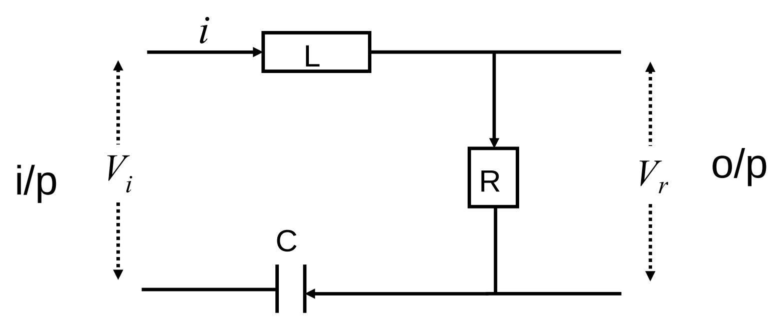
Differential equation



Transfer function (*T.f*): the T.f of a linear time invariant system is defined to be the relation of the laplace transform of the o/p (response function) to the laplace transform of the i/p (deriving force) under the assumption that all initial conditions are zero.

by L.T, $V_c = \frac{1}{RcD+1} V_i$ $\Longrightarrow$ $\boxed{\frac{V_c(s)}{V_i(s)} = \frac{1}{Rcs+1}}$

Ex(2).



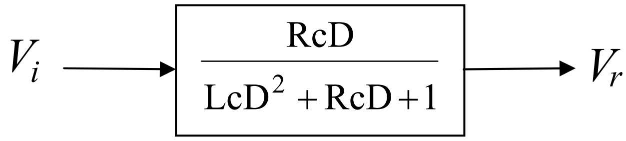
$$V_i = i\,(LD + R + \frac{1}{cD})$$

$$i = \frac{1}{LD + R + \frac{1}{cD}} V_i$$

$$V_r = i\,R$$

$$V_r = \frac{R}{LD + R + \frac{1}{cD}} V_i$$

$$V_r = \frac{RcD}{LcD^2 + RcD + 1} V_i$$



2. *Mechanical system:*

a) Translational mechanical system:

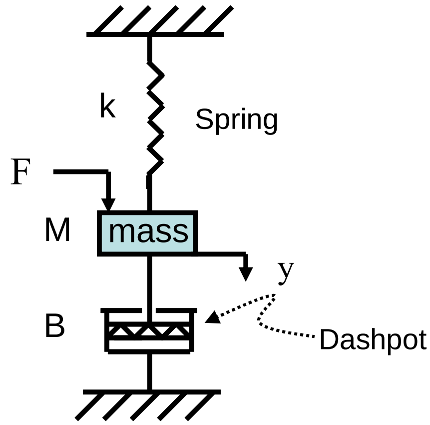
Ex(1).

$$\sum F = ma$$

$$F = ky + mD^2y + BDy$$

$$F = mD^2y + BDy + ky$$

$$\frac{y(s)}{F(s)} = \frac{1}{ms^2 + Bs + k}$$

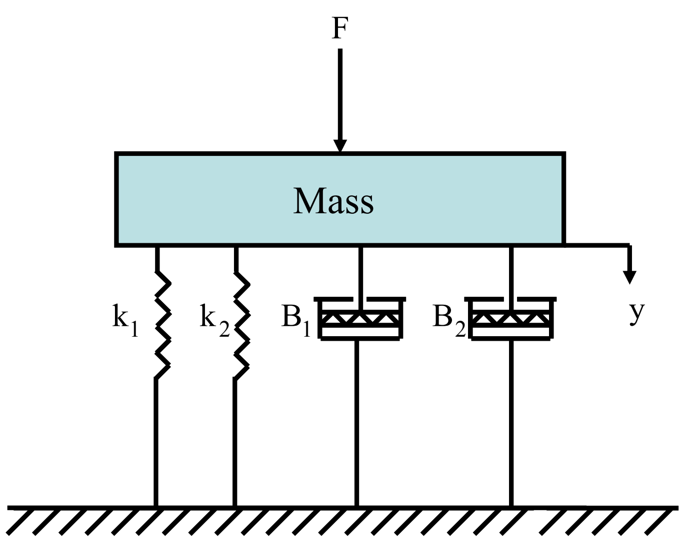


k: spring constant (stiffness coefficient)
B: viscous friction coefficient

Ex(2).

$$F = mD^2y + (k_2 + k_1)y + (B_1 + B_2)Dy$$

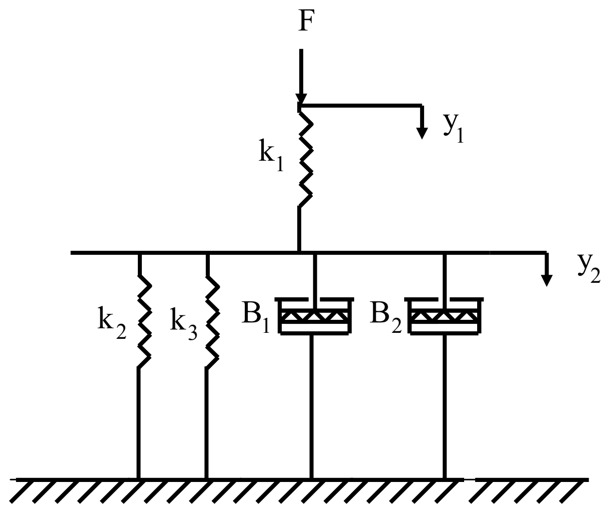
$$F = mD^2y + (B_1 + B_2)Dy + (k_2 + k_1)y$$



Ex(3).

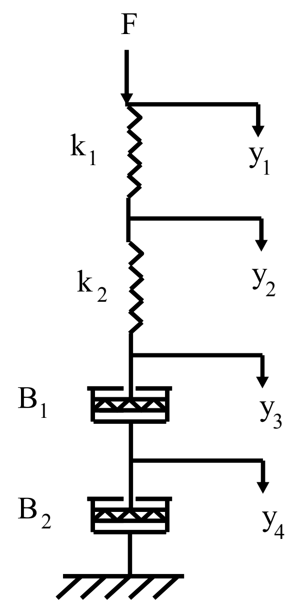
$$F = k_1(y_1 - y_2)$$

$$k_1(y_1 - y_2) = (k_2 + k_3)y_2 + (B_1 + B_2)Dy_2$$



Ex(4).

$$F = k_1(y_1 - y_2) = k_2(y_2 - y_3) = B_1D(y_3 - y_4) = B_2Dy_4$$



b) Rotational mechanical systems:

$J\alpha = \sum T$

J: Moment of inertia

α *: Rotational acceleration*

T: Torque

Ex(1).

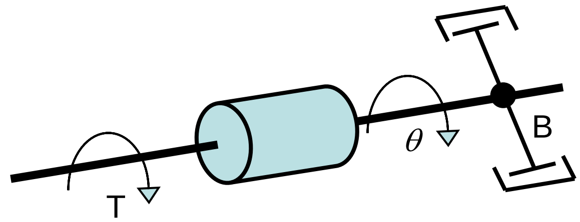
$$T = JD^2\theta + BD\theta$$

$$T = JD\omega + B\omega$$

where,

$$\omega = \dot{\theta} = D\theta$$

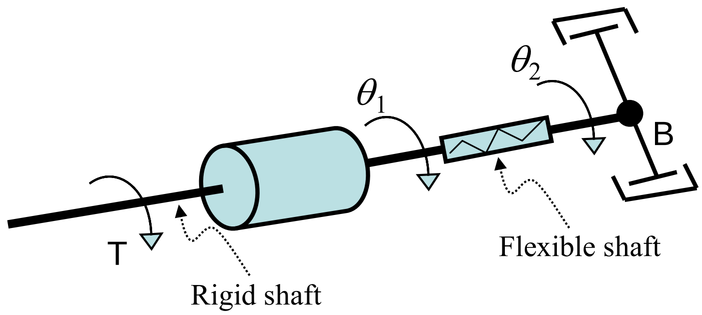
$$\alpha = \dot{\omega} = \ddot{\theta} = D^2\theta$$



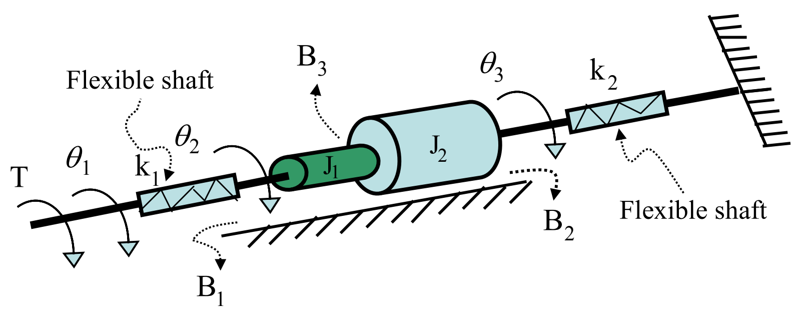
Ex(2).

$$T = JD^2\theta_1 + k(\theta_1 - \theta_2)$$

$$k(\theta_1 - \theta_2) = BD\theta_2$$



Ex(3).



$$T = k_1(\theta_1 - \theta_2)$$

$$k_1(\theta_1 - \theta_2) = J_1D^2\theta_2 + B_3D(\theta_2 - \theta_3) + B_1D\theta_2$$

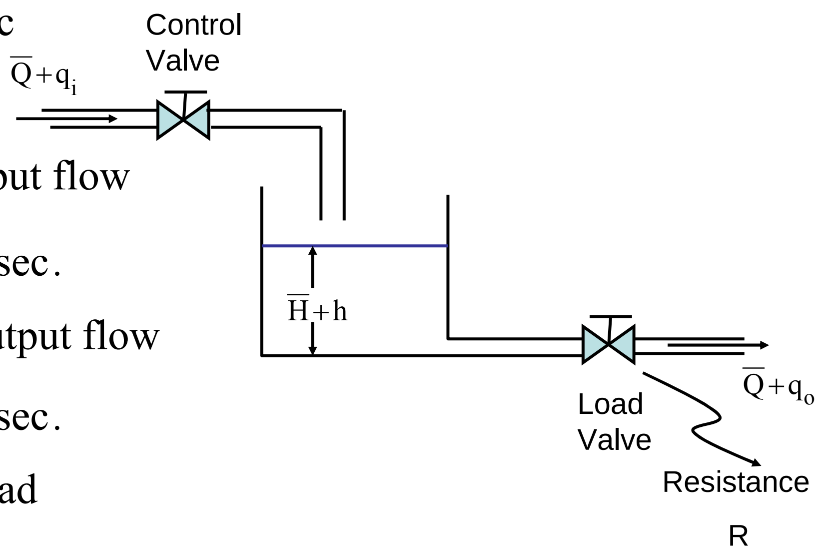
$$B_3D(\theta_2 - \theta_3) = J_2D^2\theta_3 + B_2D\theta_3 + k_2\theta_3$$

3. Liquid level systems:

Ex(1).

$\overline{Q}$**:** S.S liquid flow rate ft^3/sec

$\overline{H}$**:** S.S head.ft



q_i : Small deviation of the input flow rate from its S.S value ft^3/sec.

q_o : Small deviation of the output flow rate from its S.S value ft^3/sec.

h : Small deviation of the head from its S.S value ft.

c : *area*

$$q_i - q_o = c\frac{dh}{dt}$$

$$q_o = \frac{h}{R} \quad \text{(for laminar flow)}$$

$$q_o = k\sqrt{h} \quad \text{(for turbulent flow)}$$

$$q_i - \frac{h}{R} = c\frac{dh}{dt}$$

$$\boxed{Rc\frac{dh}{dt} + h = Rq_i}$$ Differential equation

By , laplace transform.

$$(Rcs+1)H(s) = RQ_i(s)$$

$$\boxed{\frac{H_o(s)}{Q_i(s)} = \frac{R}{Rcs+1}}$$

We can find that:-

$$\boxed{\frac{Q_o(s)}{Q_i(s)} = \frac{1}{Rcs+1}}$$

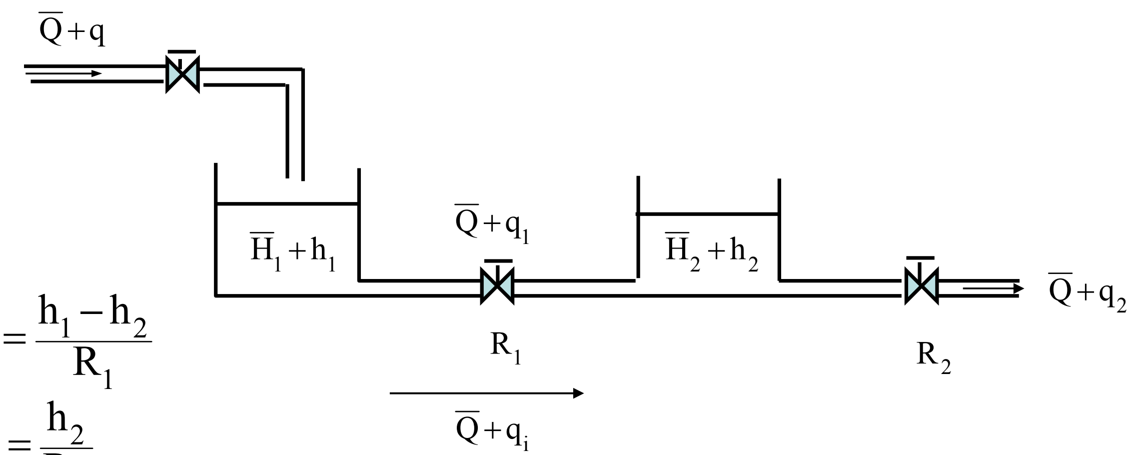
Transfer function in case when Q_o =o/p and Q_i = i/p

where, $q_o = \dfrac{h}{R}$

by L.T $Q_o(s) = \dfrac{H(s)}{R}$

$H(s) = RQ_o(s)$

Ex(2). Liquid level systems with interaction



$$q_1 = \frac{h_1 - h_2}{R_1}$$

$$q_2 = \frac{h_2}{R_2}$$

$$c_1 \frac{dh_1}{dt} = q - q_1$$

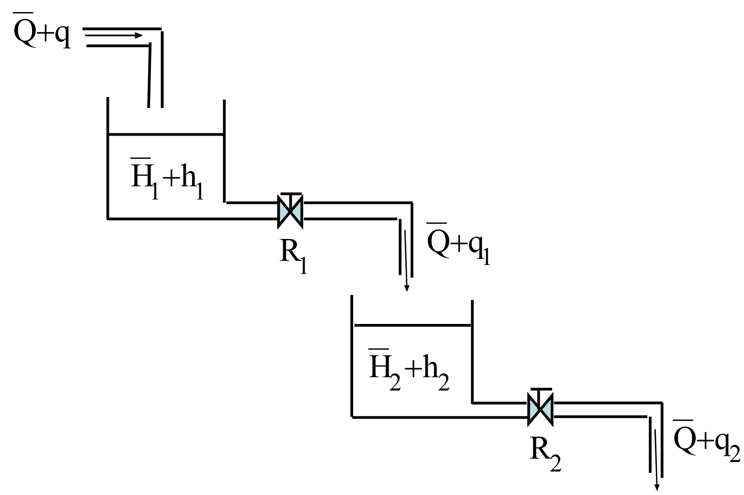
$$c_2 \frac{dh_2}{dt} = q_1 - q_2$$

$$\frac{Q_2(s)}{Q(s)} = \frac{1}{R_1c_1R_2c_2s^2 + s(R_1c_1 + R_2c_2 + R_2c_1) + 1}$$

H.W find the T.Fs ; $\dfrac{Q_1(s)}{Q(s)}, \dfrac{H_1(s)}{Q(s)}, \dfrac{H_2(s)}{Q(s)}$

Ex(3). Non interaction liquid level system:

$$q_1 = \frac{h_1}{R_1}$$



$$q_2 = \frac{h_2}{R_2}$$

$$q - q_1 = c_1 \frac{dh_1}{dt}$$

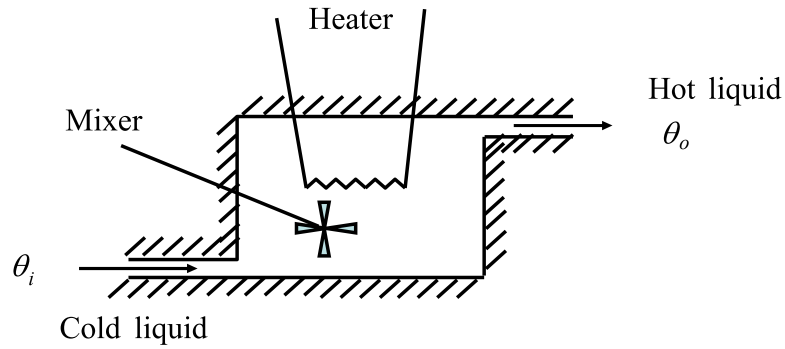
$$q_1 - q_2 = c_2 \frac{dh_2}{dt}$$

H.w. Find $\frac{Q_2(s)}{Q(s)}$,

Control Theory

Second Class **Control and System Engineering Department**
By M. J. Mohamed

4. Thermal system



θ_i : S.S Temperature of inflowing liquid, F^o
θ_o : S.S Temperature of outflowing liquid, F^o
G : S.S liquid flow rate lb/sec.
M : mass of liquid in tank , lb
c : specific heat of liquid , B tu/lb. F^o
R : thermal resistance , F^o sec/B tu.
Q : thermal capacitance, B tu/F^o.
$\overline{H}$: S.S heat i/p rate , B tu/sec.

Consider that heat input rate changes from $\overline{H}$ to $\overline{H}+h_i$ then heat outflow will change from $\overline{H}$ to $\overline{H}+h_o$ also the temperature of the outfollwing liquid will change from $\overline{\theta}_o$ to $\overline{\theta}_o+\theta$.
Considering change only:

$$h_i - h_o = Q\frac{d\theta}{dt} \qquad , \quad \theta = h.R$$

or, $$RQ\frac{d\theta}{dt} + \theta = Rh_i$$

Note: $h_o = G.c.\theta$, $G.c = \frac{1}{R}$, Q=M.c

By L.T : $\dfrac{\theta(s)}{H_i(s)} = \dfrac{R}{RQs+1}$ ⟹ This is the T.f between changes in h and θ

If we consider that the driving function (i.e) i/p was a change in θ_i then:-

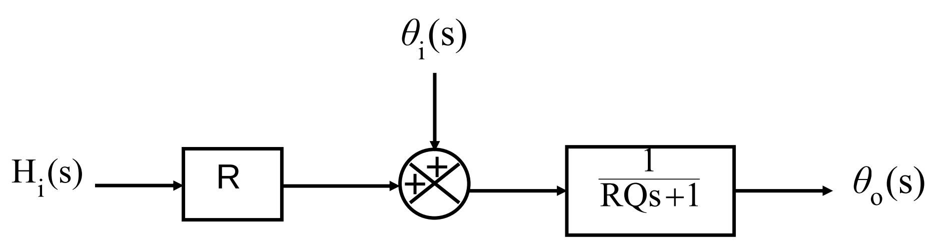
$$Q\frac{d\theta}{dt} = G.c.\theta_i - h_o$$

$$Q\frac{d\theta}{dt} = \frac{1}{R}.\theta_i - \frac{\theta}{R}$$

$$RQ\frac{d\theta}{dt} + \theta = \theta_i \quad \Longrightarrow \quad \frac{\theta(s)}{\theta_i(s)} = \frac{1}{RQs+1}$$

In case of changes in both h_i and θ_i then we have:-

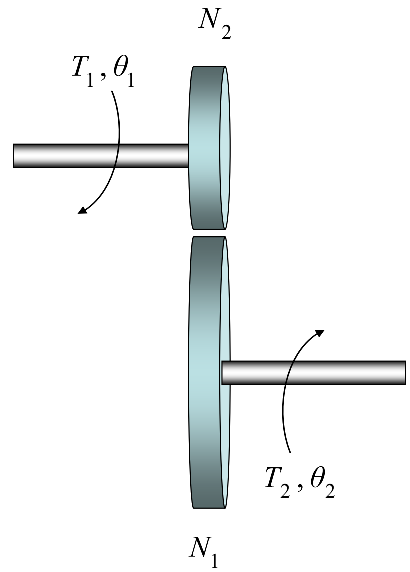
$$R.c\frac{d\theta}{dt} + \theta = \theta_i + Rh_i$$



5) ***Gear Trains:***

A gear train is a mechanical device that transmits energy from one part of a system to another in such a way that force, torque, speed, and displacement are altered. Two gears are shown coupled together in following figure. The inertia and friction of the gears are neglected in the ideal case considered.

The relationships between the torque T_1 and T_2 , angular displacements θ_1 and θ_2, and the teeth numbers N_1 and N_2 , of the gear train are derived from the following facts.



1- The number of teeth on the surface of the gear is proportional to the radius r_1 and r_2 of the gears, that is.

$$r_1 N_2 = r_2 N_1$$

2- The distance traveled along the surface of each gear is the same. Therefore,

$$\theta_1 . r_1 = \theta_2 . r_2$$

3- The work done by one gear is equal to that of the other since there is assumed to be no loss, thus

$$T_1 \theta_1 = T_2 \theta_2$$

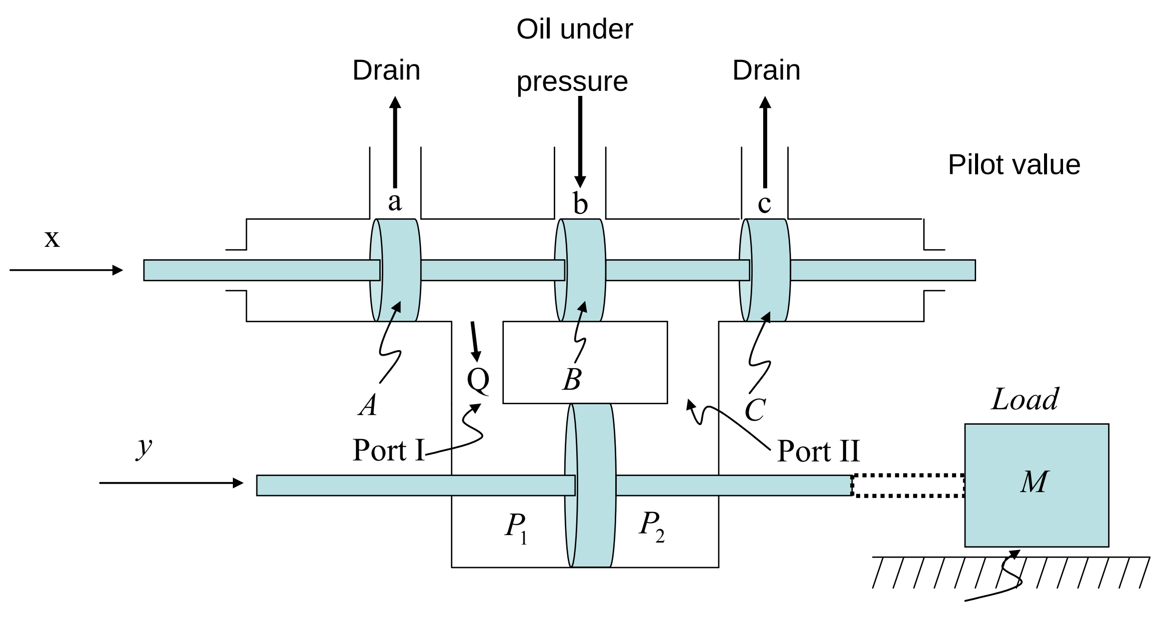
If the angular velocities of the two gears are ω_1 and ω_2.

$$\frac{T_1}{T_2} = \frac{\theta_2}{\theta_1} = \frac{N_1}{N_2} = \frac{\omega_2}{\omega_1} = \frac{r_1}{r_2}$$

6) *Hydraulic servomotor*.

