أمثلة تطبيقية على التوازن

* Example (1)

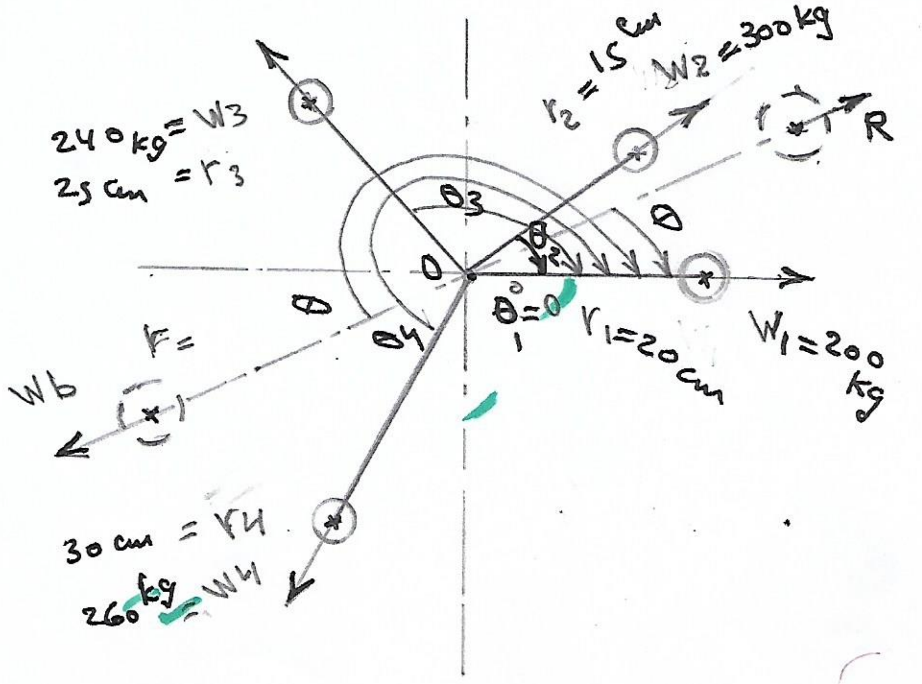
The weights of four masses W_1, W_2, W_3, and W_4 are (200 kg), (300 kg), (240 kg) and (260 kg) respectively. The corresponding radii of rotation are (20 cm), (15 cm), (25 cm) and (30 cm) respectively, and the angles between these masses are (45°), (75°) and (135°). Find the position and magnitude of the balancing weight required, if it's redius of rotation is (20 cm), and all masses in the same plane.

* Solution:

1- Tablete the given date:

Weights	mass (kg)	radius (cm)	angles (deg)	Wr kg.cm
W_1	200	20	0°	4000
W_2	300	15	45°	4500
W_3	240	25	$\theta_3 = 45° + 75° = 120°$	6000
W_4	260	30	$\theta = 45° + 75° + 135° = 255°$	7800
W_b	=?	20	$\theta_b = ?$	20 Wb

2- As shown in Fig(a)



Fig(a)

The magnitude and position (direction) of the balancing weight may be found by two ways :

1, Analitical Method: الطريقة التحليلية

is done by resolving the centrifugal forces horizontally and vertically, and find there resultant, as follow:

$$\Sigma R_H = W_1 r_1 \cos\theta_1 + W_2 r_2 \cos\theta_2 + W_3 r_3 \cos\theta_3 + W_4 r_4 \cos\theta_4$$

$$\Sigma R_V = W_1 r_1 \sin\theta_1 + W_2 r_2 \sin\theta_2 + W_3 r_3 \sin\theta_3 + W_4 r_4 \sin\theta_4$$

$$\therefore \Sigma R_H = 200 * 20 * \cos 0^\circ + 300 * 15 * \cos 45^\circ + 240 * 25 \times \cos 120^\circ + 260 * 30 * \cos 255^\circ$$

$= 2123$ kg.cm.

$$\Sigma R_V = 200 * 20 \sin 0^\circ + 300 * 15 * \sin 45^\circ + 240 * 25 \times \sin 120^\circ + 260 * 30 * \sin 255^\circ$$

$= \cancel{811}$ 2862,7 kg.cm

$\therefore$ The resultant c. force.