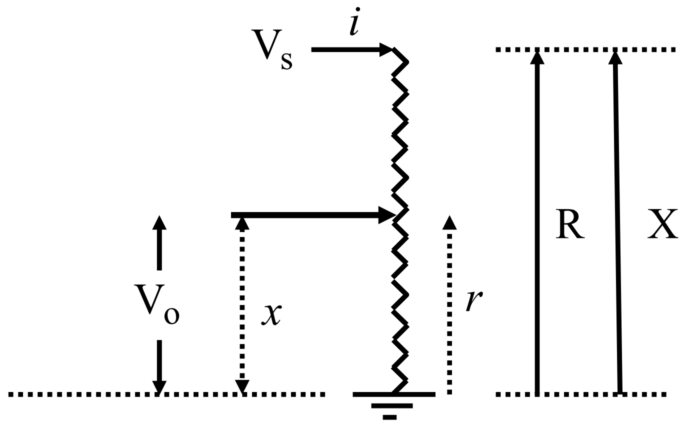
The following figure shows the hydraulic servomotor. It is essentially a pilot valve controlled hydraulic power amplifier and actuator. The pilot valve is a balanced valve, in the sense that the pressure forces acting on it are all balanced. A very large power output can be controlled by a pilot valve, which can be positioned with very little power.

The operation of this hydraulic servomotor is as follows: if the pilot valve is moved to the right, then port I is connected to the supply port, and the pressured oil enters the left hand side of the power piston. Since port II is connected to the drain port, the oil in the right hand side of the power piston is returned to the drain. The oil flowing into the power cylinder is at high pressure, and the oil flowing out from the power cylinder into the drain is at low pressure. The resulting difference in pressure on both sides of the power piston will cause it to move to the right. The returned oil is pressurized by a pump and is recirculated in the system. When the pilot piston is moved to the left, the power piston will move to the left.



Control system components (Transducer and error detectors)

1) ***Potentionmeter (transducer).***



Consider linear resistance

$x = \mathrm{k}r$

$\mathrm{X} = \mathrm{kR}$

$i = \dfrac{V_s}{R}$..................................(1)

$V_o = i \, . \, r$ ....(2)

From (1) and (2):

$V_o = \dfrac{V_s}{R} . \, r$ ....(3)

$V_o = \dfrac{V_s}{\frac{X}{k}} . \dfrac{x}{k} = \dfrac{V_s}{X} . x$

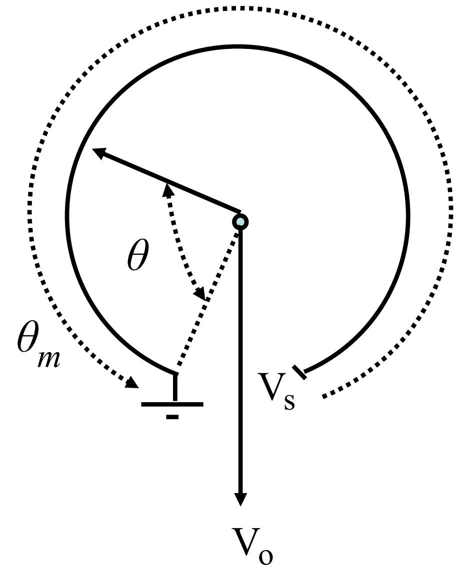
If , $K_p = \dfrac{V_s}{X}$

$\therefore \; V_o = K_p . x$

2) ***Rotational pot.***

$$V_o = \frac{V_s}{\theta_m} . \ \theta$$

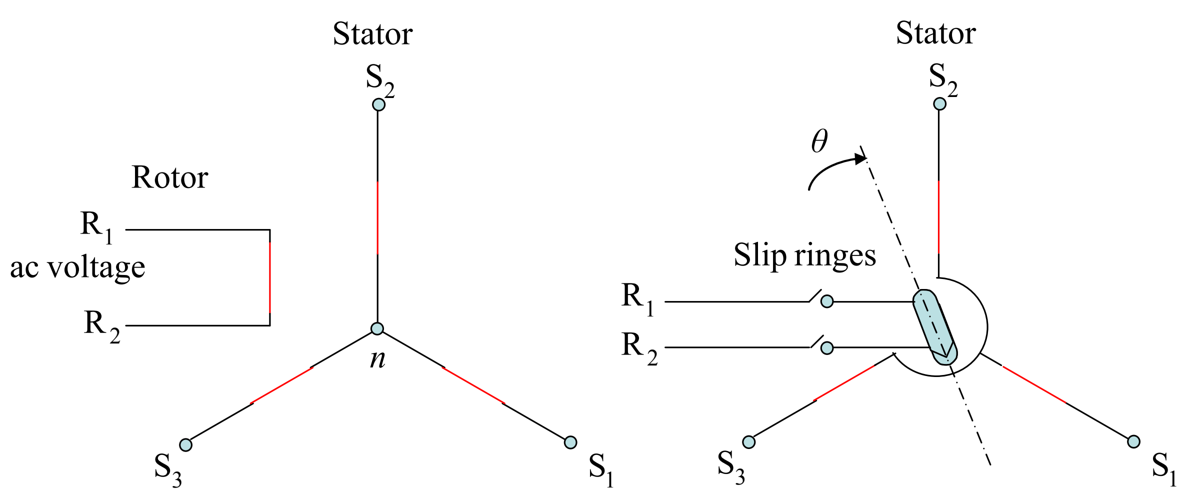
$$\therefore \ V_o = K_p . \theta$$



3) ***Synchro***

Synchos are electromechanical device used for position transducer application in ac control system. Synchros are used widely in control systems as detectors and encoders because of their ruggedness in construction and high reliability. Basically, a synchro is a rotary device that operates on the same principle as a transformer and produces a correlation between an angular position and a voltage or set of voltages.

Synchro Transmitter: A synchro transmitter has a Y-connected stator winding which resembles the stator of a three-phase induction motor. The rotor is dumbellshaped magnet with a single winding the schematic diagram of a synchro transmitter is shown in figure below. A single phase ac voltage is applied to the rotor through two slip rings. The symbol G is often used to design a synchro transmitter.



Let the ac voltage applied to the rotor of a synchro transmitter be
$e_r = E_r \sin(w_c .t)$
When the rotor is at the position of $\theta = 0$ with reference to figure, which is defined as the electrical zero, the voltage induced a cross the stator winding between S_2 and the neutral n is maximum and is written,
$e_{S2n}(t) = K.E_r .\sin(w.t)$
where, K is proportional constant. The voltages a cross the terminals S_{1n} and S_{3n}

$$e_{S1n}(t) = K.E_r \cos(240^o)\sin(w.t) = -0.5.K.E_r.\sin(w.t)$$
$$e_{S3n}(t) = K.E_r \cos(120^o)\sin(w.t) = -0.5.K.E_r.\sin(w.t)$$

Then the terminal voltages of the stator are,

$$e_{S1S2} = e_{S1n} - e_{S2n} = -1.5.K.E_r.\sin(w.t)$$
$$e_{S2S3} = e_{S2n} - e_{S3n} = 1.5.K.E_r.\sin(w.t)$$
$$e_{S3S1} = e_{S3n} - e_{S1n} = 0$$

The forgoing equations show that, despite the similarity between the construction of the stator of a synchro and that of a three phase machine, there are only single phase voltages induced in the stator.

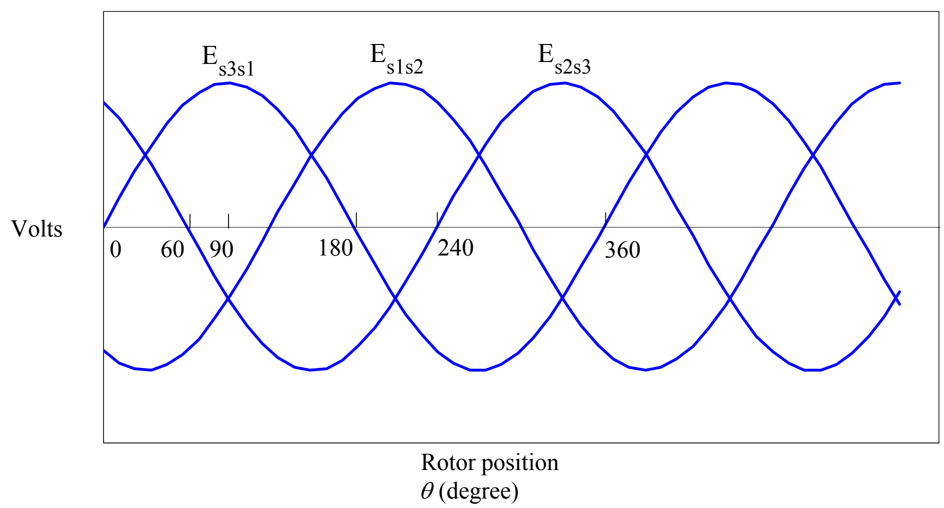
Consider now that the rotor of the synchro transmitter is at an angle of θ with reference to electrical zero, as shown in above figure. The voltages in each stator winding will be vary as a function of the cosine of the rotor displacement θ ; that is, the voltage magnitude are,

$$E_{S1n} = K.E_r.\cos(\theta - 240^o)$$
$$E_{S2n} = K.E_r.\cos(\theta)$$
$$E_{S3n} = K.E_r.\sin(\theta - 120^o)$$

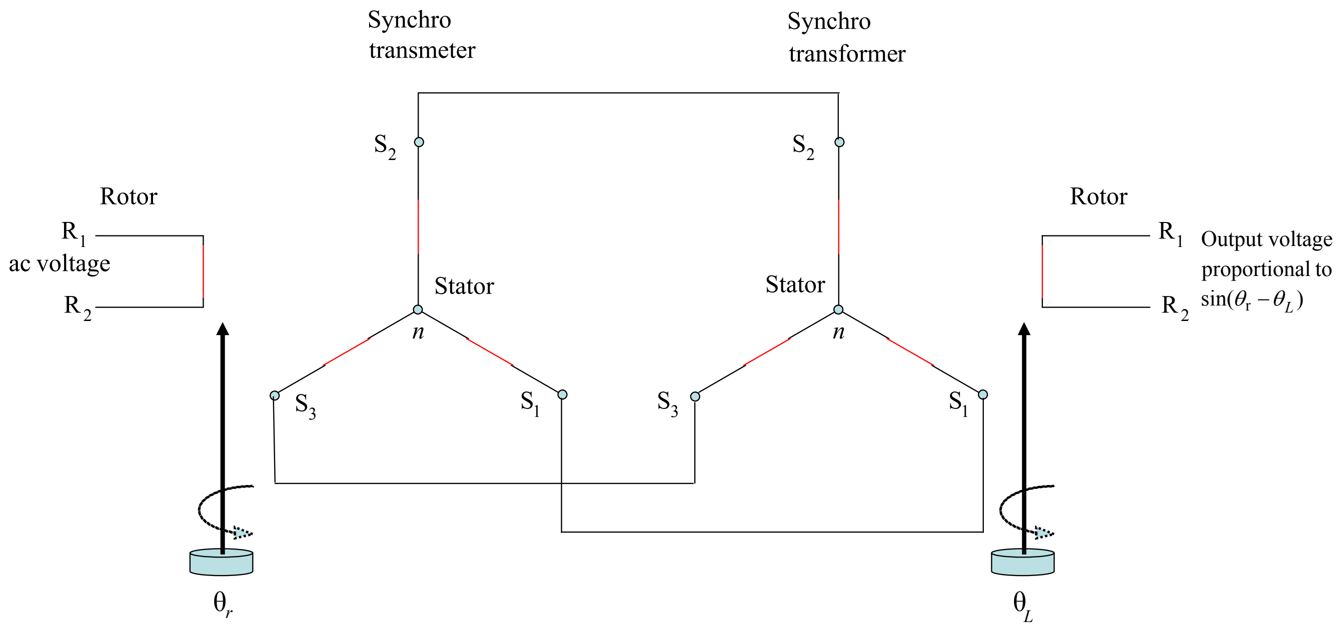
The magnitudes of the stator terminal voltages become

$$E_{S1S2} = E_{S1n} - E_{S2n} = \sqrt{3}.K.E_r.\sin(\theta + 240^o)$$
$$E_{S2S3} = E_{S1n} - E_{S3n} = \sqrt{3}.K.E_r.\sin(\theta + 120^o)$$
$$E_{S3S1} = E_{S3n} - E_{S1n} = \sqrt{3}.K.E_r.\sin(\theta)$$

A plot of these terminal voltages as a function of the rotor shaft position is shown in the following figure. Notice that each rotor position corresponds to one unique set of stator voltages. This leads to the use of the synchro transmitter to identify angular positions by measuring and identifying the sets of voltages at the three stator terminals.



Synchro control Transformer: since the function of an error detector is to convert the difference of two shaft positions into an electrical signal, a signal synchro transmitter is apparently inadequate. A typical arrangement of a synchro error detector involves the use of two synchros: a transmitter and a control transformer, as shown in the following figure.

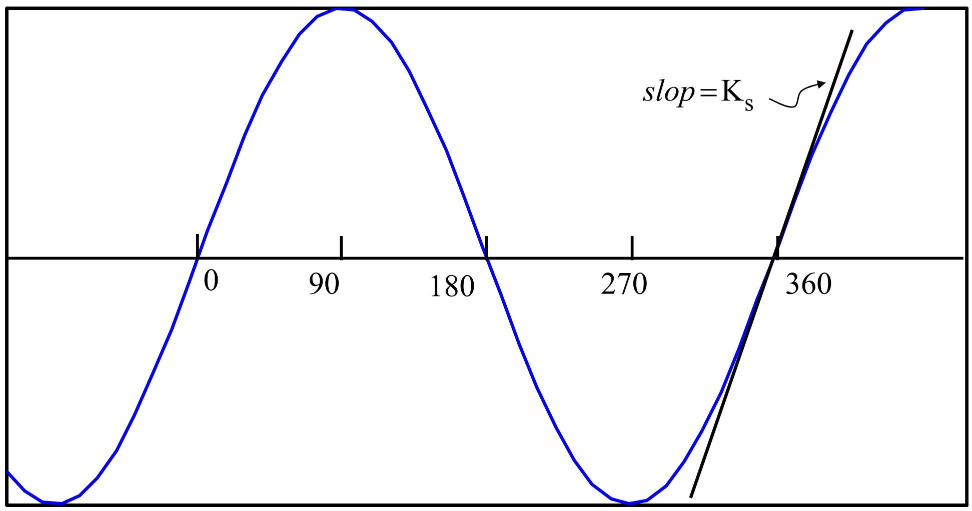


Basically, the principle of operation of a synchro control transformer is identical to that of the synchro transmitter, except that the rotor is cylindrically shaped so that the air cap flux is uniformly distributed around the rotor. This feature is essential for a control transformer, since its rotor terminals are usually connected to an amplifier or similar electrical device, in order that the latter sees constant impedance. The change in the rotor impedance with rotations of the shaft position should be minimized. The symbol CT is often used to designate a synchro control transformer.

Referring to the arrangement shown in figure , the voltage given by Eqs

$$E_{S1S2} = E_{S1n} - E_{S2n} = \sqrt{3}KE_r \sin(\theta + 240^o)$$
$$E_{S2S3} = E_{S1n} - E_{S3n} = \sqrt{3}KE_r \sin(\theta + 120^o)$$
$$E_{S3S1} = E_{S3n} - E_{S1n} = \sqrt{3}KE_r \sin(\theta)$$

are now impressed across the corresponding stator terminals of the control transformer. When the rotor positions of two synchros are in perfect alignment, the voltage generated across the terminals of the rotor windings is zero. When the two rotor shafts are not in alignment, the rotor voltage of the CT is approximately a sine function of the difference between the two shaft angles, as shown in figure.



Form figure above it is apparent that the synchro error detector is a nonlinear device. However, for small angular deviations of up to 15^o degree, the rotor voltage of the CT is approximately proportional to the difference between the positions of the rotors of the transmitter and the control transformer. Therefore for small deviation, the transfer function of the synchro error detector can be approximated by a constant K_s.

$$K_s = \frac{E}{\theta_r - \theta_L} = \frac{E}{\theta_e}$$

Where, E = error voltage.

θ_r = shaft position of synchro transmitter, degrees.

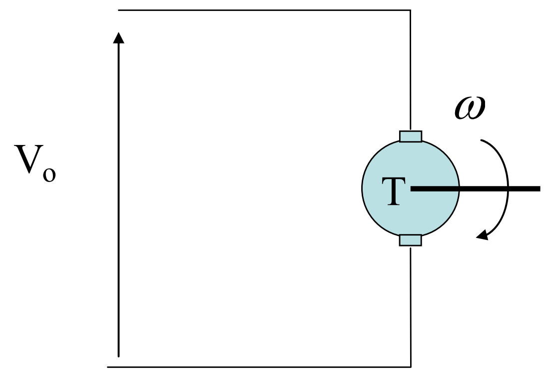
θ_L = shaft position of synchro control transformer, degree.

θ_e = error in shaft positions.

K_s = sensitivity of the error detector, volt per degree.

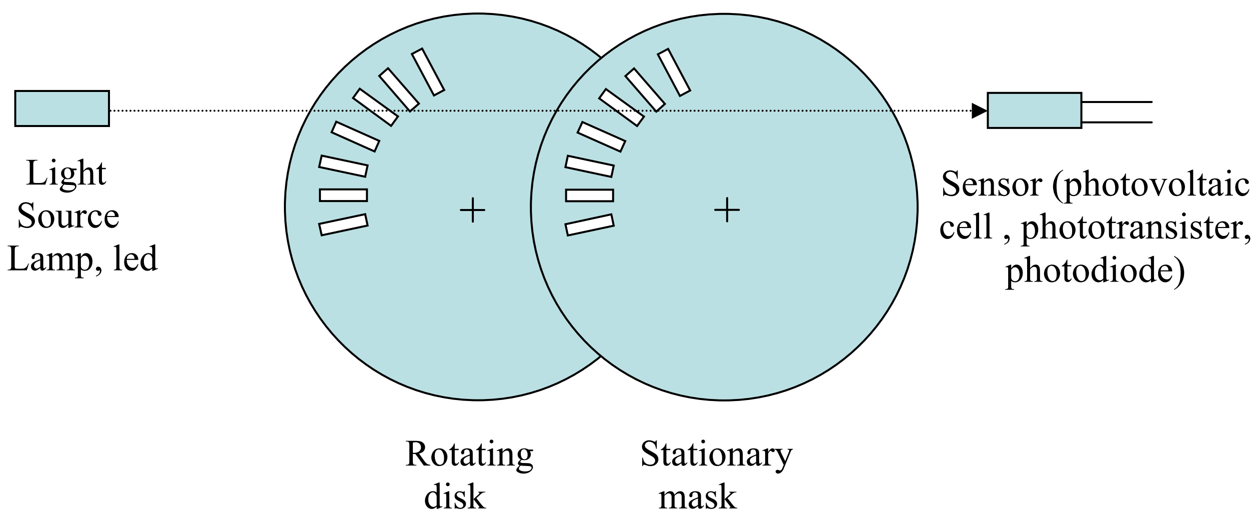
4) ***Tachometers***

Tachometers are electromechanical device that convert mechanical energy into electrical energy. The device works essentially as a generator with the output voltages proportional to the magnitude of the angular velocity.

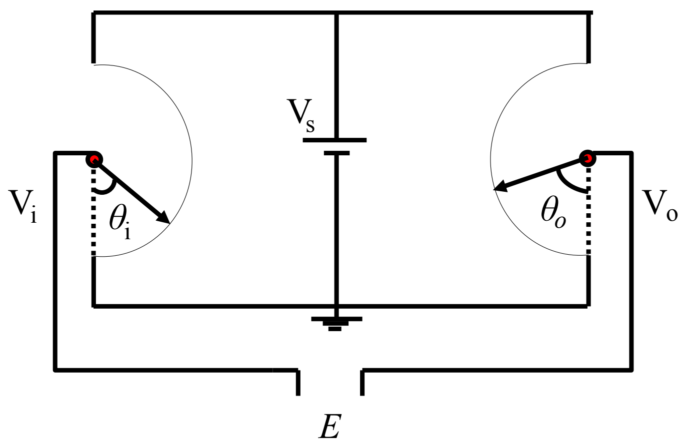


5) ***Incremental encoder:***

One type of encoder that is frequently found in modern control systems converts linear or rotary displacements into digitally coded or pulse signals. The encoders that output a digital signal are known as the absolute encoders.

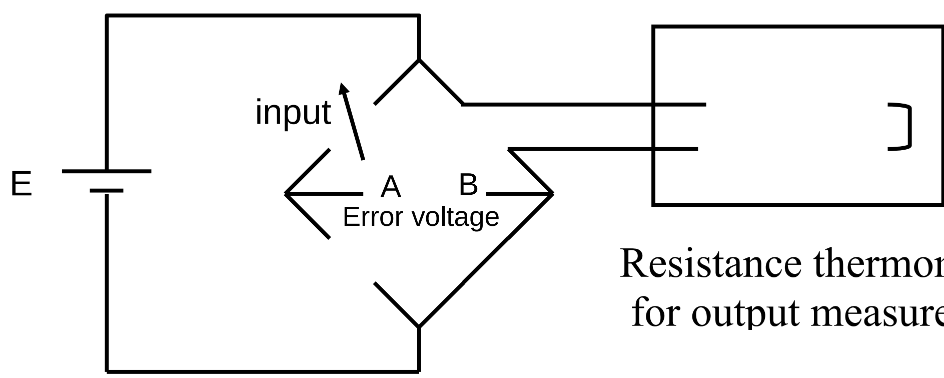


error detector



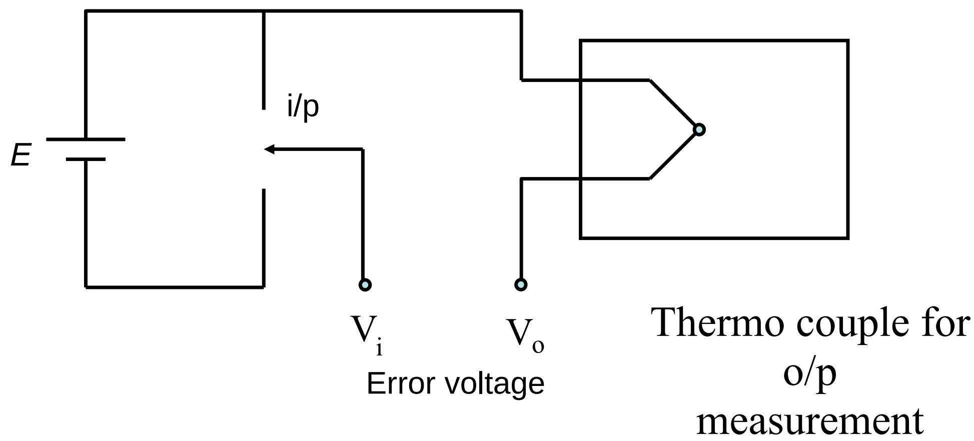
$$V_i = k_1 . \theta_1$$
$$V_o = k_2 . \theta_o$$
$$E = k_1 . \theta_i - k_2 . \theta_o$$
$$\text{If } \; k = k_1 = k_2$$
$$E = k\,(\theta_i - \theta_o)$$

7) ***Resistance thermometer bridge error detector***

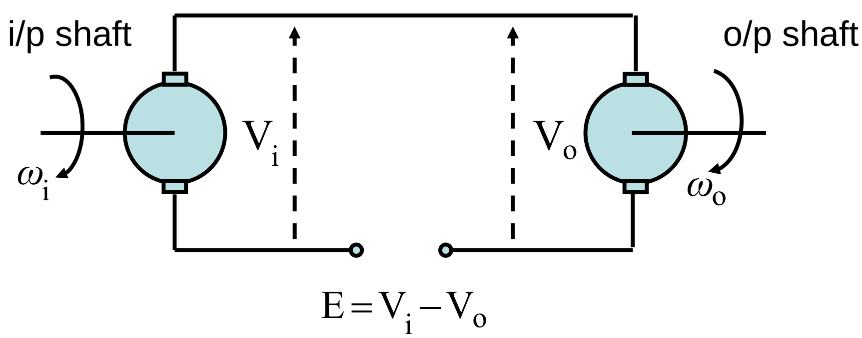


Resistance thermometer
for output measurement

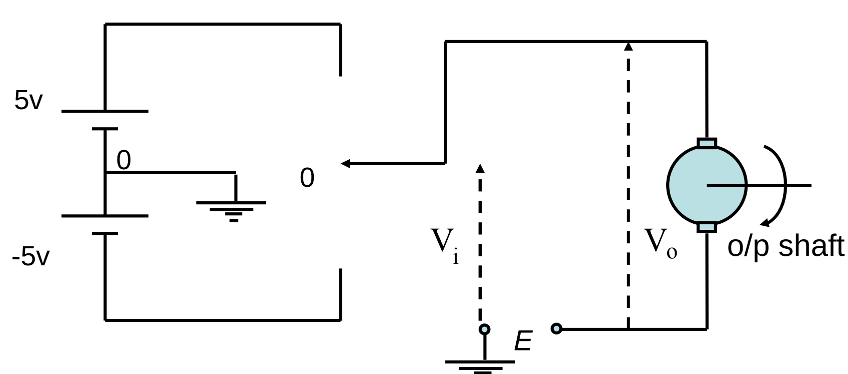
8) ***Thermo couple bridge error detector***



9) ***Tachogenerator bridge error detector***



10) ***Potentionmeter tachogenerator bridge error detector***



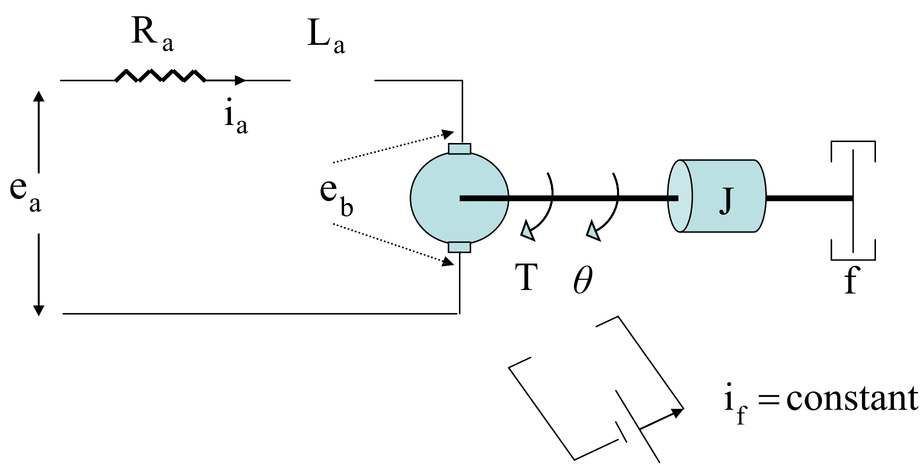
Control Theory

Second Class **Control and System Engineering Department**
By M. J. Mohamed

Output elements:

Ex. (electrical example) D.C servo motor

a) Armature control .



R_a : Armature winding resistance.
L_a : Armature winding inductance.
i_a : Armature winding current.
i_f : Field current.
e_a : applied armature voltage.
e_b : back emf.
θ : Angular displacement of the motor shaft.
T : Torque delivered by the motor.
J : equivalent moment of inertia of the motor and load referred to the motor shaft.
f : equivalent viscous friction coefficient of the motor and load referred to the motor shaft.

$$T \propto \psi_1 \propto \psi_2$$

$$T = k\,\psi_1\,\psi_2$$

$$T = k\,(k_a i_a)(k_f i_f)$$

$$T = K\,i_a \Longrightarrow K = k\,k_a\,k_f\,i_f$$

$$e_b = k_b \frac{d\theta}{dt} \Longrightarrow \dot{\theta} = \omega$$

$$e_a = L_a \frac{di_a}{dt} + R_a i_a + e_b$$

$$T = J \frac{d^2\theta}{dt^2} + f \frac{d\theta}{dt}$$

$$K\,i_a = T = J \frac{d^2\theta}{dt^2} + f \frac{d\theta}{dt}$$

$$e_a\,(i/p) \Longrightarrow \theta\,(o/p)$$

$$e_a = L_a D i_a + R_a i_a + e_b$$

$$e_a - e_b = i_a (L_a D + R_a)$$

$$i_a = \frac{e_a - e_b}{L_a D + R_a} = \frac{e_a - (k_b D\theta)}{L_a D + R_a}$$

$$\frac{K e_a - K(k_b D\theta)}{L_a D + R_a} = J\,D^2\theta + f\,D\theta$$

By L.T and arrangement

$$\frac{\theta(s)}{E_a(s)} = \frac{K}{s(L_a J s^2 + (L_a f + R_a J)s + R_a f + K k_b)}$$

If we consider L_a is small enough to be neglected.

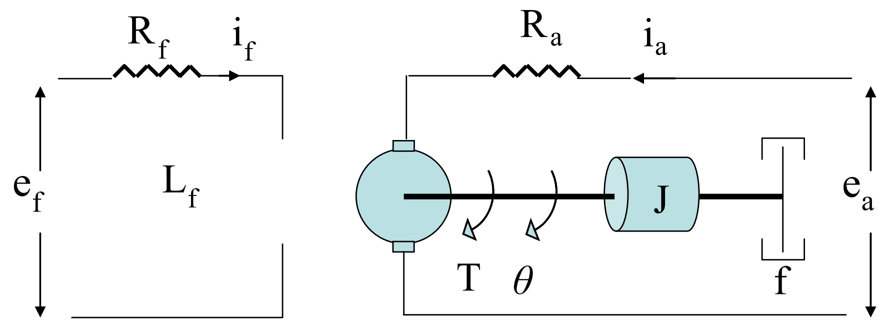
$$\therefore \frac{\theta(s)}{E_a(s)} = \frac{k_m}{s(T_m s + 1)}$$

where,

$$k_m = \frac{K}{(R_a f + K\,k_b)} \qquad \text{(motor gain constant)}$$

$$T_m = \frac{R_a J}{(R_a f + K\,k_b)} \qquad \text{(motor time constant)}$$

b) Field control.



$$T \propto \psi_1 \propto \psi_2$$

$$T = k\,\psi_1\,\psi_2$$

$$T = k\,(k_a i_a)(k_f i_f)$$

$$T = K\,i_f \quad \Longrightarrow \quad K = k\,k_a\,k_f\,i_a$$

$$L_f \frac{di_f}{dt} + R_f i_f = e_f$$

$$J\frac{d^2\theta}{dt^2} + f\frac{d\theta}{dt} = T = Ki_f$$

$$(L_f s + R_f)I_f(s) = E_f(s)$$

$$(Js^2 + fs)\theta(s) = KI_f(s)$$

$$\frac{\theta(s)}{E_f(s)} = \frac{K}{s(L_f s + R_f)(Js + f)} = \frac{K_m}{s(T_f s + 1)(T_m s + 1)}$$

where, $K_m = \dfrac{K}{R_f f}$ motor time constant

$T_f = \dfrac{L_f}{R_f}$ time constant of field cct.

$T_m = \dfrac{J}{f}$ time constant of inertia-friction element

Block Diagrams and Signal Flow Graphs:

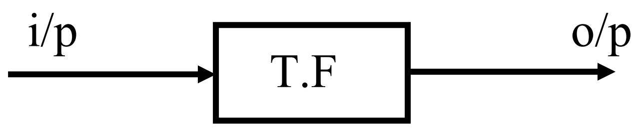
In general it is extremely difficult to handle, analysis, or find the mathematical representation of composite and complicated systems. That is why we need some tools to simplify these requirements. Among these tools are the block diagrams and signal flow graphs.

Block diagram:

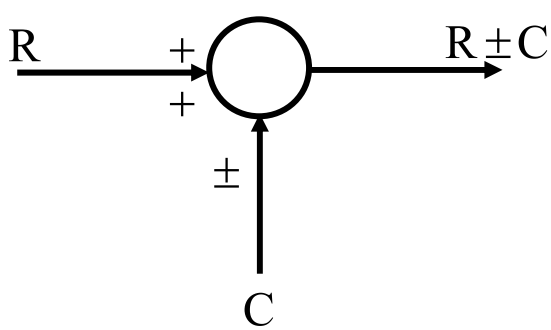
Beside what is mentioned above, block diagrams illustrate the operation and interrelationships of different system components since the block diagram gives the relationship between the i/p and o/p of the component.

Symbols used in block diagrams(B.D):

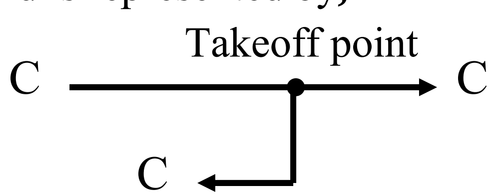
1) Block: The T.f of the system element is placed in the block symbolized by,



2) Summing points: The operation of addition or subtraction is performed by this system element and is symbolized by.

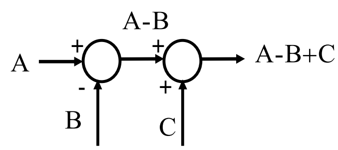
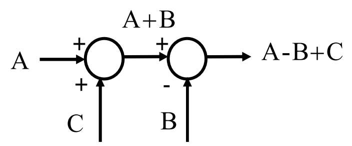
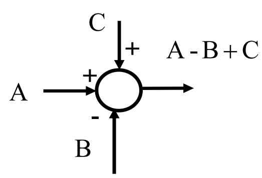
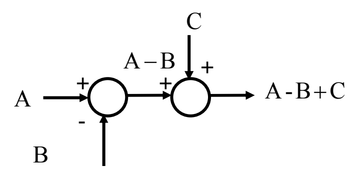
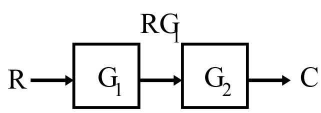
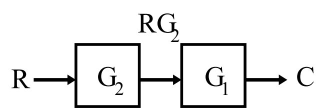
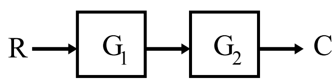
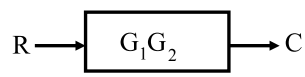
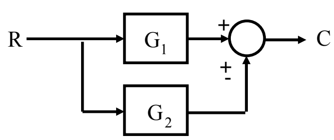
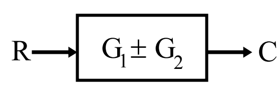
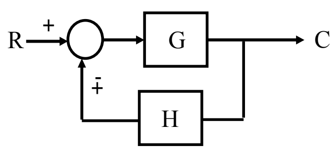
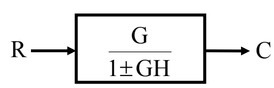
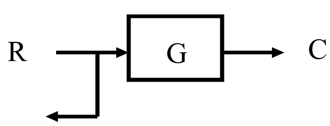
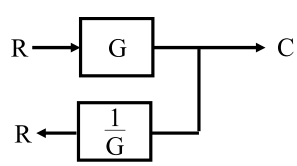


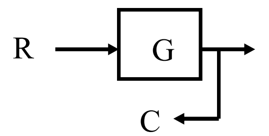
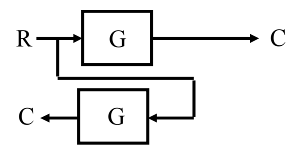
3) Take off point: This operation is used to provide a dual i/p or o/p to a system element and is represented by**,**

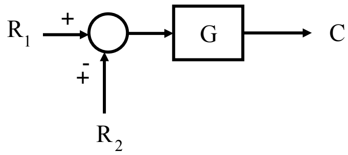
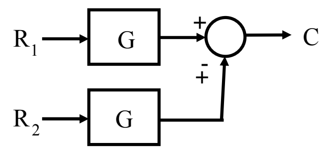
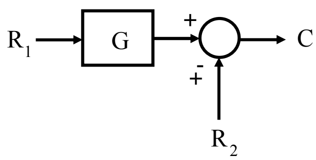
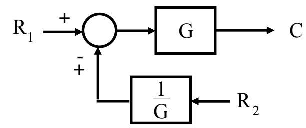
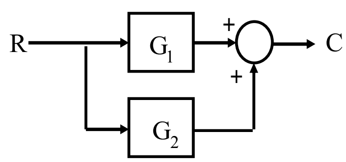
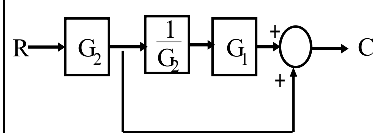
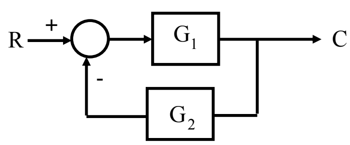
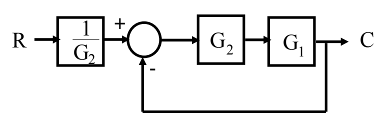


4) Directional arrows: This symbol defines unidirectional flow of the signal.

Rules of B.D

Transformation	B.D	Equivalent B.D	Equation(T.f)
Summing operation			....
Summing operation			....
Swapping cascaded blocks			$\frac{C}{R} = G_1G_2$
Cascaded block			$\frac{C}{R} = G_1G_2$
Eliminating a forward loop			$\frac{C}{R} = G_1 \pm G_2$
Eliminating a f/b loop			$\frac{C}{R} = \frac{G}{1 \pm GH}$
Moving pickoff point beyond a block			$\frac{C}{R} = G$

Moving pickoff point a head a block			$\frac{C}{R}=G$

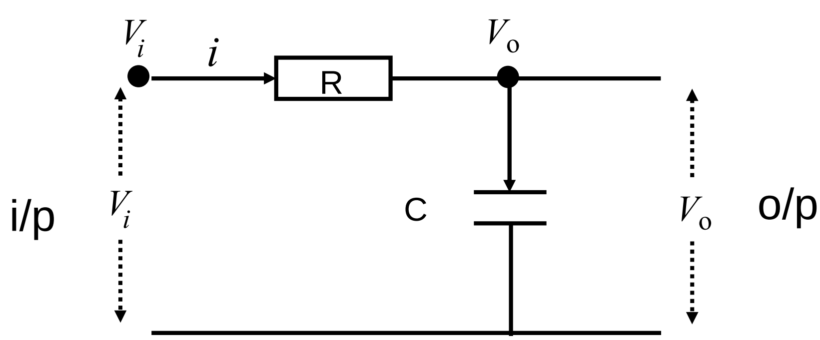
Moving summing point beyond a block			$\frac{C}{R_1 \mp R_2}=G$
Moving summing point a head of a block			$C=R_1G\mp R_2$
Moving block To the forward path			$C=R(G_1+G_2)$
Moving f/b block to forward path			$\frac{C}{R}=\frac{G_1}{1+G_1G_2}$

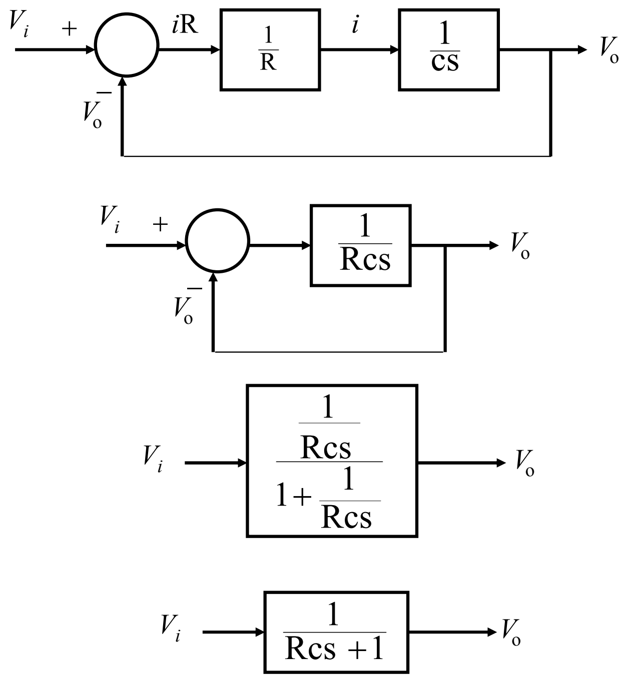
Constructing B.Ds of systems:

Ex- Construct B.D of the system shown below.

$V_i - V_o = iR$ ....(1)

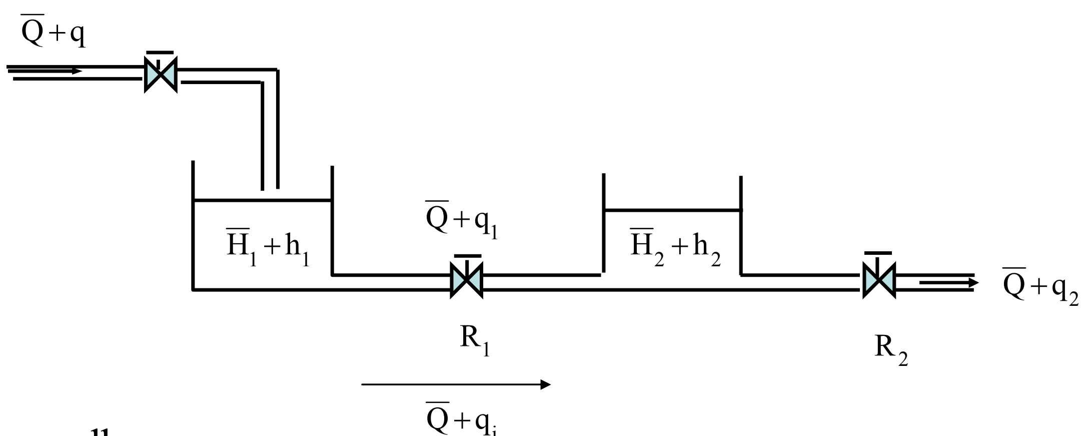
$V_o = \frac{1}{sc} i$ ....(2)





$$\frac{V_o}{V_i} = \frac{1}{Rcs+1}$$

Ex. Construct the B.D for the system shown below.(q, q_1, q_2, h_1, h_2 are changes from steady state).



$$q - q_1 = c_1 \frac{dh_1}{dt}$$

$$q_1 = \frac{h_1 - h_2}{R_1}$$

$$q_1 - q_2 = c_2 \frac{dh_2}{dt}$$

$$q_2 = \frac{h_2}{R_2}$$

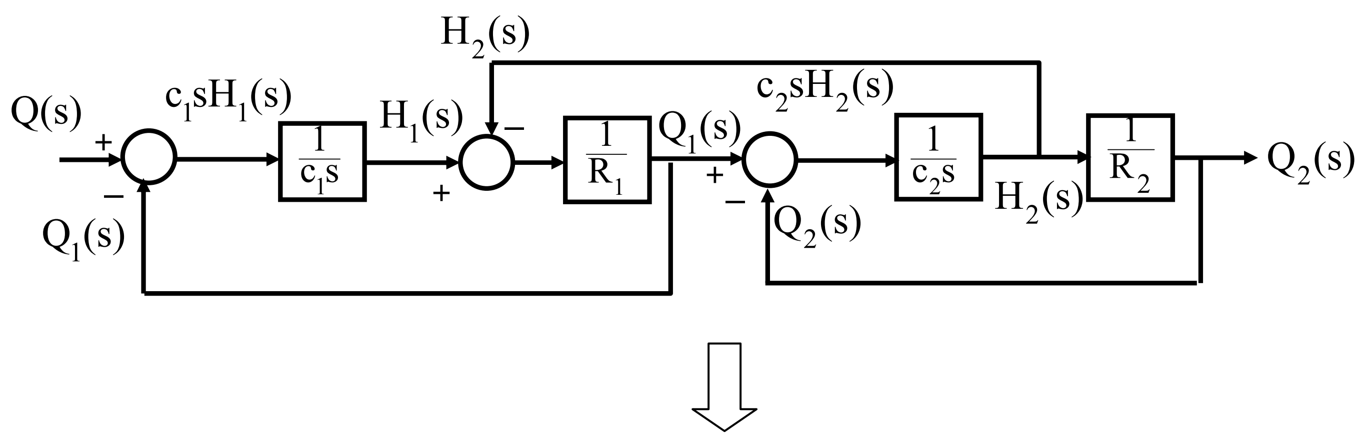
....