$$R = \sqrt{(\Sigma R_H)^2 + (\Sigma R_V)^2} = \sqrt{(2122)^2 + (\cancel{811}\ 28637)^2} = 2200 \text{ kg.cm}$$

But we know that

$R = W * r = 2200$ kg.cm.

$$\therefore W = \frac{R}{r} = \frac{2200}{20} = \boxed{110 \text{ kg}} \rightarrow \text{balance Weight}$$

and $\tan\theta^\circ = \frac{\Sigma V}{\Sigma H} = \frac{811}{2122} = 0{,}384$

$$\therefore \theta_R^\circ = \boxed{21^\circ} \rightarrow \text{position of resultants force.}$$

$\therefore \theta_B = 180^\circ + 21^\circ = 201^\circ \rightarrow$ Position of balancing weight, which is equal and opposite direction of resultant

$W_b = \leftrightarrows R$

* 2- Graphical Method — الطريقة البيانية

The magnitude and position of balancing weight may be found ass follow:

* I, Draw space diagram; for all masses and there positions, with suitable scale

$\because W_1 r_1 = 200 * 20 = 4000$ kg.cm

$W_2 r_2 = 300 * 15 = 4500$ kg.cm

$W_3 r_3 = 240 * 25 = 6000$ kg.cm

$W_4 r_4 = 260 * 30 = 7800$ kg.cm.

II, Take scal: 1000 kg.Cm = 5 mm

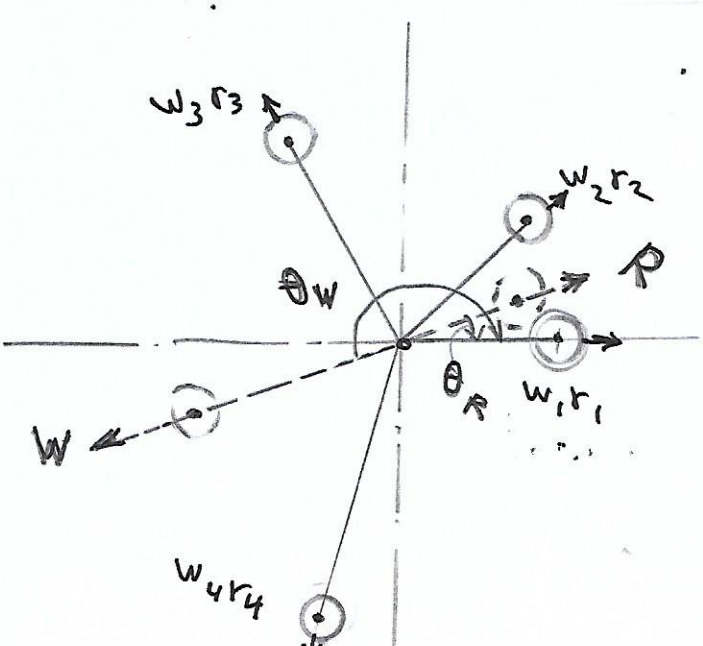
$\therefore K_e = \dfrac{W_1 r_1}{\overline{W_1 r_1}} = \dfrac{W_1 r_1}{W_1 r_1 * \frac{5}{1000}} = \dfrac{4000}{4000 * \frac{5}{1000}} = 200$

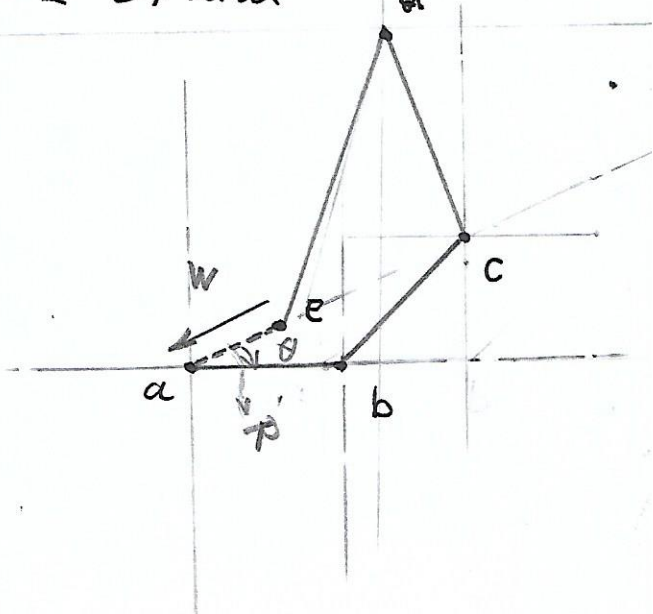
$\therefore \overline{W_1 r_1} = \dfrac{W_1 r_1}{K_e} = \dfrac{4000}{200} = 20$ mm