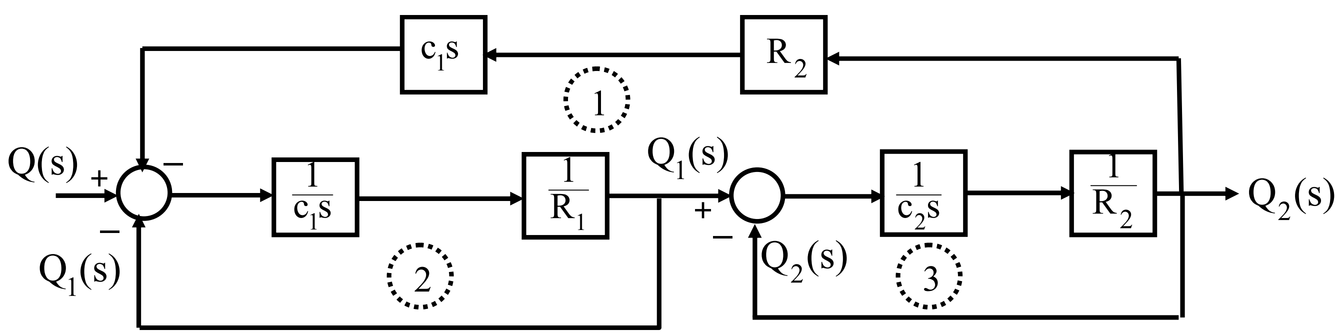
$$Q(s) - Q_1(s) = c_1 s H_1(s)$$

$$Q_1(s) = \frac{H_1(s) - H_2(s)}{R_1}$$

$$Q_1(s) - Q_2(s) = c_2 s H_2(s)$$

$$Q_2(s) = \frac{H_2(s)}{R_2}$$



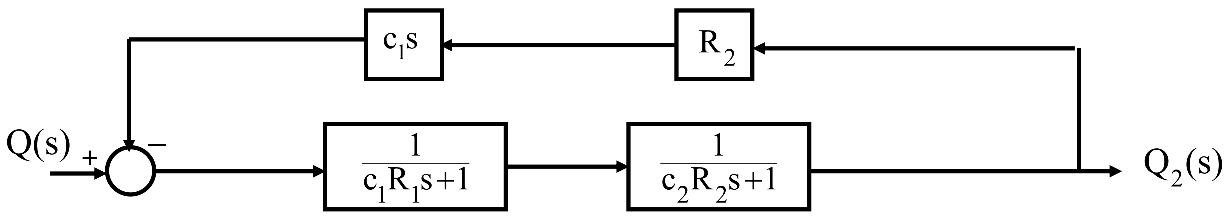


loop(3)

$$\frac{Q_2(s)}{Q_1(s)} = \frac{\frac{1}{c_2 s}\frac{1}{R_2}}{1+\frac{1}{c_2 s}\frac{1}{R_2}} = \frac{1}{c_2 R_2 s + 1}$$

loop(2)

$$\frac{Q(s)}{Q_1(s)}=\frac{\frac{1}{c_1 s}\frac{1}{R_1}}{1+\frac{1}{c_1 s}\frac{1}{R_1}}=\frac{1}{c_1 R_1 s+1}$$



loop(1)

$$\frac{Q_2(s)}{Q_1(s)}=\frac{\frac{1}{(c_1R_1s+1)}\frac{1}{(c_2R_2s+1)}}{1+\frac{1}{(c_1R_1s+1)}\frac{1}{(c_2R_2s+1)}c_1R_2s}$$

$$\frac{Q_2(s)}{Q_1(s)}=\frac{1}{c_1c_2R_1R_2s^2+(c_1R_1+c_2R_2+c_1R_2)s+1}$$

Signal Flow Graphs: The block diagram is useful for graphically representing control systems. For a very complicated system, however the block diagrams reduction process becomes quite time consuming. An alternate approach for finding the relationships among the system variables of a complicated control system is the signal flow graph approach.

A signal flow graph consists of a network in which nodes are connected by directed braches. Each node represents a system variable, and each branch connected between two nodes acts as a signal multiplier. Note that the signal flows in only one direction. The direction of signal flow is indicated by an arrow placed on the branch , and the multiplication factor is indicated along the branch. The signal flow graph depicts the flow of signals from one point of a system to another and gives the relationships among the signals.

Definitions:

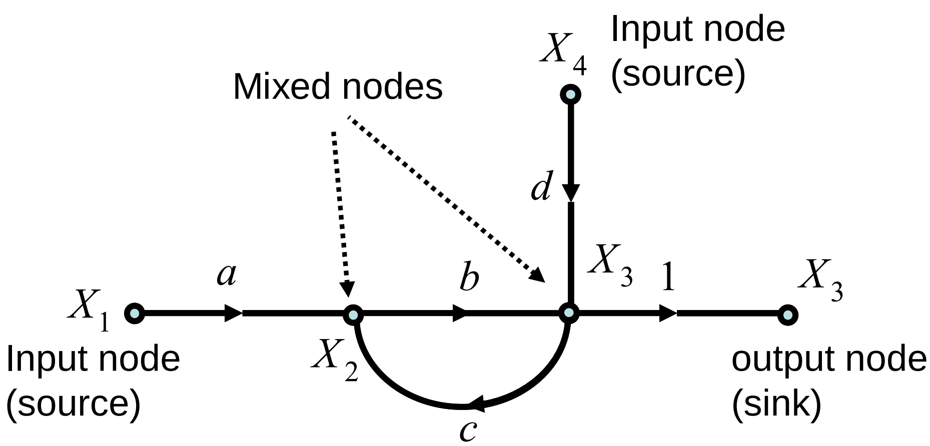
Node : A node is a point representing a variable or signal.
Transmittance: The transmittance is a gain between two nodes. The gain of a branch is a transmittance.
Branch: A branch is a directed line segment joining two nodes . the gain of a branch is a transmittance.
Input node or source: An input node or source is a node which has only outgoing branches. This corresponds to a dependent variable.
Output node or sink: An output node or sink is anode which has only incoming branches. This corresponds to a dependent variable.
Mixed node: A mixed node is anode which has both incoming and outgoing braches.
Path: A path is a traversal of connected branches in the direction of the branch arrow.
Loop: A loop is a closed path.
Loop gain: The loop gain is the product of the branch transmittances of a loop.
Nontouching loops: Loops are nontouching if they do not posses any common nodes.
Forward path: A forward path is a path from an input node (source) to an output node (sink) which does not cross any nodes more that once.
Forward path gain: A forward path gain is the product of the branch transmittances of a forward path.

Properties of signal flow graph:

1) A branch indicates the functional dependence of one signal upon another. A signal passes through only in the direction specified by the arrow of the branch.
2) A node adds the signals of all incoming branches and transmits this sum to all outgoing branches**.**

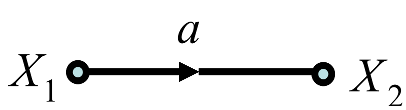
3) A mixed node, which has both incoming and outgoing branches, may be treated as an output node (sink) by adding an outgoing branch of unity transmittance. (See figure below. Notice that branches with unity transmittance is directed from X_3 to another node, also denoted by X_3) Note, however, that we cannot change a mixed to a source by this method.

4) For a given system, a signal flow graph is not unique. Many different signal flow graphs can be drawn for a given system by writing the system equations differently.

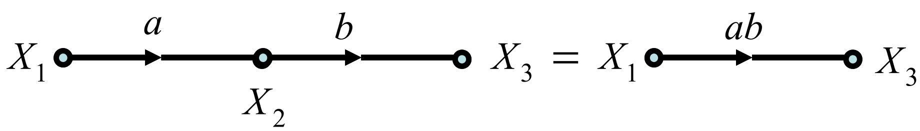


<u>Rules of Signal flow graph</u>:

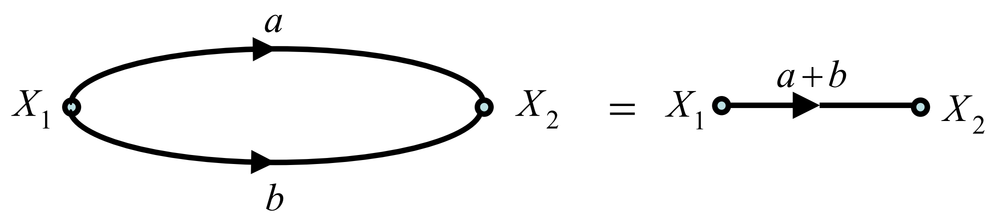
1) The value of a node with one incoming branch, as shown below is $X_2 = aX_1$.



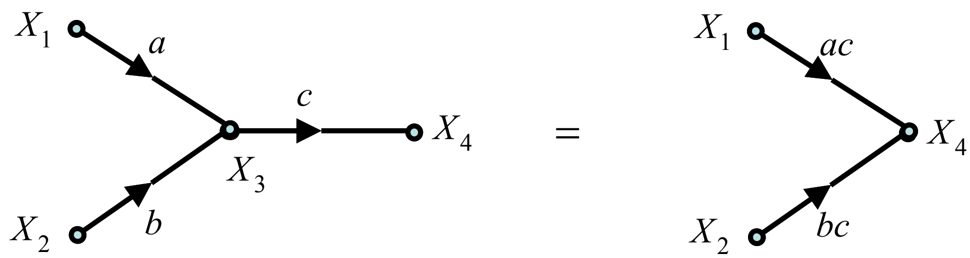
2) The total transmittance of cascaded branches is equal to the product of all the branch transmittances.Cascaded braches can thus be combined into a single branch by multiplying the transmittances, as shown below.



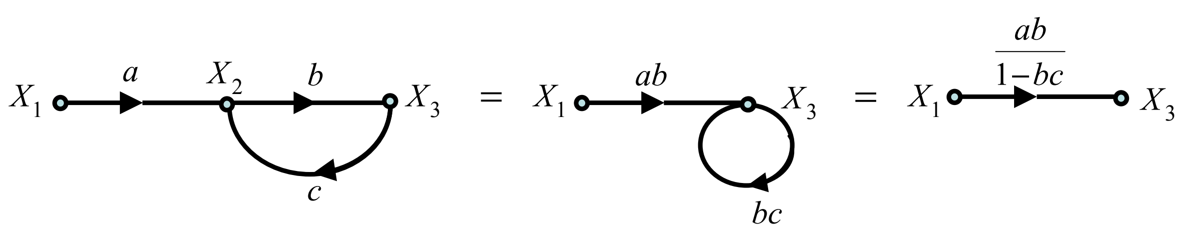
3) Parallel branches may be combined by adding the transmittances , as shown below.



4) A mixed node may be eliminated, as shown below.



5) A loop may be eliminated, as shown below.



Hence,

$$X_3 = bX_2$$
$$X_2 = aX_1 + cX_3$$

$$X_3 = abX_1 + bcX_3 \text{....................}(*)$$
$$X_3 = \frac{ab}{1-bc} X_1 \text{..........................}(**)$$

Equation$(*)$ corresponds to a diagram having a self-loop of transmittance *bc*. Elimination of the self-loop yields Equation$(**)$,which clearly shows that the overall transmittance is *ab*/(1-*bc*).

Ex. Consider a system defined by the following set of equations:

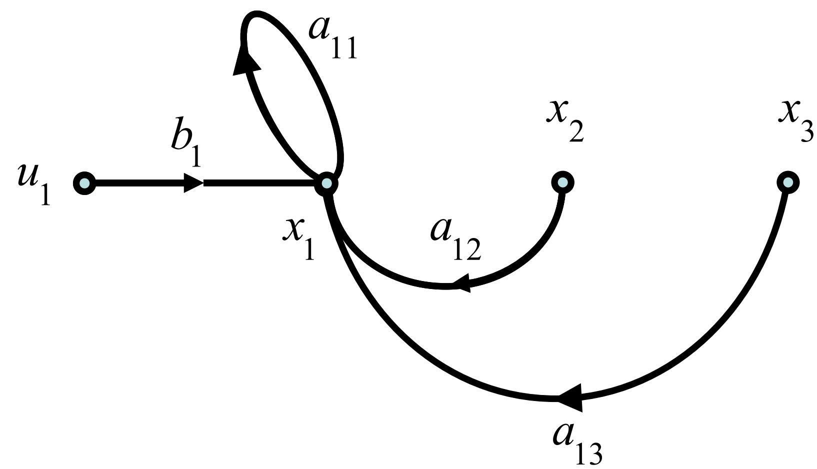
$$x_1 = a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + b_1u_1 \quad \ldots\ldots\ldots\ldots(1)$$

$$x_2 = a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + b_2u_2 \quad \ldots\ldots\ldots\ldots(2)$$

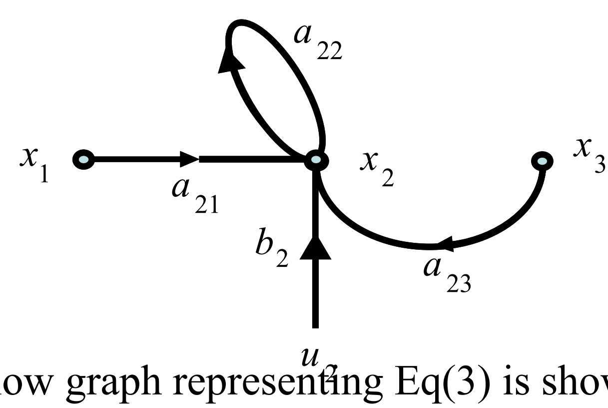
$$x_3 = a_{31}x_1 + a_{32}x_2 + a_{33}x_3 \quad \ldots\ldots\ldots\ldots(3)$$

where, u_1 and u_2 are input variables; x_1, x_2, and x_3 are output variables. A signal flow graph for this system, a graphical representation of these three simultaneous equations, indicating the interdependence of the variables, can be obtained as follows;

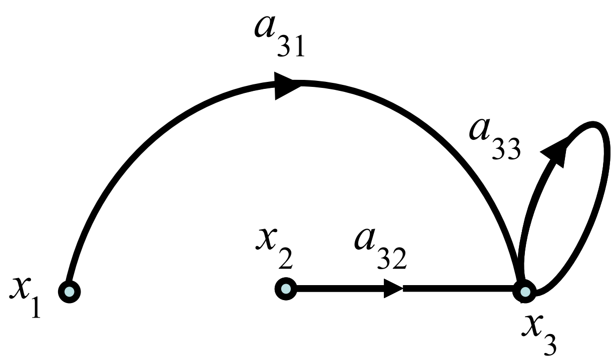
First locate the nodes x_1, x_2, and x_3, as shown in below.



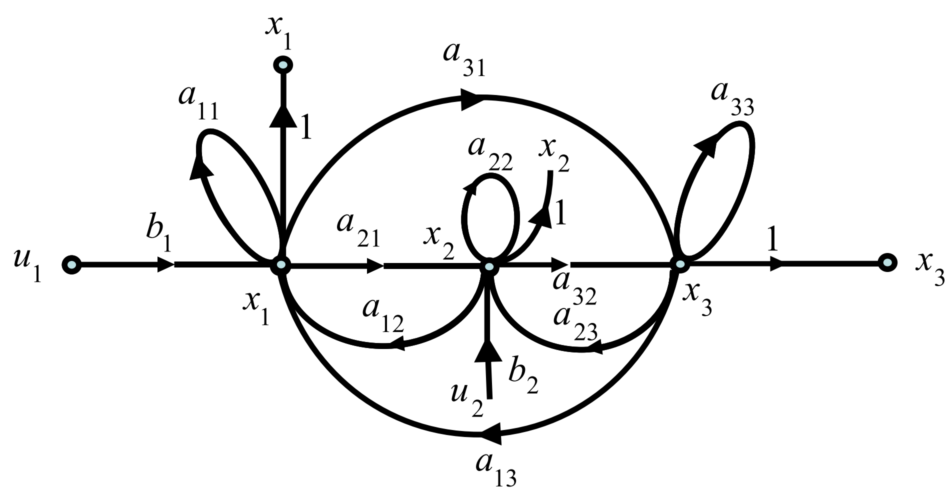
Note that a_{ij} is the transmittance between x_j and x_i. Equation (1) state that x_1 is equal to the sum of the four signals, $a_{11}x_1, a_{12}x_2, a_{13}x_3$, *and* b_1u_1. The signal flow graph representing Eq(1) is shown above. Eq(2) states that x_2 is equal to the sum of $a_{21}x_1, a_{22}x_2, a_{23}x_3$, *and* b_2u_2. The corresponding signal flow graph is shown below.



The signal flow graph representing Eq(3) is shown below.



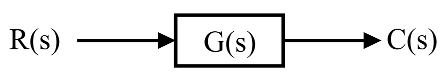
The signal flow graph representing Eq(1),Eq(2), and Eq(3) is then obtained by combining the above three figures. Finally the complete signal flow graph for the given simultaneous equations is shown below.

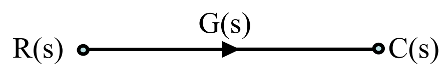


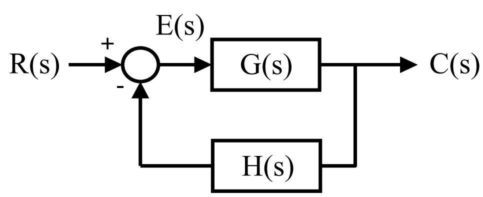
Control Theory

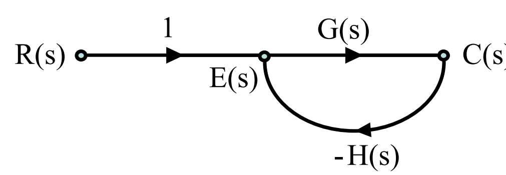
Second Class **Control and System Engineering Department**
By M. J. Mohamed

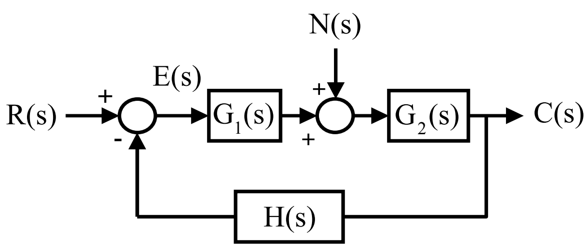
Signal flow graph of control system: some signal flow graphs of simple control system are shown below. For such simple graphs, the closed loop transfer function C(s)/R(s) can be obtained easily by inspection. For more complicated signal flow graphs, Mason's gain formula is quite useful.

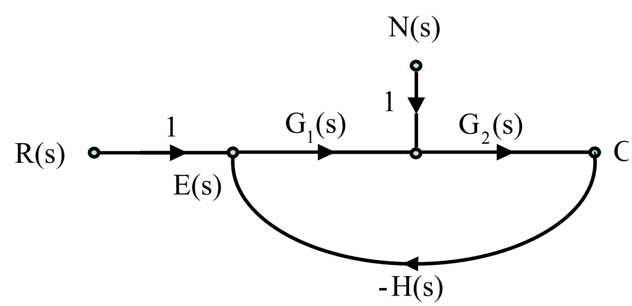


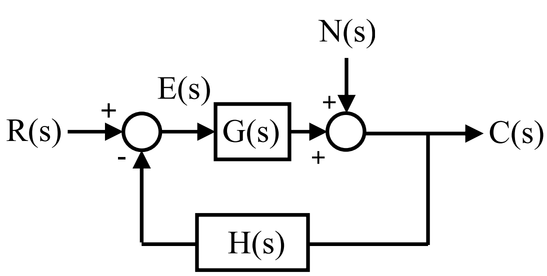


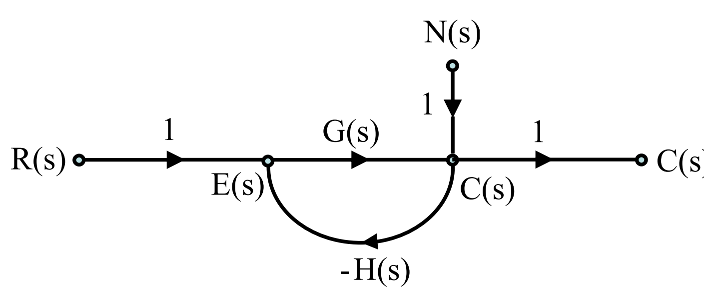


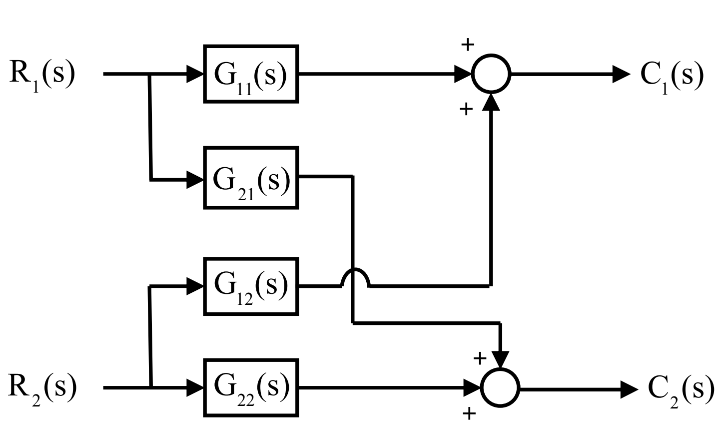


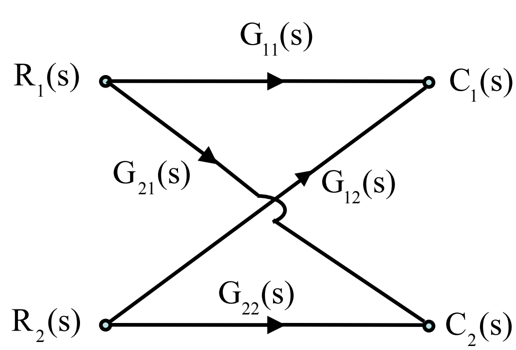












Mason's gain formula for signal flow graphs: In many practical cases we wish to determine the relationship between an input variable and an output variable of the signal flow graph. The transmittance between an input node and an output node is the overall gain, or overall transmittance, between these two nodes. Mason's gain formula, which is applicable to the overall gain, is given by;

$$P = \frac{1}{\Delta}\sum_{k} P_k \Delta_k$$

where, $P_k =$ path gain or transmittance of the *k*th forward path.

Δ_k =cofactor of the *k*th forward path determinant for the graph with the loops touching the *k*th forward path removed.

$\Delta =$ determinant of graph.

$\Delta = 1 -$ (sum of all different loop gains) $+$ (sum of gain products of all possible combinations of two nontouching loops) - (sum of gain products of all possible combination of three nontouching loops) $+$........

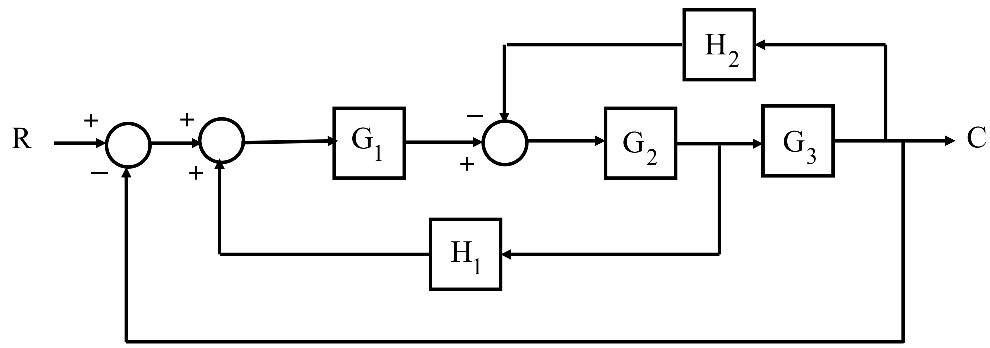
$$\Delta = 1 - \sum_{a} L_a + \sum_{b,c} L_b L_c - \sum_{d,e,f} L_d L_e L_f + \ldots\ldots\ldots\ldots$$

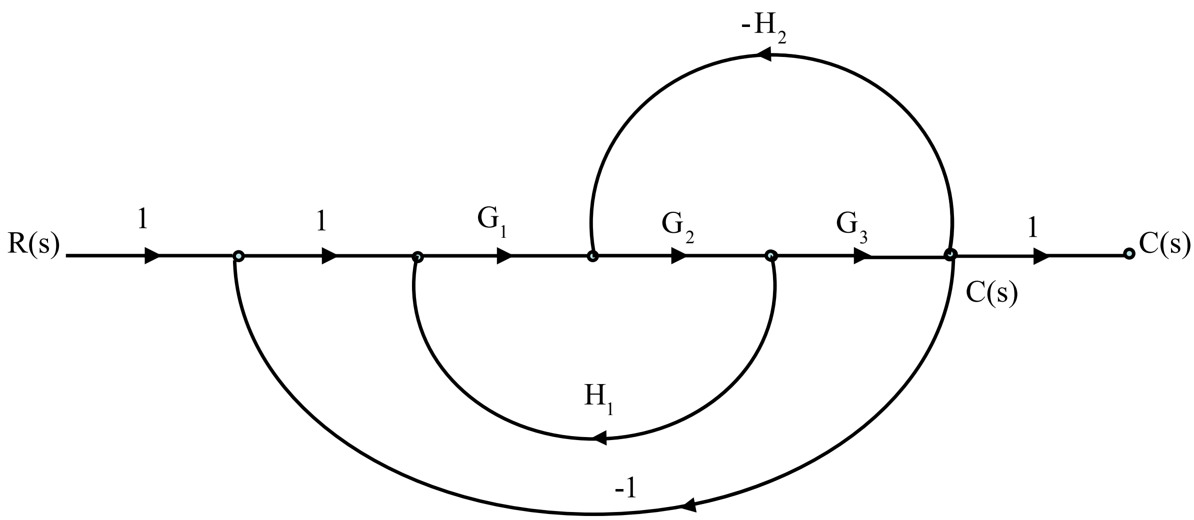
$\sum_{a} L_a$ =sum of all different loop gains

$\sum_{b,c} L_b L_c$ = sum of gain products of all possible combinations of two non-touching loops.

$\sum_{d,e,f} L_d L_e L_f$ =sum of gain products of all possible combinations of three non-touching loops.

Ex. Consider the system shown below. a signal flow graph for this system is also shown. Let us obtain the closed-loop transfer function C(s)/R(s) by use of Mason's gain formula.





In the system there is only one forward path between the input R(s) and the output C(s). The forward path gain is,

$$P_1 = G_1G_2G_3$$

From the signal flow graph, we see that there are three individual loops. The gains of these loops are;

$$L_1 = G_1G_2H_1$$
$$L_2 = -G_2G_3H_2$$
$$L_3 = -G_1G_2G_3$$

Note that since all three loops have a common branch, there are no non-touching loops; hence, the determinant Δ is given by;

$$\Delta = 1 - (L_1 + L_2 + L_3)$$
$$= 1 - G_1G_2H_1 + G_2G_3H_2 + G_1G_2G_3$$

The factor Δ_1 of the determinant along the forward path connecting the input node and output node is obtained by removing the loops that touch this path. Since path P_1 touches all three loops, we obtain;

$\Delta_1 = 1$

Therefore, the overall gain between the input R(s) and the output C(s), or the closed-loop transfer function, is given by,

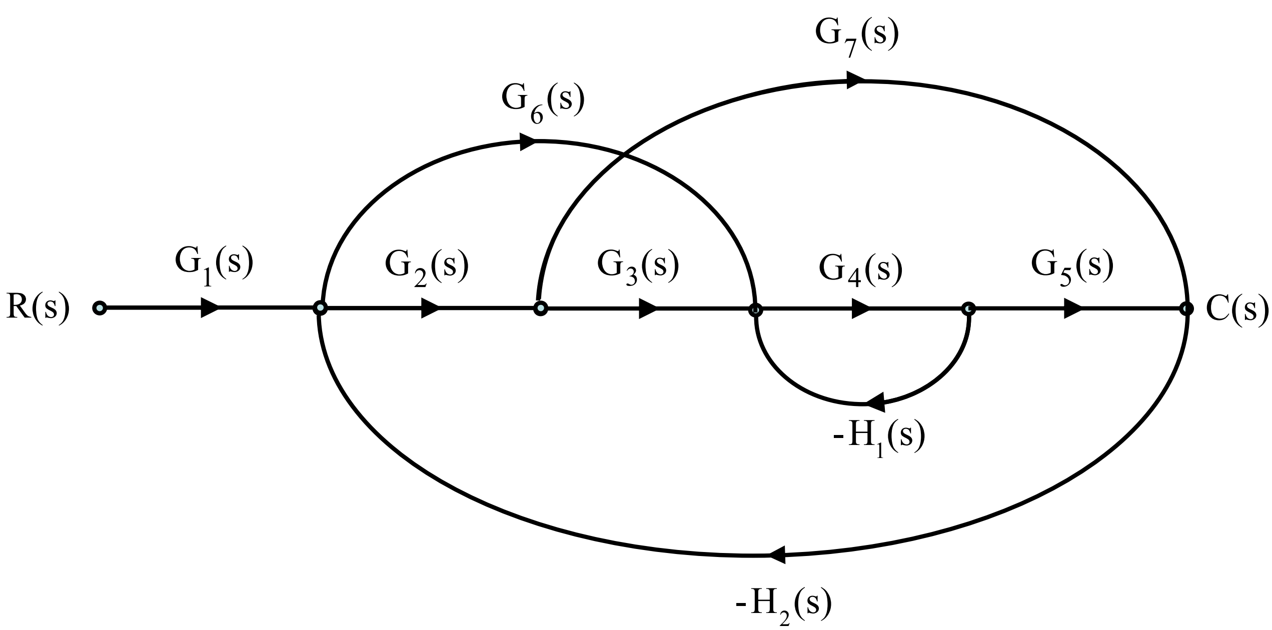
$$\frac{C(s)}{R(s)} = \frac{P_1\Delta_1}{\Delta}$$

$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3}{1 - G_1G_2H_1 + G_2G_3H_2 + G_1G_2G_3}$$

which is the same as the closed-loop transfer function obtained by block diagram reduction. Mason's gain formula thus gives the overall gain C(s)/R(s) without a reduction of the graph.

H.w. Find T.f using B.D reduction operations?

Ex. Consider the system shown in the following figure. Obtain the closed-loop transfer function C(s)/R(s) by use of Mason's gain formula.



In this system, there are three forward paths between the inputs R(s) and the output C(s). The forward path gains are;

$$P_1 = G_1G_2G_3G_4G_5$$
$$P_2 = G_1G_6G_4G_5$$
$$P_3 = G_1G_2G_7$$

There are four individual loops; the gains of these loops are,

$$L_1 = -G_4H_1$$
$$L_2 = -G_2G_7H_2$$
$$L_3 = -G_6G_4G_5H_2$$
$$L_4 = -G_2G_3G_4G_5H_2$$

Loop L_1 does not touch loop L_2. Hence, the determinant Δ is given by,

$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4) + L_1L_2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots..(*)$$

The factor Δ_1 is obtained from Δ by removing the loops that touch path P_1. Therefore, by removing L_1, L_2, L_3, L_4 and $L_1 L_2$ from equation(*), we obtain.

$$\Delta_1 = 1$$

Similarly, the factor Δ_2 is,

$$\Delta_2 = 1$$

The factor Δ_3 is obtain by removing L_2, L_3, L_4 and $L_1 L_2$ from equation (*), giving.

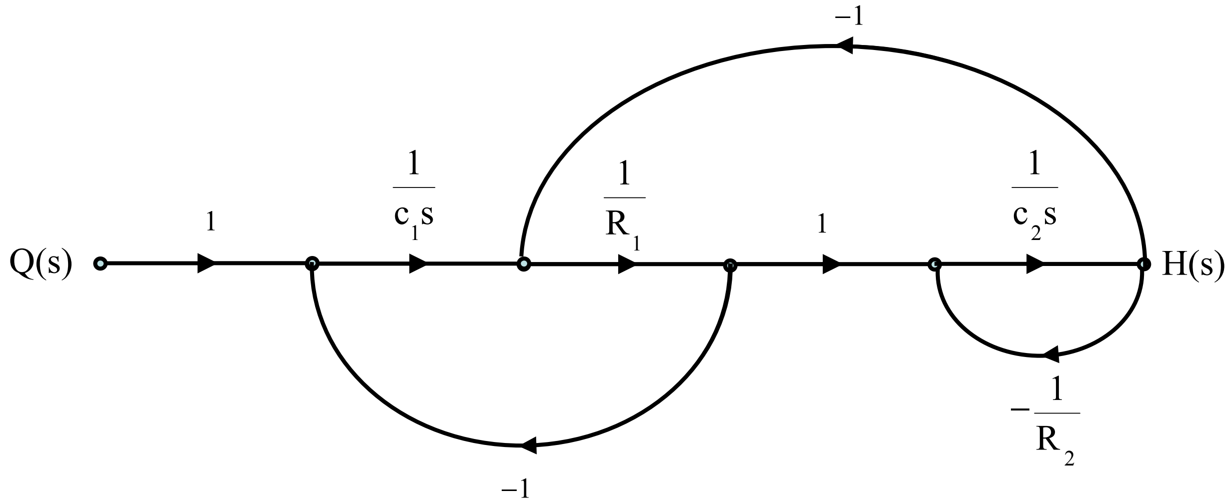
$$\Delta_3 = 1 - L_1$$

The close-loop transfer function C(s)/R(s) is then,

$$\frac{C(s)}{R(s)} = \frac{1}{\Delta}(P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3)$$

$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_4G_5 + G_1G_6G_4G_5 + G_1G_2G_7(1 + G_4H_1)}{1 + G_4H_1 + G_2G_7H_2 + G_6G_4G_5H_2 + G_4H_1G_2G_7H_2 + G_2G_3G_4G_5H_2}$$

Ex. Consider the system shown in the following figure. Obtain the closed-loop transfer function H(s)/Q(s).



Sol: in the given system, there is only one forward path that connects the input Q(s) and the output H(s). thus,

$$P_1 = \frac{1}{c_1 s}\frac{1}{R_1}\frac{1}{c_2 s}$$

There are three individual loops, thus,

$$L_1 = -\frac{1}{c_1 s}\frac{1}{R_1}$$

$$L_2 = -\frac{1}{c_2 s}\frac{1}{R_2}$$

$$L_3 = -\frac{1}{R_1}\frac{1}{c_2 s}$$

Loop L_1 does not touch loop L_2. (loop L_1 touches loop L_3, and loop L_2 touches loop L_3). Hence the determinant Δ is given by.

$$\Delta = 1 - (L_1 + L_2 + L_3) + L_1 L_2$$

$$\Delta = 1 + \frac{1}{R_1c_1s} + \frac{1}{R_2c_2s} + \frac{1}{R_1c_2s} + \frac{1}{R_1c_1R_2c_2s^2}$$

Since all three loops touch the forward path P_1, we remove L_1, L_2, and L_3 from Δ and evaluate the cofactor Δ_1 as follows,

$$\Delta_1 = 1$$

Thus, we obtain the closed-loop transfer function as shown.

$$\frac{H(s)}{Q(s)} = \frac{1}{\Delta}P_1\Delta_1$$

$$\frac{H(s)}{Q(s)} = \frac{\dfrac{1}{R_1c_1c_2s^2}}{1 + \dfrac{1}{R_1c_1s} + \dfrac{1}{R_2c_2s} + \dfrac{1}{R_1c_2s} + \dfrac{1}{R_1c_1R_2c_2s^2}}$$

$$\frac{H(s)}{Q(s)} = \frac{R_2}{R_1c_1R_2c_2s^2 + (R_1c_1 + R_2c_2 + R_2c_1)s + 1}$$

State Space Representation of Systems:

Modern control theory adopts what is known as state space representation for mathematical representation of systems. Among its different advantages: it makes it possible to deal with:

1) Nonlinear systems.
2) Time variant systems.
3) Multi i/p multi o/p systems.

State: The state of dynamic system is the smallest set of variables (called state variables) such that the knowledge of these variables at $t = t_o$ together with the i/p for $t > t_o$ completely determines the behavior of the system for any time $t \geq t_o$.

State space: the n-dimensional space whose coordinate axes consist of x_1 axis, x_2 axis, x_3 axis,…., x_n axis, is called a state space (where $x_1, x_2, x_3, \ldots, x_n$ represents state variables). Any state can be represented by a point in the state space.

***State space representation of nth order linear systems in which the forcing i/p function does not involve derivative terms*:**

Consider the following nth order system.

$$y^n + a_1 y^{n-1} + a_{n-1}\dot{y} + a_n y = u$$

Let us define the state variables;

$$\begin{aligned}
x_1 &= y & \dot{x}_1 &= x_2 \\
x_2 &= \dot{y} & \dot{x}_2 &= x_3 \\
x_3 &= \ddot{y} & \dot{x}_3 &= x_4 \\
&\vdots & &\vdots \\
&\vdots & \dot{x}_{n-1} &= x_n \\
x_n &= y^{n-1} & \dot{x}_n &= -a_n x_1 - a_{n-1} x_2 \ldots\ldots - a_1 x_n + u
\end{aligned}$$

Our aim is the following form:

$$\dot{\vec{X}} = \vec{A}\,\vec{X} + \vec{B}\,u$$

where, $\vec{X}$ is state vector.
$\vec{A}$ is state matrix.
$\vec{B}$ is the input vector.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \vdots \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ -a_n & -a_{n-1} & -a_3 & -a_2 & -a_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$\vec{A} \qquad\qquad\qquad \vec{B}$$

The output equation is;

$$Y = \vec{C}\,\vec{X}$$

$$Y = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix}$$

Ex. Consider the system defined by;

$$\dddot{y} + 6\ddot{y} + 11\dot{y} + 6y = 6u$$

Obtain the state space representation of the system.

$$\begin{aligned} x_1 &= y & \qquad \dot{x}_1 &= x_2 \\ x_2 &= \dot{y} & \dot{x}_2 &= x_3 \\ x_3 &= \ddot{y} & \dot{x}_3 &= -6x_1 - 11x_2 - 6x_3 + 6u \end{aligned}$$

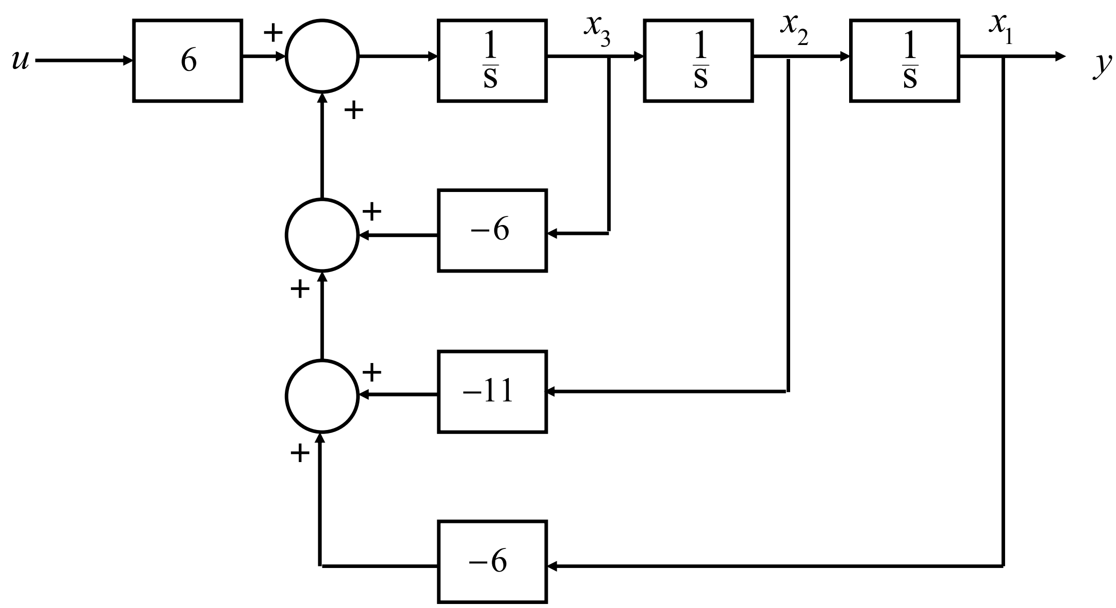
$$\dot{\vec{X}} = \vec{A}\,\vec{X} + \vec{B}\,u$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 6 \end{bmatrix} u$$

The output equation is,

$$Y = \vec{C}\,\vec{X}$$

$$Y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$



Ex. Consider the system defined by T.F;

$$G(s) = \frac{1}{s^2 + 2s + 2}$$

Find the state space representation.

$$G(s) = \frac{y(s)}{U(s)} = \frac{1}{s^2 + 2s + 2}$$

$$\ddot{y} + 2\dot{y} + 2y = u$$

$x_1 = y$ $\qquad\qquad$ $\dot{x}_1 = x_2$

$x_2 = \dot{y}$ $\qquad\qquad$ $\dot{x}_2 = -2x_1 - 2x_2 + u$

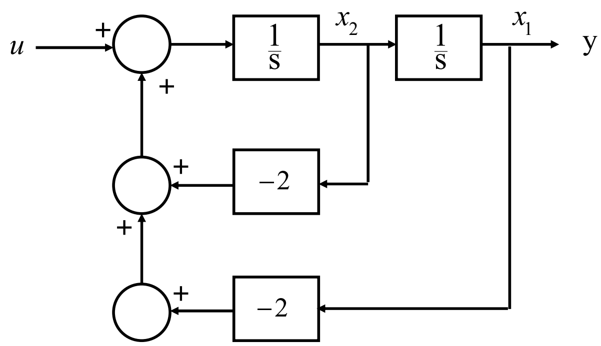
$$\dot{\vec{X}} = \vec{A}\,\vec{X} + \vec{B}\,u$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

The output equation is,

$$Y = \vec{C}\,\vec{X}$$

$$Y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$



***State space representation of nth order linear systems in which the forcing i/p function does involve derivative terms*:**

Consider the following *n*th order system.

$$y^n + a_1 y^{n-1} + a_{n-1}\dot{y} + a_n y = b_o u^n + b_1 u^{n-1} + b_{n-1}\dot{u} + b_n u$$

Let us define the state variable:-

$$x_1 = y - B_o u$$
$$x_2 = \dot{y} - B_o \dot{u} - B_1 u = \dot{x}_1 - B_1 u$$
$$x_3 = \ddot{y} - B_o \ddot{u} - B_1 \dot{u} - B_2 u = \dot{x}_2 - B_2 u$$
$$\vdots$$
$$x_n = y^{n-1} - B_o u^{n-1} - B_1 u^{n-2} \ldots\ldots\ldots - B_{n-2}\dot{u} - B_{n-1} u = \dot{x}_{n-1} - B_{n-1} u$$

where,

$$B_o = b_o$$
$$B_1 = b_1 - a_1 B_o$$
$$B_2 = b_2 - a_1 B_1 - a_2 B_o$$
$$B_3 = b_3 - a_1 B_2 - a_2 B_1 - a_3 B_o$$
$$\vdots$$
$$B_n = b_n - a_1 B_{n-1} \ldots\ldots\ldots\ldots\ldots - a_{n-1} B_1 - a_n B_o$$

The state equation $\dot{x}_n$

$$\dot{x}_n = \sum_{i=1}^{n} - a_{(n+1-i)}\, x_i + \mathrm{B}_n\, u$$

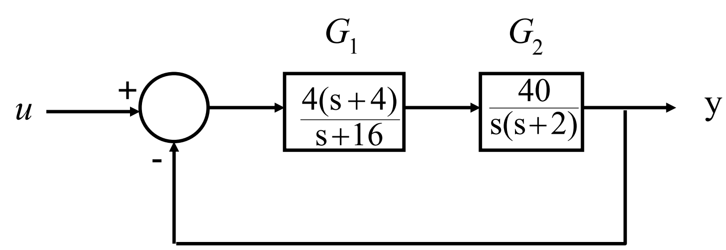
$$\dot{\vec{\mathrm{X}}} = \vec{\mathrm{A}}\,\vec{\mathrm{X}} + \vec{\mathrm{B}}\, u$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \vdots \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ -a_n & -a_{n-1} & -a_{n-2} & -a_2 & -a_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} \mathrm{B}_1 \\ \mathrm{B}_2 \\ \mathrm{B}_3 \\ \mathrm{B}_{n-1} \\ \mathrm{B}_n \end{bmatrix} u$$

$$\mathrm{Y} = \vec{\mathrm{C}}\,\vec{\mathrm{X}} + \vec{\mathrm{D}} u$$

$$\mathrm{Y} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix} + \mathrm{B_o}\, u$$