$\overline{W_2 r_2} = \dfrac{W_2 r_2}{K_e} = \dfrac{4500}{200} = 22,5$ mm

$\overline{W_3 r_3} = \dfrac{W_3 r_3}{K_e} = \dfrac{6000}{200} = 30$ mm

$\overline{W_4 r_4} = \dfrac{W_4 r_4}{K_e} = \dfrac{7800}{200} = 39$ mm





III) Take pol (a), and from a draw all the Vector $W_1 r_1 = 20$ mm length, and with angle $\theta = 0$, ---

then from Vector diagram measure

$W \cdot r = \bar{ae} = 11 \text{ mm} * K\ell = 11 * 200 = 2200 \text{ kg.cm}$

$\therefore W = \frac{2200}{r} = \frac{2200}{20} = \boxed{110 \text{ kg}} \rightarrow$ balancing weight

also we measure angle θ between $\bar{ae}$ and $\bar{ab}$

$\therefore \theta_R = 21^\circ \rightarrow$ angle of inclination of R-froce

$\therefore$ inclination of balancing weight = and opposit direction.

$\theta_W = 180^\circ + 21^\circ = 201^\circ$

*· example (2):

A shaft carries four rotating masses A, B, C and D in this order along its axis. The mass A may be assumed to be concentr-ated at radius of (18 cm), B at (24 cm), C at (12 cm), and D at (15 cm). The weights of B, C and D are (30 kg), (50 kg), and (40 kg) res-pectively. The planes containing B, and C are (30 cm) aparat. The angular spacing of the planes containing C and D are (90°, and (210°) respectively relative to B mea-sured in the same sense. If the shaft and masses are to be in complete dynamic balance: Find:

1- The weight and angular position of mass (A)

2- The position of the planes A and D.

Solution:

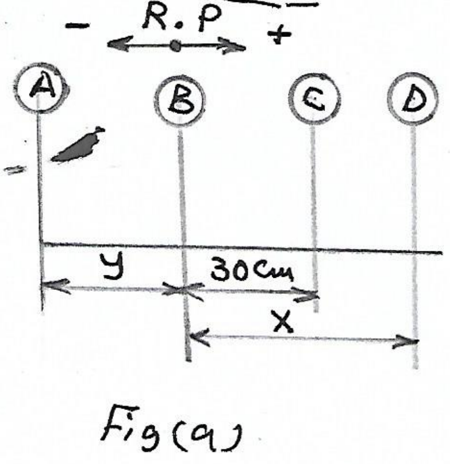


Fig (a)

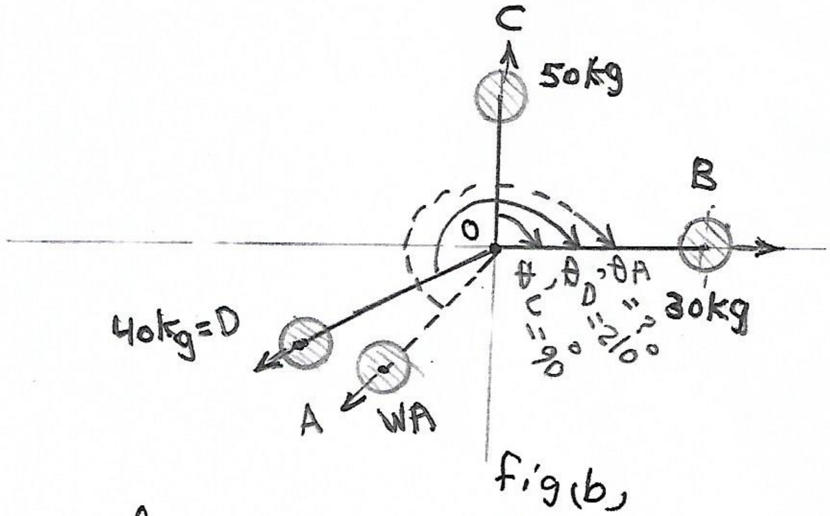


fig (b)

1) Assume; plane B a reference plane.