Ex. Obtain a state space representation for the following system:-



$$\frac{Y(\mathrm{s})}{U(\mathrm{s})} = \frac{G_1 G_2}{1 + G_1 G_2}$$

$$\frac{Y(\mathrm{s})}{U(\mathrm{s})} = \frac{160(\mathrm{s}+4)}{\mathrm{s}^3 + 18\mathrm{s}^2 + 192\mathrm{s} + 640}$$

$$\dddot{\mathrm{y}} + 18\ddot{\mathrm{y}} + 192\dot{\mathrm{y}} + 640\mathrm{y} = 160\dot{u} + 640u$$

$$a_1 \qquad a_2 \qquad a_3 \qquad b_2 \qquad b_3$$

Let us define :-

$$x_1 = y - B_o u$$
$$x_2 = \dot{x}_1 - B_1 u$$
$$x_3 = \dot{x}_2 - B_2 u$$

$$B_o = b_o = 0$$
$$B_1 = b_1 - a_1 B_o = 0\text{-}18(0)=0$$
$$B_2 = b_2 - a_1 B_1 - a_2 B_o = 160\text{-}18(0)\text{-}192(0)=160$$
$$B_3 = b_3 - a_1 B_2 - a_2 B_1 - a_3 B_o = 640\text{-}18(160)\text{-}192(0)\text{-}640(0)=-2240$$

Then the state equation becomes:-

$$\dot{\vec{X}} = \vec{A}\,\vec{X} + \vec{B}\,u$$

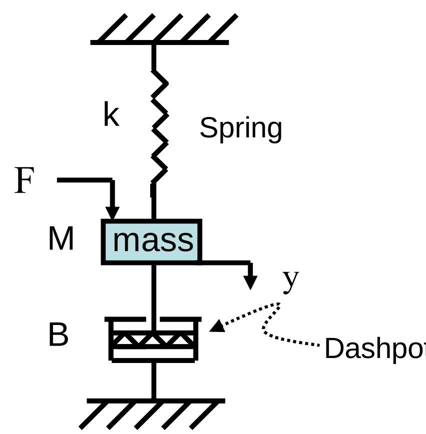
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -640 & -192 & -18 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 160 \\ -2240 \end{bmatrix} u$$

The output equation is,

$$Y = \vec{C}\,\vec{X}$$

$$Y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + B_o u$$

Ex. Find the state space representation for the following system



$$F = ky + mD^2y + BDy$$
$$F = mD^2y + BDy + ky$$
$$\ddot{y} = -\frac{B}{m}\dot{y} - \frac{k}{m}y + \frac{1}{m}f$$
$$x_1 = y \quad , \quad \dot{x}_1 = x_2$$
$$x_2 = \dot{y} \quad , \quad \dot{x}_2 = -\frac{k}{m}x_1 - \frac{B}{m}x_2 + \frac{1}{m}f$$

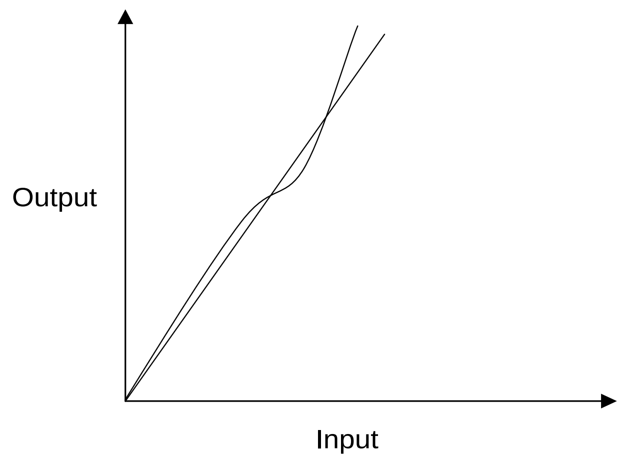
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{B}{m} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} f$$

The output equation is, $Y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

Design Principles of Automatic Control System:

Steps for the analysis and design of systems:-

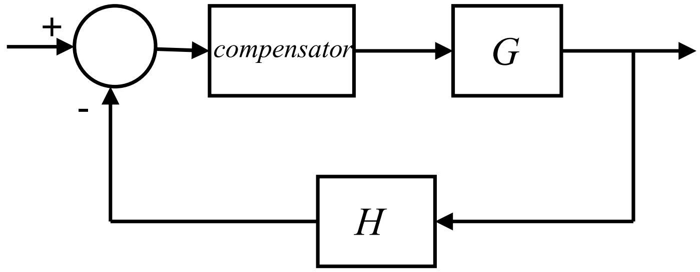
Step i: define the objective of the problem including specifications.
Step ii: decide a scheme for the above problem using existing hardware and new components required.
Step iii: write down the mathematical equations of all the components /subsystems by making suitable simplifying assumptions e.g regarding linearity.



Note that these mathematical equations may be:

i) Algebraic equations $x=ky$
ii) Differential equation $\frac{dy}{dt} = ky$
iii) Difference equation

Step iv: combine the equations obtained above so that the system is represented as:-



Step v: test the system behaviour by using impulse, step, or ramp inputs. Testing may be done by;

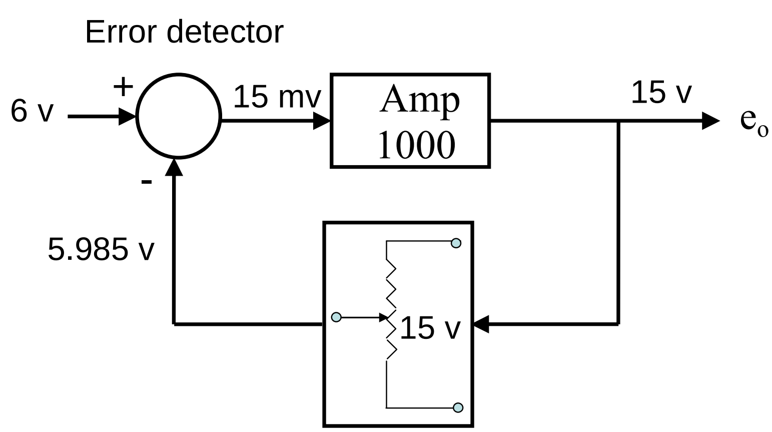
1) direct calculations
2) computer simulation

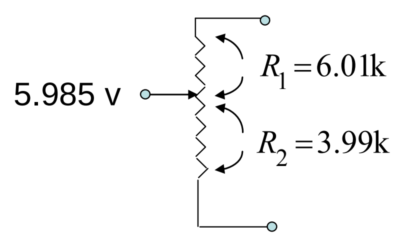
Step vi: if the response is unsatisfactory. A compensation is designed by following the techniques available in control theory.

Ex. A feedback voltage regulator to produce 15 volt D.C output is required. The available components are;

i) An amplifier of gain 1000.
ii) A comparator (error detector).
iii) A constant voltage reference of 6 volt D.C.
iv) A potentiometer of $10k\Omega$.

a) Propose a scheme for the C.L voltage regulator.
b) Calculate the potentiometer setting.
c) If the amplifier gain decreases by 10% what will be its effect on the output, if the potentiometer is not readjusted.



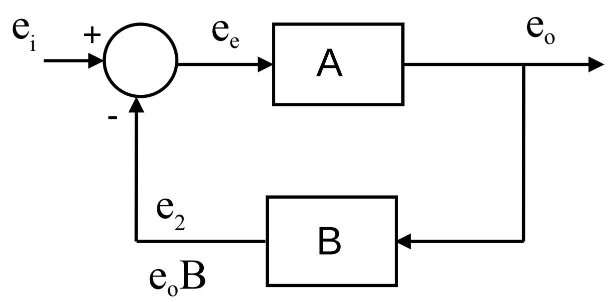


Let e_o is the output

$$(6 - e_o \frac{3.99}{10})900 = 0$$

$$e_o = 14.9958$$

Ex. Consider a feedback control system as shown.



1) Show that the ratio of output to input for C.L system is given by.

$$\frac{e_o}{e_i} = \frac{A}{1+AB}$$

2) Show that if A changes by $\pm x\%$, the percentage change in $\frac{e_o}{e_i}$ is given by $\pm \frac{x}{1+AB(1\pm 0.01x)}\%$.

3) Show that if B changes by $\pm y\%$, the percentage change in $\frac{e_o}{e_i}$ is given by $\pm \frac{AB\,y}{1+AB(1\pm 0.01x)}\%$.

Comment on the above results?

$e_i - e_oB = e_e$ ....(1)

$e_e * A = e_o$ ....(2)

$$e_i - e_oB = \frac{e_o}{A}$$

$$Ae_i - ABe_o = e_o$$

$$Ae_i = e_o(1+AB)$$

$$\frac{e_o}{e_i} = \frac{A}{1+AB}$$

....

$A \xrightarrow{\text{Changed}} (A \pm \frac{x}{100} A) = A\,(1 \pm 0.01x) = \alpha A$

where, $\alpha = (1 \pm 0.01x)$

$$\left(\frac{e_o}{e_i}\right)_{new} = \frac{(A \pm 0.01xA)}{1+(A \pm 0.01xA)B}$$

$$\frac{\left(\frac{e_o}{e_i}\right)_{new} - \left(\frac{e_o}{e_i}\right)_{original}}{\left(\frac{e_o}{e_i}\right)_{original}} 100\% = \frac{\left(\frac{\alpha A}{1+\alpha AB}\right) - \left(\frac{A}{1+AB}\right)}{\left(\frac{A}{1+AB}\right)} 100\%$$

$$\frac{\frac{\alpha A(1+AB) - A(1+\alpha AB)}{(1+\alpha AB)(1+AB)}}{\left(\frac{A}{1+AB}\right)} 100\% = \frac{\alpha + \alpha AB - 1 - \alpha AB}{1+\alpha AB} = \frac{\alpha - 1}{1+\alpha AB}$$

$$= \frac{(1 \pm 0.01x) - 1}{1+\alpha AB} = \frac{\pm 0.01x}{1+AB(1 \pm 0.01x)} = \frac{\pm x}{1+AB(1 \pm 0.01x)} \%$$

....

$$B \xrightarrow{\text{Changed}} (B \pm \frac{y}{100} B) = B\,(1 \pm 0.01y) = \Theta B$$

where, $\Theta = (1 \pm 0.01y)$

$$\frac{\left(\frac{A}{1+\Theta AB}\right) - \left(\frac{A}{1+AB}\right)}{\left(\frac{A}{1+AB}\right)} 100\% = \frac{\frac{A(1+AB) - A(1+\Theta AB)}{(1+\Theta AB)(1+AB)}}{\left(\frac{A}{1+AB}\right)} 100\%$$

$$= \frac{1+AB-1-\Theta AB}{1+\Theta AB} = \frac{AB(1-\Theta)}{1+\Theta AB} = \frac{AB(1-1 \pm 0.01y)}{1+\Theta AB}$$

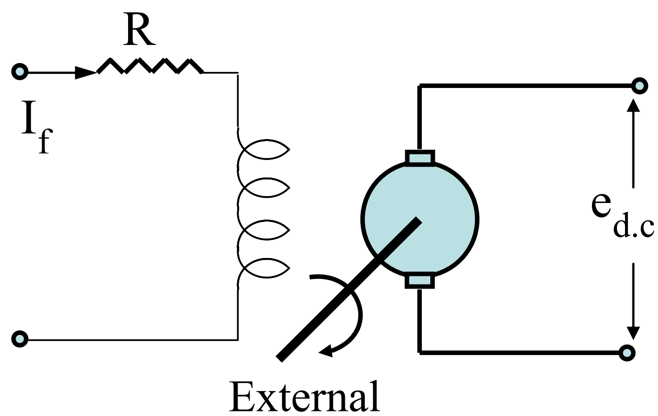
$$= \frac{\pm 0.01yAB}{1+AB(1 \pm 0.01y)} = \pm \frac{AB\,y}{1+AB(1 \pm 0.01y)} \%$$

Ex. The output voltage of a D.C generator is directly proportional to its field current and speed. A D.C generator develops 220 volt at 1500 r.p.m with a field current of 2.2 Amp. It is required to regulate the output voltage by a close loop feedback control.

a) Develop a suitable scheme. Given that the generator filed resistance is 100 ohm, a 12 volt car battery is available as reference, and an amplifier of gain 1000, capable of supplying the necessary filed current.

b) Calculate the potentiometer setting to give the correct output voltage of 220 volts (speed=1500 r.p.m).

c) How would you set 110 volt out put in this system?



a)

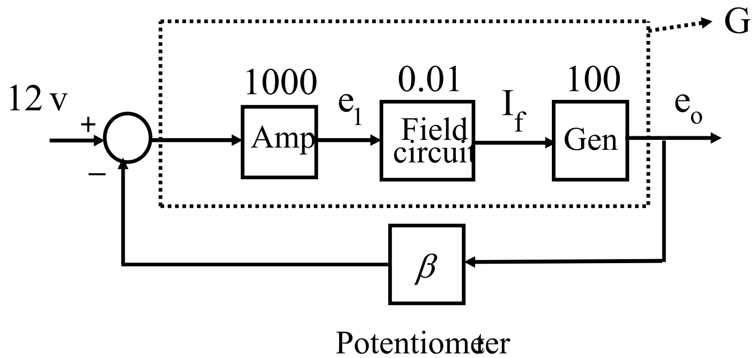
Generator voltage $= k \cdot I_f \cdot$ speed

$$220 = k * 2.2 * 1500$$

$$k = 0.06667$$

A constant speed of 1500 r.p.m

$e_o = k * I_f * 1500 = 100 * I_f$



b) $\frac{e_o}{12} \equiv \frac{G}{1+G\beta} \Longrightarrow \frac{220}{12} \equiv \frac{1000}{1+1000\beta} \Longrightarrow \beta = 0.0535$

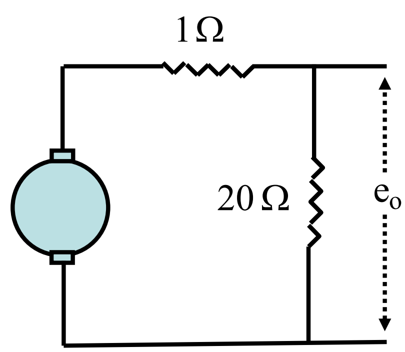
c) $\frac{110}{12} \equiv \frac{1000}{1+1000\beta} \Longrightarrow \beta = 0.108$

d) If the generator speed drops to 1400 r.p.m , what would be changed output voltage ? (system was originally set for 220 volt).

$$e_o = k * I_f * speed$$
$$e_o = 0.06667 * I_f * 1400$$
$$e_o = 93.333 * I_f$$
$$\frac{e_o}{I_f} = 93.333 \quad \text{, the gain of the last block}$$
$$e_o = e_i \frac{G}{1+GH}$$
$$e_o = 12 * \frac{933.33}{1+933.33 * 0.0535}$$
$$e_o = 219.895$$

e) If the generator has an armature resistance of $1\,\Omega$ and it is supplying a load of $20\,\Omega$. What will be the output voltage of the regulated system?. (speed=1500 r.p.m and the system set for 220 volt at no load)

$$e_o = \frac{20}{21} * 220 = 209.5$$



f) The inductance of the filed winding is given as 50H. Develop the dynamic equation of the complete system.

$$e_1 = 50\frac{dI_f}{dt} + 100 * I_f$$

$$\frac{I_f}{e_1} = \frac{1}{50D+100}$$

$$G=\frac{100000}{50D+100} \quad , \quad H=0.0535$$

$$e_o=\frac{\frac{100000}{50D+100}}{1+\frac{100000}{50D+100}*0.0535}*12 \quad , \quad e_o=\frac{100000}{50D+100+5350}*12$$

$$e_o=\frac{100000}{50D+5450}*12 \quad , \quad e_o=\frac{100000}{50*(D+109)}*12 \quad , \quad e_o=\frac{24000}{(D+109)}$$

$$\frac{de_o}{dt}+109e_o=24000$$

g) What is the time constant of the close loop system for an amplifier gain of (i) 1000 (ii) 200?

$$\frac{dx(t)}{dt}+ax(t)=u(t)$$

For gain =1000 , $T=\frac{1}{109}=9.17\text{ msec}$

For gain =200 ,

$$e_o=\frac{\frac{20000}{50D+100}}{1+\frac{20000}{50D+100}*0.0535}*12$$

$$e_o=\frac{20000}{50D+100+20000*0.0535}*12$$

$$e_o=\frac{20000}{50D+1170}*12$$

$$e_o=\frac{4800}{D+23.4}$$

$$\frac{de_o}{dt}+23.4\,e_o=4800$$

$$T=\frac{1}{23.4}=42.73\text{ msec}$$

Control Theory

Second Class **Control and System Engineering Department**
By M. J. Mohamed

Analysis of Typical Control System:

Consider a first order differential equation.

$\frac{dx(t)}{dt} + ax(t) = u(t)$ This may be the equation of a physical system with input $u(t)$ and output $x(t)$.

Solution:

i) Transient part (Tr) (complementary function) is that part of the response which occurs near t=0 and then decays. This part of the response is due to the characteristics of the system only.

ii) Steady state part (S.S) (particular integral) is that part of the response which is present through out the period t=0 to t=∞. But at $t \to \infty$ this is the complete solution because the transient part is absent.

The nature of the steady state response depends on external input only.

Complete solution = Tr part + S.S part

i) Auxiliary equation (characteristic equation)

m+a=0 $\Rightarrow$ m=-a

Transient part = Ae^{-at}

ii) S.S part

let, $u(t)$ = U (constant)

$$\frac{dx}{dt} = 0 \text{ at S.S}$$

$$a\, x_{ss} = \text{U} \Rightarrow x_{ss} = \frac{\text{U}}{a}$$

$$x(t) = Ae^{-at} + \frac{\text{U}}{a}$$

If we know $x(t) = 0$ at $t=0$.

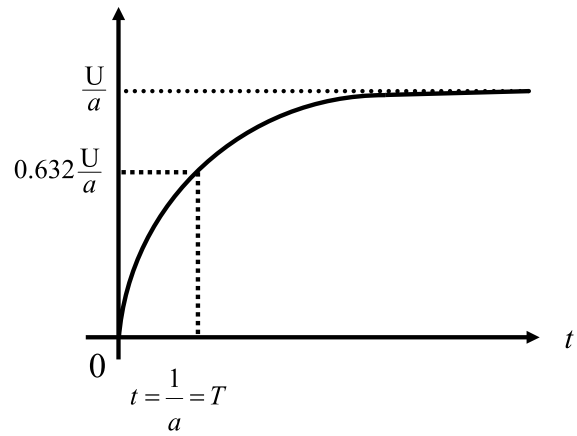
$$x(t) = \frac{U}{a}(1 - e^{-at})$$

at $t = 0$, $x(t) = 0$

at $t = \infty$, $x(t) = \frac{U}{a}$

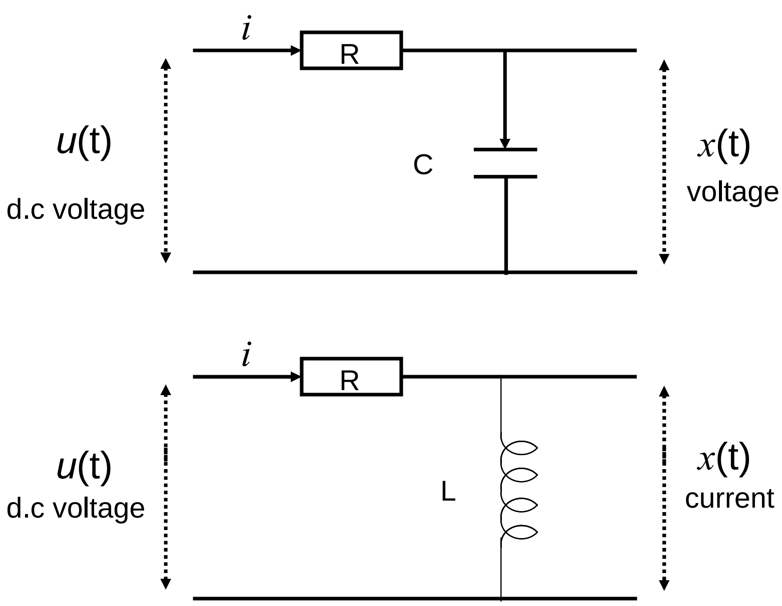
at $t = \frac{1}{a}$, $x(t) = 0.632\,\frac{U}{a}$

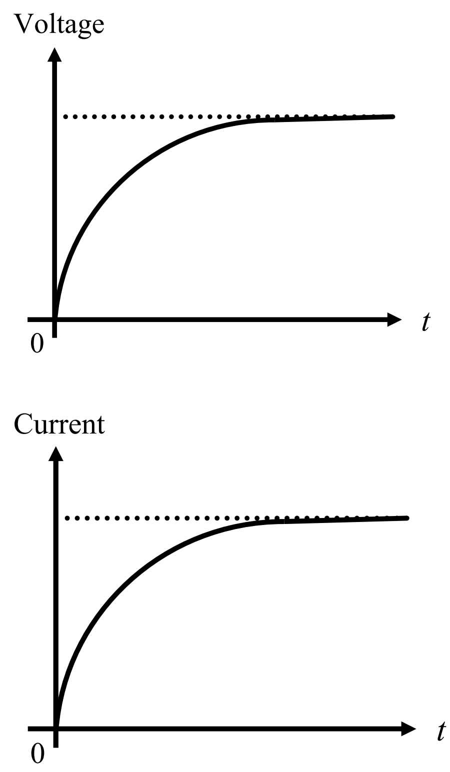
$T = \frac{1}{a} \equiv$ Time constant



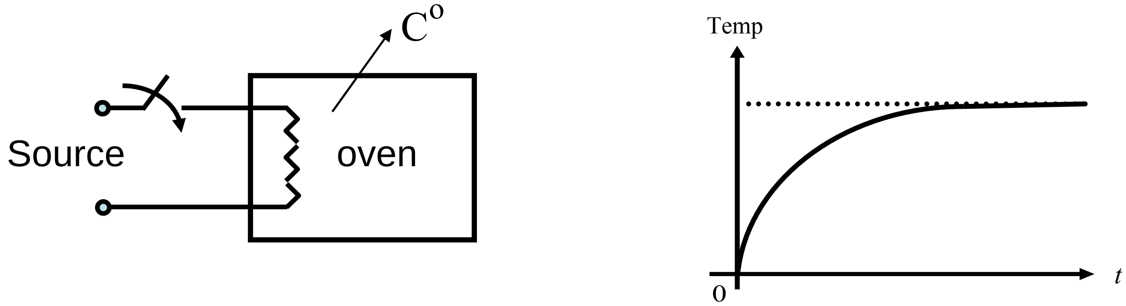
Examples of systems showing the exponential time response.

i) R-C , R-L circuit with constant voltage input.

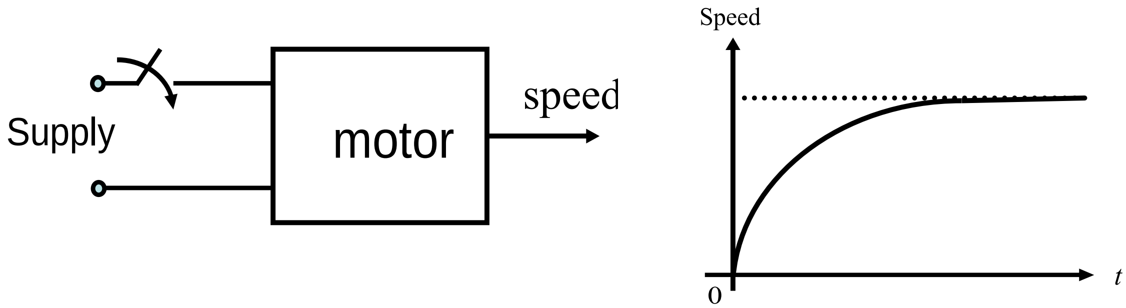




ii) A sudden voltage applied an electric oven.



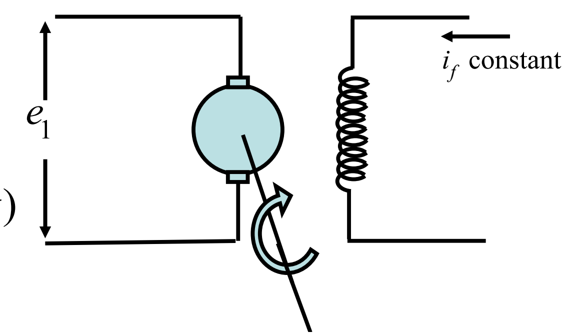
iii) Constant supply voltage switched on to motor.



All these system may be represented by differential equation of first order.

SUCH SYSTEMS ARE CALLED FIRST ORDER SYSTEM

Consider a D.C motor operating with a constant field current i_f. If the input to the motor is taken as e_1 (armature voltage) and the output is taken as speed ω. The differential equation of the motor may be written as:-

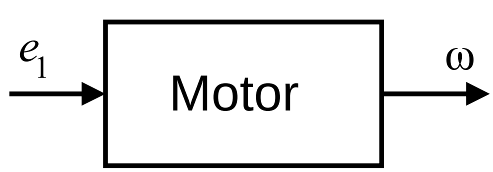


$$\frac{d\omega(t)}{dt} + a\omega(t) = e_1(t)$$

Using 'D' operator $D\omega(t) + a\omega(t) = e_1(t)$

$$\frac{\omega(t)}{e_1(t)} = \frac{1}{D+a}$$

If we define θ as the output variable,

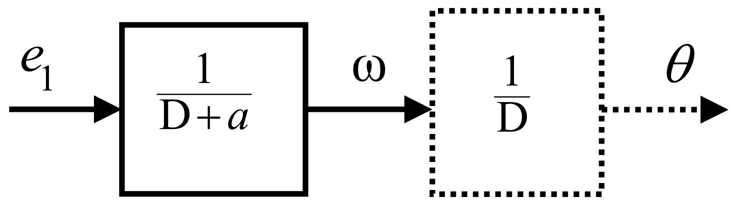


$$\omega = \frac{d\theta}{dt}$$

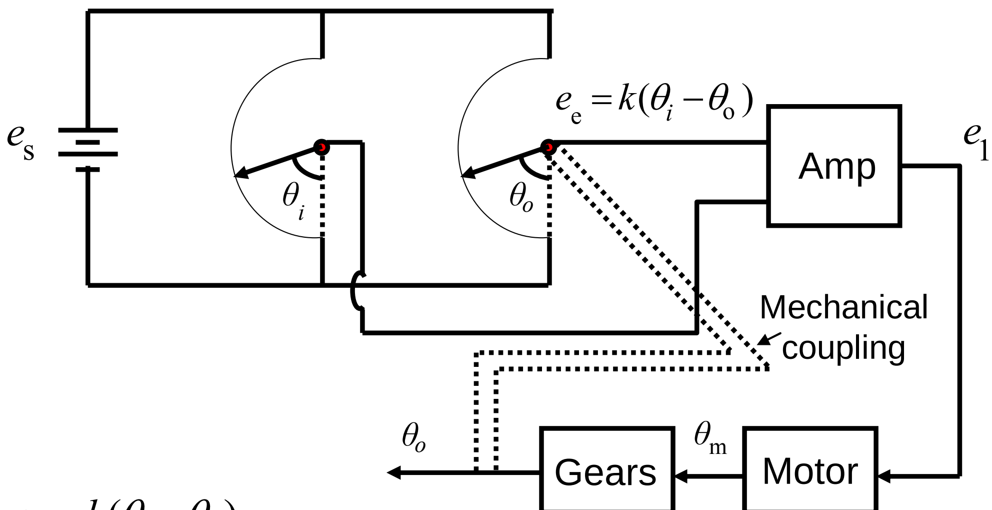
$$\frac{d^2\theta(t)}{dt^2} + a\frac{d\theta(t)}{dt} = e_1(t)$$

$$D^2\theta + aD\theta = e_1(t)$$

$$\frac{\theta(t)}{e_1(t)} = \frac{1}{D(D+a)}$$



Analysis of a position control system:

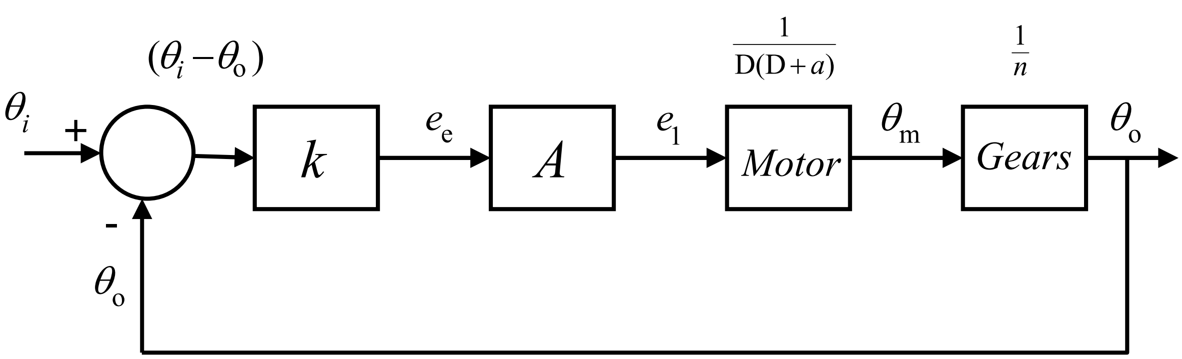


$$e_e = k(\theta_i - \theta_o)$$

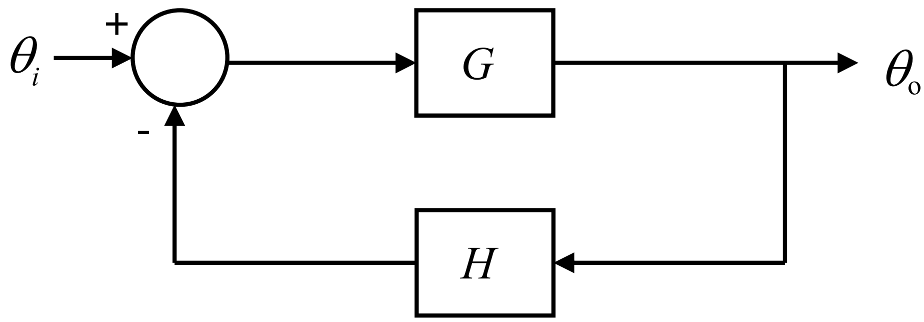
$$k = \frac{e_e}{(\theta_i - \theta_o)}$$

$$k = \frac{e_s}{\theta_{Max}}$$

Block diagram:

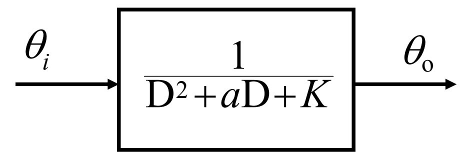


Simplified block diagram:



$$G \equiv (k\,A\,\frac{1}{D(D+a)}\frac{1}{n}) = \frac{K}{D(D+a)}$$ where K= system gain

$$\frac{\theta_o}{\theta_i} = \frac{G}{1+GH} = \frac{\frac{K}{D(D+a)}}{1+\frac{K}{D(D+a)}} = \frac{K}{D^2 + aD + K}$$



$$\frac{d^2\theta_o(t)}{dt^2} + a\frac{d\theta_o(t)}{dt} + K\theta_o(t) = k\theta_i(t)$$

a is parameter of the motor

$K = k.A\frac{1}{n}$ where A is amplifier gain and $\frac{1}{n}$ is gear ratio.

TEST INPUTS:

i) STEP FUNCTION: A step is a sudden change in the value of the physical quantity x(t) from one level (usually zero) to another level, in zero time.

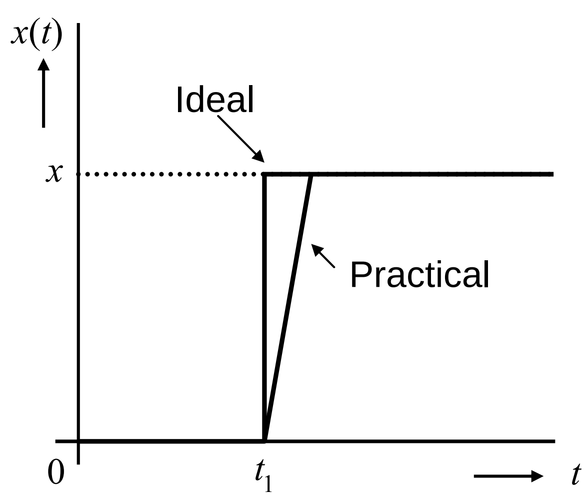
$x(t) = x \qquad t > t_1$

$\quad = 0 \qquad t \le t_1$

UNIT STEP:

$u(t) = 1 \qquad t > t_1$

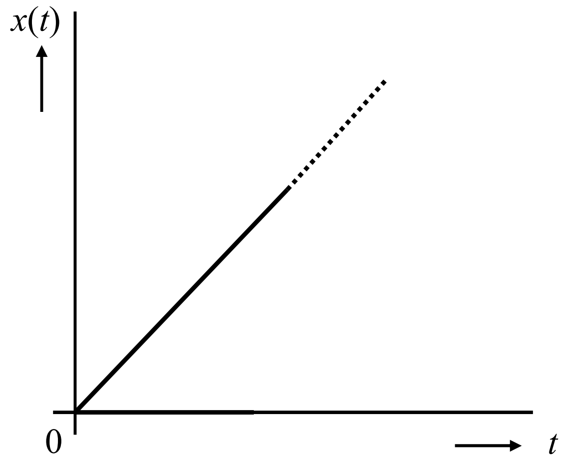
$\quad = 0 \qquad t \le t_1$



Step function at t_1

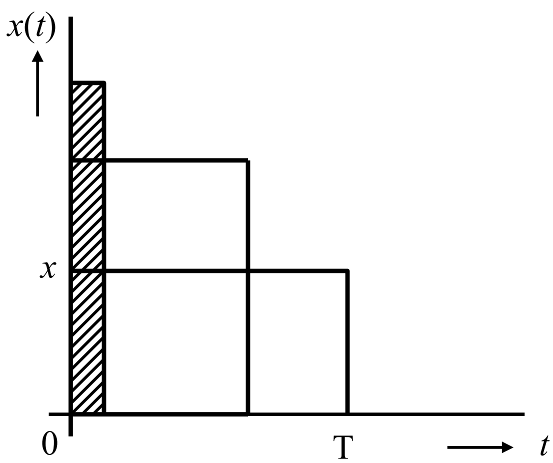
ii) RAMP FUNCTION: Ramp is a signal which starts from a zero level and increase linearly with respect to time.

$$x(t) = kt \qquad t > 0$$
$$= 0 \qquad t \le 0$$

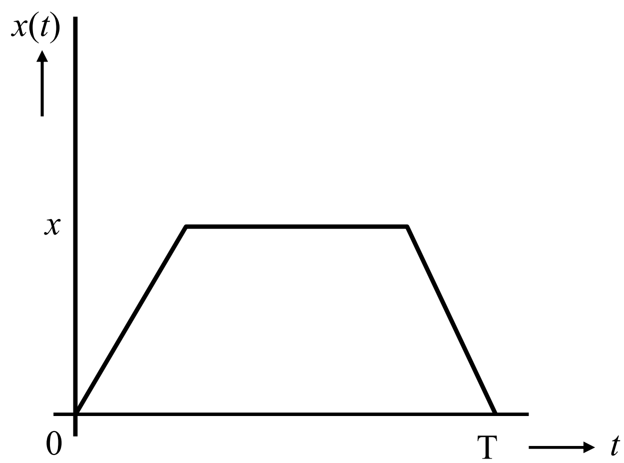


Ramp function at $t = 0$

iii) **PULSE FUNCTION**: A pulse may be considered as a step function which is present for limited period.



Ideal

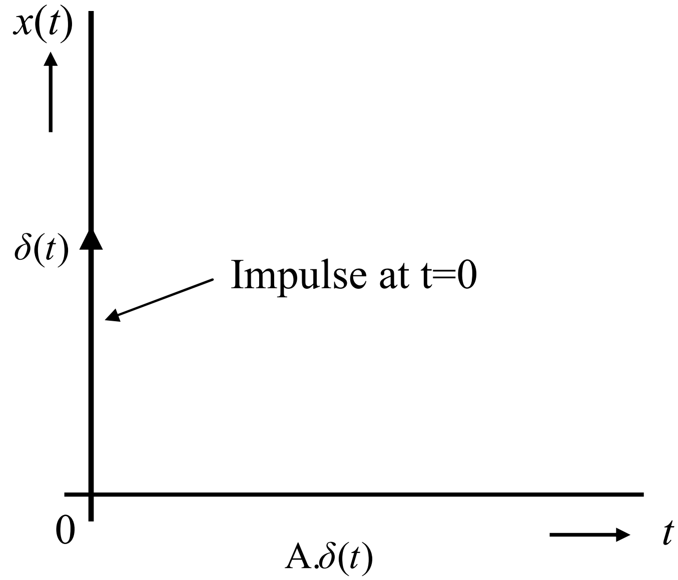


Practical

$$x(t) = x \qquad 0 < t \le \mathrm{T}$$
$$= 0 \qquad \text{elsewhere}$$

iv) **IMPULSE FUNCTION:** If in the pulse, the width is decreased and the height is increased such that.

$\lim\limits_{\substack{T \to 0 \\ x \to \infty}} x.T = A$, the resulting function is impulse A δ(t)



Differential Equation of the C.L Position Control System:

$$\frac{d^2\theta_o(t)}{dt^2} + a\frac{d\theta_o(t)}{dt} + k\theta_o(t) = k\theta_i(t)$$

For step input, $\theta_i(t) = R$, $t > 0$

$$\frac{d^2\theta_o(t)}{dt^2} + a\frac{d\theta_o(t)}{dt} + k\theta_o(t) = kR$$

Solve the differential equation.

i) S.S solution ($\dot{\theta}_o(t) = \ddot{\theta}_o(t) = 0$)

$$(\theta_o)_{s.s} = R$$

ii) Transient solution

Auxiliary equation: $r^2 + ar + k = 0$

(characteristic equation)

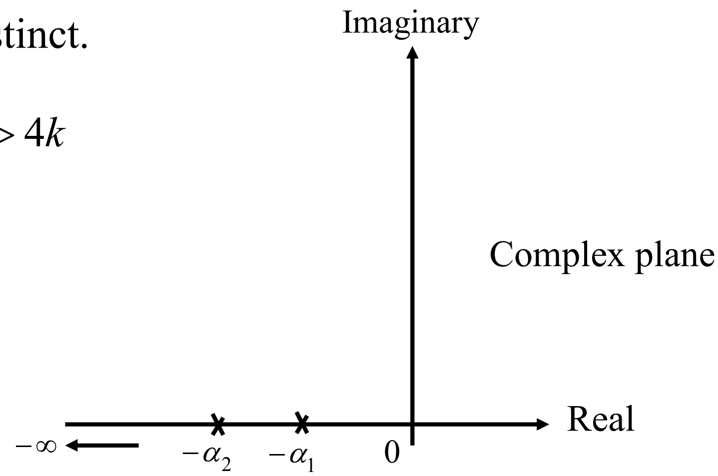
$$r_1, r_2 = \frac{-a \pm \sqrt{a^2 - 4k}}{2}$$

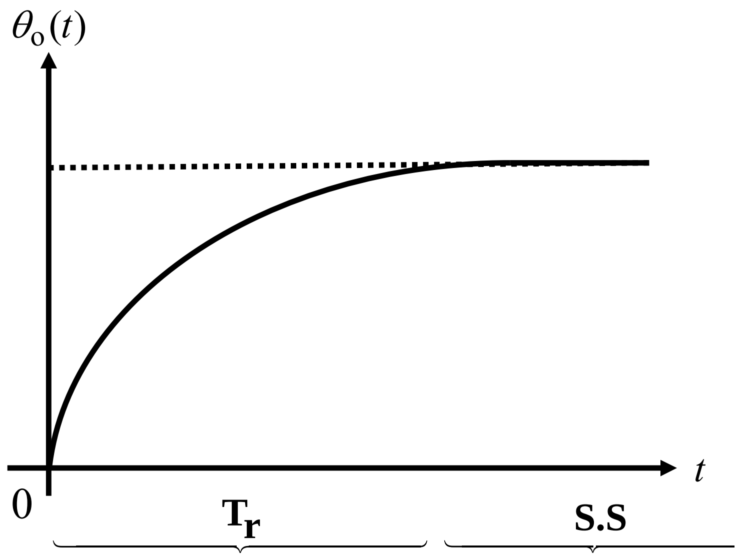
Case I : The two roots are distinct.

$$r_1, r_2 = -\alpha_1, -\alpha_2 \quad ; \quad a^2 > 4k$$

$$(\theta_o)_{Tr} = C_o e^{-\alpha_1 t} + C_1 e^{-\alpha_2 t}$$

$$\theta_o(t) = R + C_o e^{-\alpha_1 t} + C_1 e^{-\alpha_2 t}$$



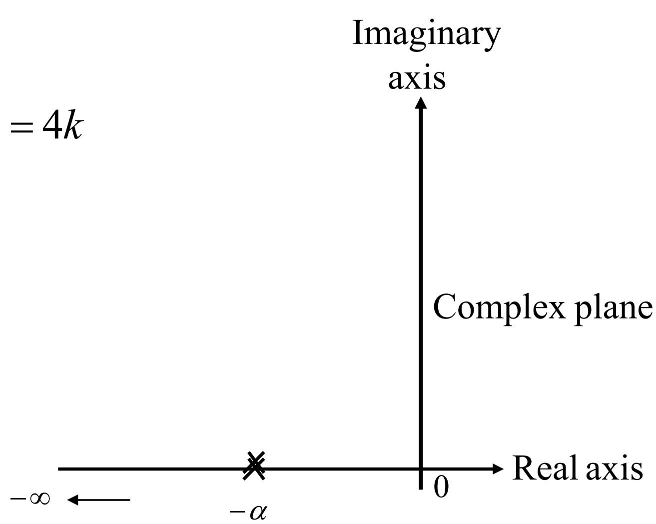


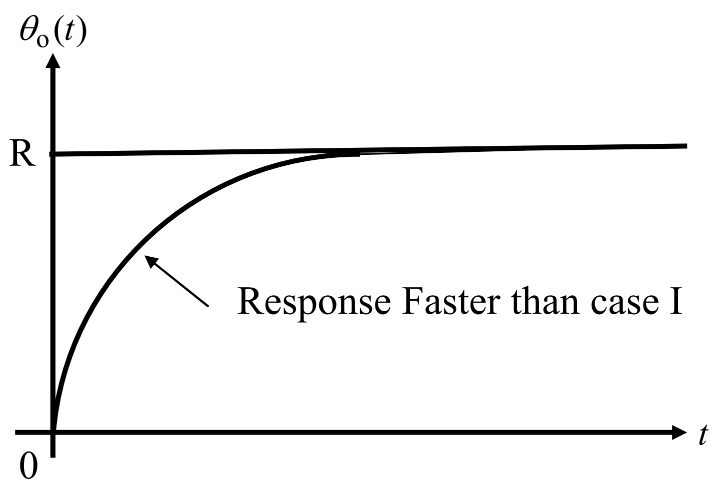
Case II: Repeated Roots.

$$r_1, r_2 = -\alpha, -\alpha \quad ; \quad a^2 = 4k$$

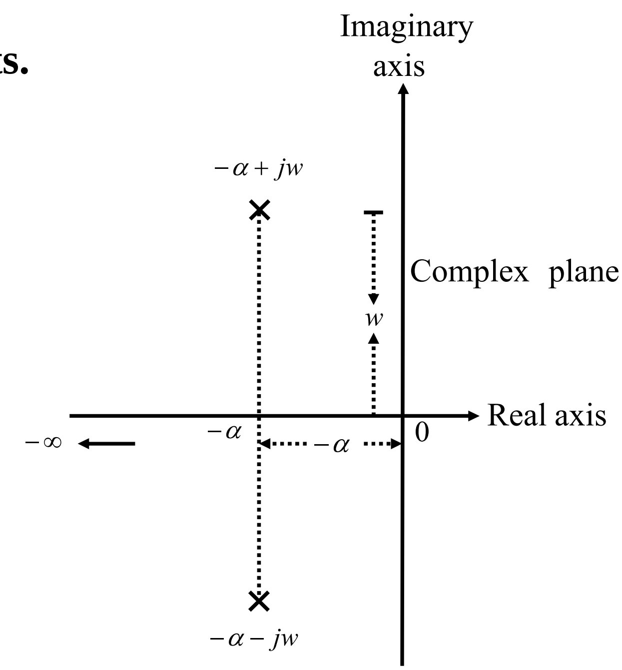
$$(\theta_o)_{Tr} = (C_o + C_1 t)\, e^{-\alpha t}$$

$$\theta_o(t) = R + (C_o + C_1 t)\, e^{-\alpha t}$$





Case III: Complex conjugate Roots.