X - distance between plane D and B

y - distance between plane A and B

W_A - Weight of mass A

$\theta_A = \measuredangle AOB$ - angular position of mass A.

as shown in fig (a) and fig (b)

2) Tabulate the given date as follow:

Plane -1-	Weight W (kg) 2	radius r (cm) 3	centrifugal force $\div \omega^2/g$ W.r (kg.cm) -4-	Distance from Plane B L (cm) -5-	Couple $\div \omega^2/g$ W.r.L (kg.cm^2) -6-	θ
A	W_A	18	18 W_A	-y	-18 $W_A \cdot y$	θ_A
B (R-P)	300	24	720	0	0	0
C	50	12	600	30	18000	90°
D	40	15	600	+X	600X	[illegible]

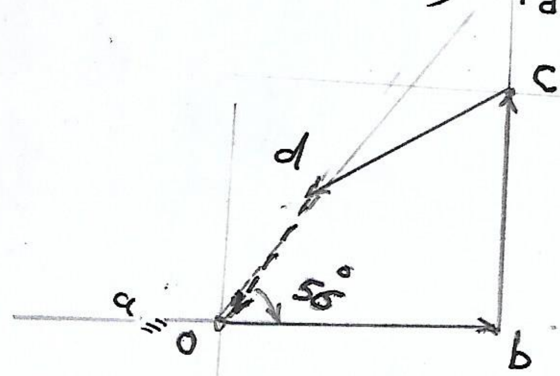
3- The weight of mass A may be determine by drawing the force polygon from column (4) of the table, with suitable scale as shown in fig (c)

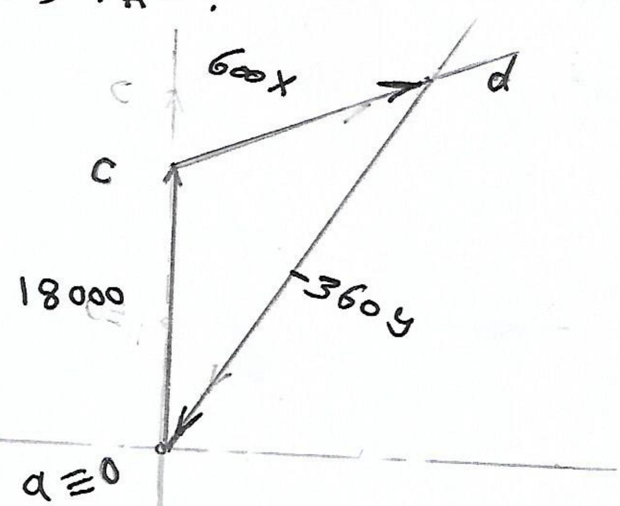
I) assume scale 20kg/cm = 1 mm or $\rightarrow$ 1 kg.cm = 20 mm

$$\therefore K_F = \frac{W_B \cdot r_B}{\overline{W_B \cdot r_B}} = \frac{720}{720 * \frac{1}{20}} = \frac{720}{36} = \boxed{20}$$

II) Take pole o, and from o draw vector ob // and = to = 36 mm, by same way draw other forces as shown in fig (c)

where F_c = 30 mm, F_d = 30 mm, F_A = ?





Fig(c) Force polygon

$\therefore$ From Fig (C) we get

(1) The closing side do = $W_A r_A$ = 18 W_A = 18 mm * K_F = 18*20

$$\therefore W_A = \frac{18 * 20}{18} = \boxed{20 \text{ kg}}$$

2- By measuring the angle between vectors od and ob we get $\measuredangle dob = 56^\circ$

$\therefore \theta_A = 180 + 56 = \boxed{236^\circ}$

4 - Position of plane A, D may be determined by drawing Couple Polygon with suitable scale from Column (6) as follow.