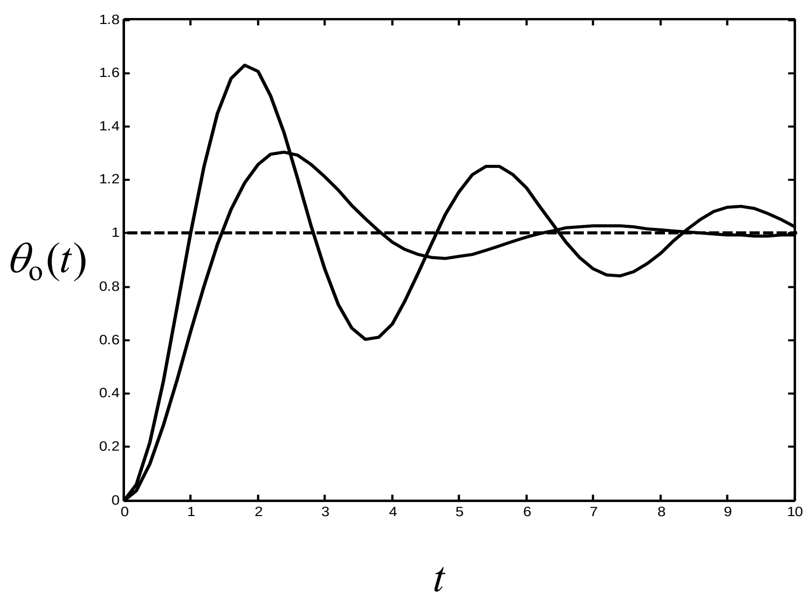


$$r_1, r_2 = -\alpha \pm jw \quad ; \quad a^2 < 4k$$

$$(\theta_o)_{Tr} = e^{-\alpha t}(C_o \cos wt + C_1 \sin wt)$$

$$(\theta_o)_{Tr} = C_2\, e^{-\alpha t} \sin(wt + C_3)$$

$$\theta_o(t) = R + C_2\, e^{-\alpha t} \sin(wt + C_3)$$

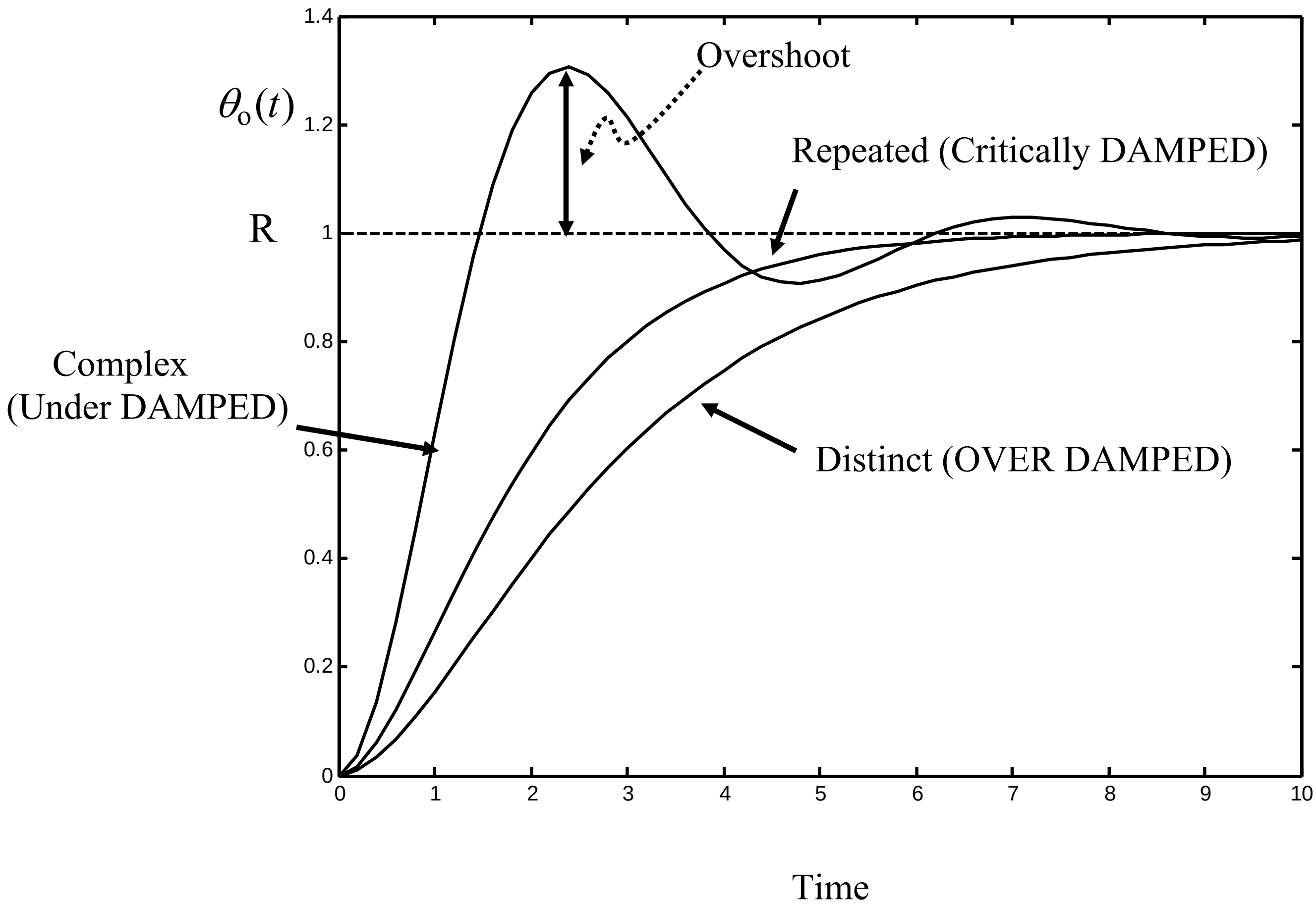


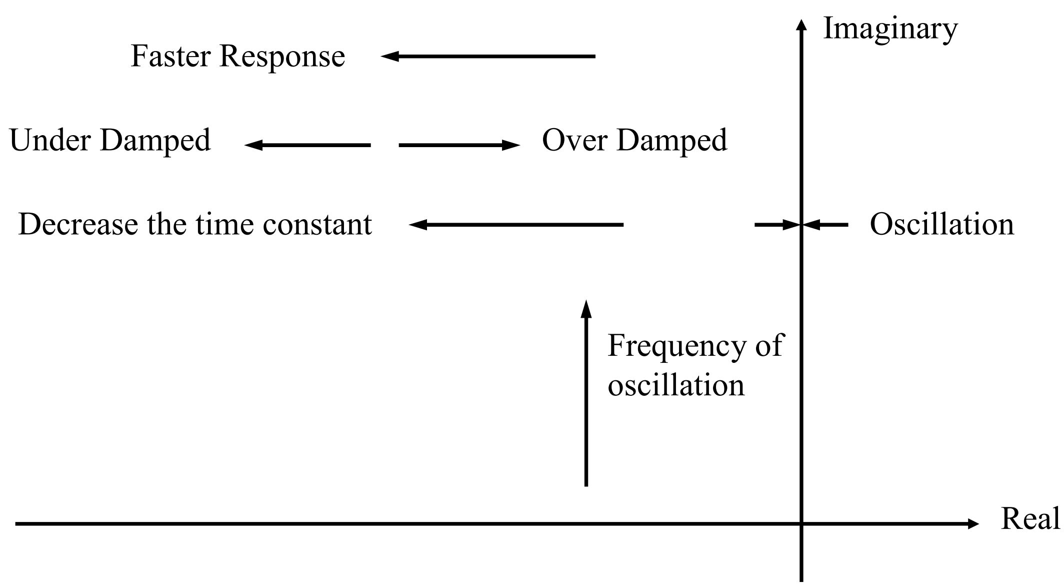
Response of Position Control System

i) Distinct Roots. ⇨ $\theta_o(t) = R + C_o e^{-\alpha_1 t} + C_1 e^{-\alpha_2 t}$

ii) Repeated Roots. ⇨ $\theta_o(t) = R + (C_o + C_1\, t)\, e^{-\alpha_1 t}$

iii) Complex Conjugate Roots. ⇨ $\theta_o(t) = R + C_2\ e^{-\alpha t} \sin(wt + C_3)$





1) Response becomes faster and faster as the roots moved along the –ve real axis. The time constant $\frac{1}{\alpha}$ also decreases progressively.
2) Damping increase as the roots moves away in the –ve real dirction.
3) Frequency of oscillation increases as the roots move away from the real axis (along the imaginary axis dirction).

All control system design methods attempt to shift the roots of the characteristic equation from an undesirable location to a dersirable location.

Ex. A field controlled d.c motor is characterized by the following differential equation.

$$0.5\,\frac{dw(t)}{dt} + w(t) = 1.57\, i_f(t)$$

Where, w(t) is the angular velocity of the motor in radians/second and i_f is the field current in mA.

a) if the motor is supplied with a step input of 100mA what is the steady state speed in r.p.m.

at S.S $\Longrightarrow$ $\dot{w} = 0$

$$w_{ss} = 1.57 * 100 = 157 \text{ rad/second} = 157\frac{60}{2\pi} \text{ r.p.m} = 1499.23 \text{ r.p.m}$$

b) in (a) how much time would be taken by the motor to reach i) 25% , ii) 50 % and iii) 75% of the steady state speed?

Characteristic equation

$(0.5m+1)=0$

$m = -2$

$w_{Tr} = A\mathrm{e}^{-2t}$

$w(t) = 157 + A\mathrm{e}^{-2t}$

$at \quad t=0\ ,\ w(0)=0$

$0=157+A$

$A=-157$

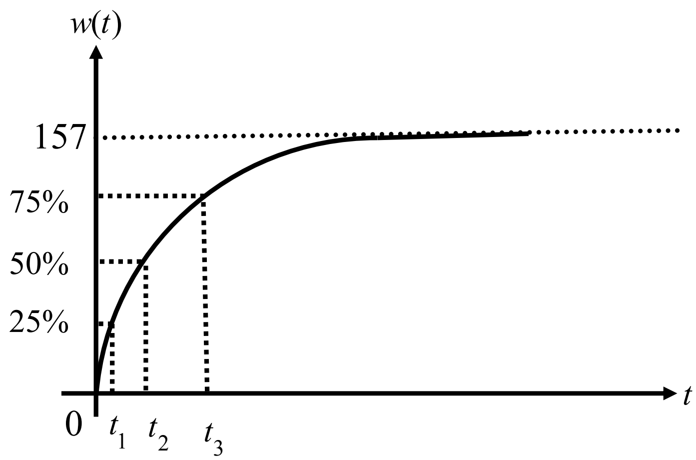
$$w(t) = 157 * (1 - e^{-2t})$$

i) $w(t_1) = 25\%$ of the S.S speed (157 rad/second)

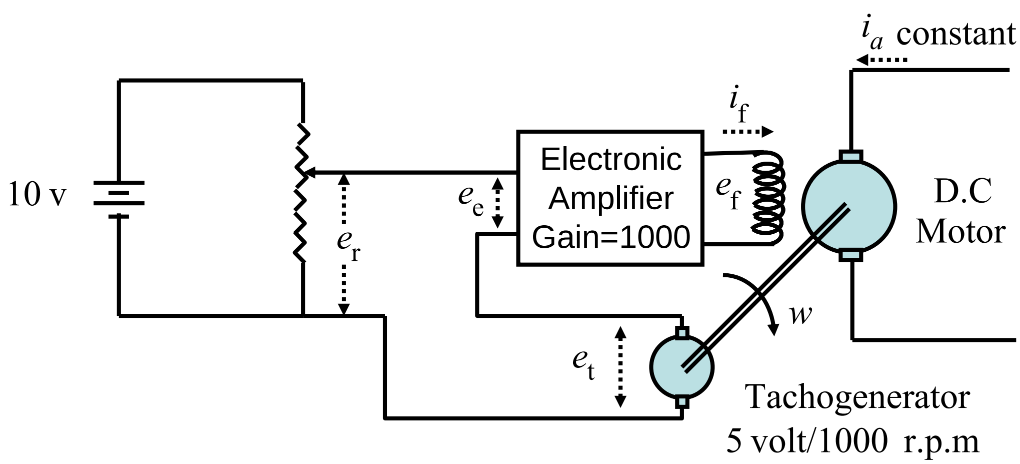
$$\frac{25}{100} * 157 = 157 * (1 - e^{-2t_1}) \Longrightarrow t_1 = 0.1438 \text{sec}$$

$$\frac{50}{100} * 157 = 157 * (1 - e^{-2t_2}) \Longrightarrow t_2 = 0.3466 \text{sec}$$

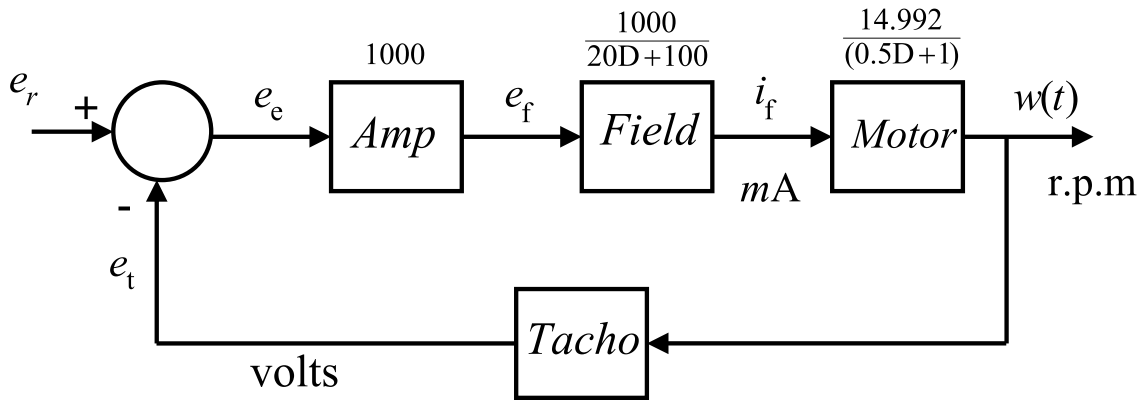
$$\frac{75}{100} * 157 = 157 * (1 - e^{-2t_3}) \Longrightarrow t_3 = 0.6931 \text{sec}$$



c) The above motor is used in a speed control scheme as shown in figure below.



Draw the block diagram of the system and write down the differential equation of the closed loop system. Given that field resistance =100Ω , inductance 20 H.



$$e_f = R_f i_f + L_f \frac{di_f}{dt}$$

$$e_f = 100 * i_f + 20D\, i_f$$

$$e_f = 100 * i_f + 20D\, i_f$$

$$\frac{i_f}{e_f} = \frac{1}{100+20D} = \frac{1000}{100+20D} \text{ mA}$$

from system equation.

$(0.5\ D+1\ )\ w = 1.57\ i_f$

$$\frac{w}{i_f} = \frac{1.57}{0.5\,D+1} \text{ in rad/second} = \frac{14.992}{0.5\,D+1} \text{ in r.p.m}$$

d) calculate the setting of the potentiometer to get a steady state speed of i) 900 r.p.m , ii) 1100 r.p.m.

$$G = 1000 * \frac{1000}{20D+100} * \frac{14.992}{0.5D+1} = \frac{1499200}{(D+2)(D+5)}$$

H=0.005 volt/r.p.m

$$\frac{w(t)}{e_r(t)} = \frac{G}{1+GH} = \frac{1499200}{D^2+7D+7506}$$

$$\frac{d^2w(t)}{dt^2} + 7\frac{dw(t)}{dt} + 7506w(t) = 1499200\, e_r(t)$$ Differential Equation of the C.L System

i) For $w(t)\big|_{t=\infty} = 900$ r.p.m

$D = D^2 = 0$ at steady state

$$w(t)_{\text{s.s}} = 900 = e_{\text{r}} \frac{1499200}{7506}$$

$e_{\text{r}} = 4.506$ volts

Potentiometer factor = 0.4506

ii) For $w(t)_{\text{s.s}} = 1100$ r.p.m $\qquad e_{\text{r}} = 5.507$ volts

Potentiometer factor = 0.5507

e) if the amplifier gain suddenly decreases by 25% what would be the range in the motor speed if it was earlier running at 900 r.p.m.

when the motor is running at 900 r.p.m

$e_{\text{r}} = 4.506$ volts

Amplifier gain =750

$$\frac{w(t)}{e_{\text{r}}(t)} = \frac{1124400}{\text{D}^2 + 7\text{D} + 5632}$$

At S.S $w(t)\big|_{t=\infty} = \dfrac{4.506 * 1124400}{5632} = 899.6$ r.p.m

Ex. A small electric oven is known to have a first order differential equation as its describing equation. When the rated input of 20 volt is applied to the oven at 25^{o}C, the steady state temperature is found to be $1225\,^{\text{o}}$C and a temperature of $625\,^{\text{o}}$ is reached in 30 seconds.

a) Write down the differential equation of the oven.

General first order differential equation.

$$\frac{dT(t)}{dt} + aT(t) = b\, e_1(t)$$

Oven Temperature (↑ $T(t)$) Oven Voltage (↑ $e_1(t)$)

$t = 0 \quad , \quad T = 25^{\text{o}}$

$t = 30 \quad , \quad T = 625^{\text{o}}$

$t = \infty \quad , \quad T = 1225^{\text{o}}$

$T_{SS} = \frac{b}{a} e_1$; $T_{tr} = Ae^{-at}$

$T_{total} = Ae^{-at} + \frac{b}{a} e_1$

Initial condition at $t=0$, $T(t)=25$

$25 = A + \frac{b}{a} e_1$, $A = 25 - \frac{b}{a} e_1$

$$T(t) = (25 - \frac{b}{a} e_1) * e^{-at} + \frac{b}{a} e_1$$

At $t=\infty$ (steady state) ; $T(t) = 1225\ \mathrm{C^o}$

$$1225 = \frac{b}{a} 20$$

At $t=30$, $T(t)=625$

$$625 = (25 - \frac{b}{a} 20) * e^{-30a} + \frac{b}{a} 20$$

$$625 = (25 - 1225) * e^{-30a} + 1225$$

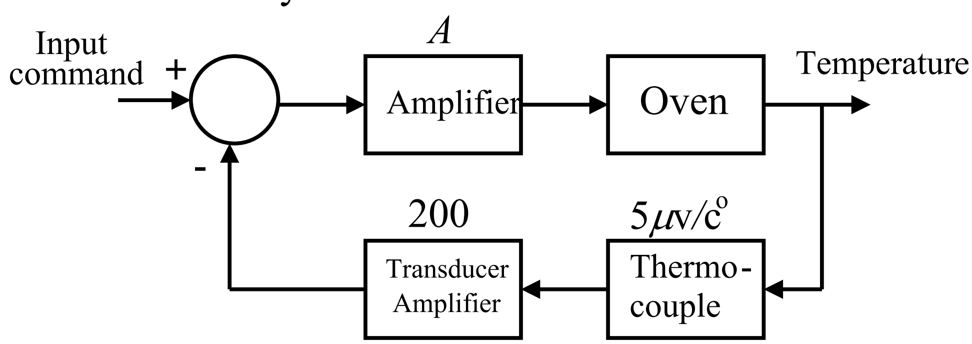
$a = 0.0231049$

$b = 1.4151755$

Oven equation is

$$\frac{dT(t)}{dt} + 0.023T(t) = 1.415\, e_1(t)$$

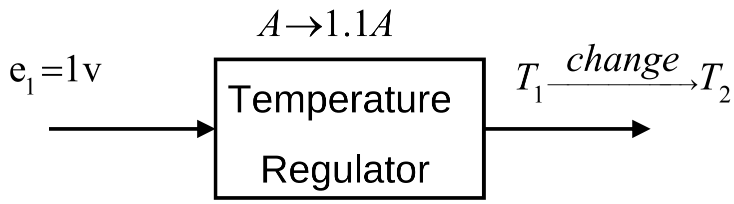
b) It is now required to control the temperature of the oven by a close loop feedback system as shown in figure below. obtain the differential equation of the overall system.



$$G \equiv A \frac{1.415}{\mathrm{D} + 0.023} \quad ; \quad H = 5 * 10^{-6} * 200 = 10^{-3}$$

$$\frac{T(t)}{e_1}=\frac{G}{1+GH}=\frac{1.415A}{\mathrm{D}+0.023+A*1.415*10^{-3}}$$

c) Calculate the value of '*A*' such that if '*A*' increases by 10% the steady state change in the oven temperature does not exceed $0.5\,\mathrm{C}^{\mathrm{o}}$ for $\mathrm{e}_1=1$ volts



$$T_2-T_1=0.5$$

$$\frac{1.415*1.1*A}{0.023+1.415*1.1*10^{-3}*A}-\frac{1.415*A}{0.023*1.415*10^{-3}*A}=0.5$$

$$1.10122375*A^2+34.172251*A+264.5=3254.5*A$$

$$1.10122375*A^2-3220.32775*A+264.5=$$

$$A=\frac{3220.3277\pm\sqrt{(3220.3277)^2-4*1.10122375*264.5}}{2*1.10122375}$$

$$A=2924.158$$

d) Calculate the time constant of the close loop system for the value of '*A*' calculated in part (c).

$$\frac{T(t)}{e_1}=\frac{1.415A}{\mathrm{D}+0.023+A*1.415*10^{-3}}=\frac{K}{(\mathrm{D}+a)}$$

$$a=0.023+2924.153*1.415*10^{-3}=4.160676$$

Time constant$=T=\dfrac{1}{a}=0.240345$ sec

e) What is the range of input command in volts required for controlling the temperature from $100\,\mathrm{C}^{\mathrm{o}}$ to $1000\,\mathrm{C}^{\mathrm{o}}$.

At S.S $T=\dfrac{1.415*2924.153}{0.023+2924.153*1.415*10^{-3}}e_1=\dfrac{4137.676496}{4.1606764}e_1$

$T = 994.472 * e_1$

at $T = 100$

$100 = 994.472 * e_1$, $e_1 = 0.100555$ volt

at $T = 1000$

$1000 = 994.472 * e_1$, $e_1 = 1.00555$ volt

The range of input command is $0.100555 \le e_1 \le 1.00555$