I) assume scale for couple $600\ kg.cm^2 = 1\ mm$

$$\therefore k_{couple} = \frac{W_c r_2 . L_c}{\overline{W_c . r_c . L_c}} = \frac{18000}{18000 * \frac{1}{600}} = \frac{18000}{30} = \boxed{600}$$

II) Take pol o, and from o draw Couple $\bar{oc} = 30\ mm$ and // oc,

From c draw // OD
From O draw // OA

$\times d$ measur $cd = 37.5\ mm * K_c$, $da = 58.75\ mm * K_c$

$\therefore 600X = cd = 37.5 * 600 = 22500\ kg.cm^2$

$$\therefore X = \frac{22500}{600} = 37.5\ cm.$$

and

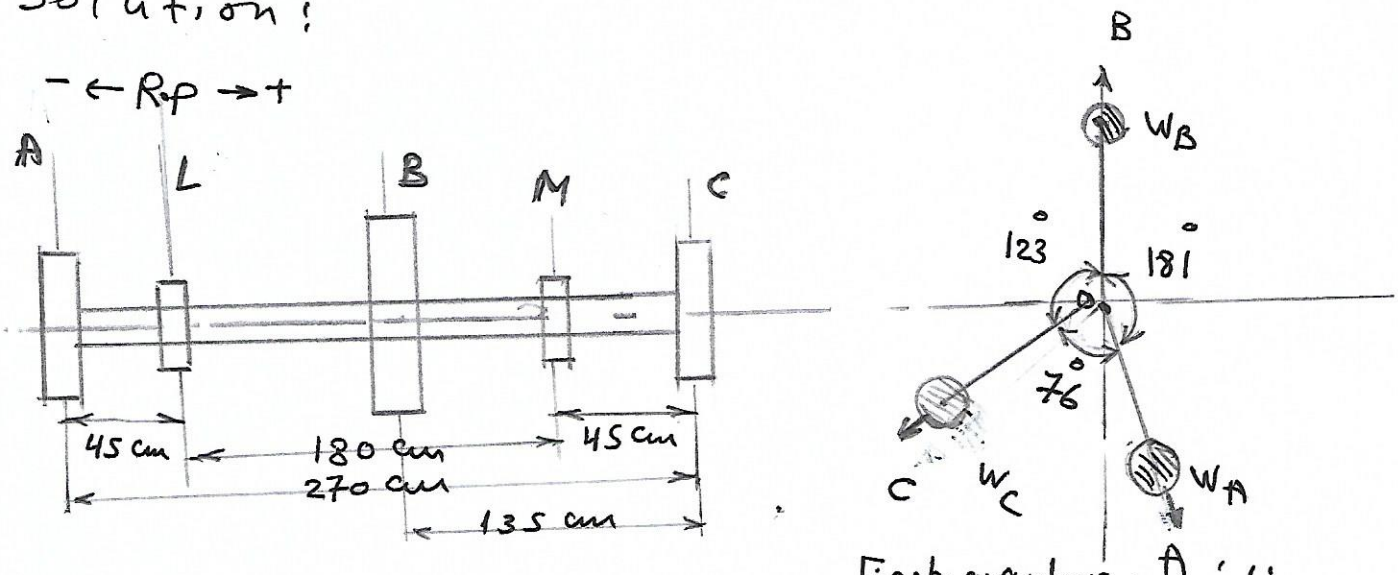
$-18 * 20 * y = da = 58.75 * 600 = 35250\ kg.cm^2$

$$\therefore y = \frac{35250}{18 * 20} = -97.9\ cm.$$ to the left of R-plane.

Example (3) :

A shaft is supported in bearings (180 cm) apart and projects (45 cm) beyond bearings at each end. The shaft carries three pulleys one at each end and one at the middle of the length. The end pulleys weight (48 kg) and (20 kg), and their centre of gravity are (1.5 cm) and (1.25 cm) respectively from the shaft axis. The centre pulley weights (56 kg) and its centre of grivety is (1.5 cm) from the shaft axis. If the pulleys are arranged so as to give static balance. Determine a) relative angular positions of the pulleys, and b) dynamic forces produced on the bearings when the shaft rotates at (n=300 r.p.m).

Solution:



Fig(b) angular position

fig (a) show the position of shaft and pulleys

1- Take plane L-R-P.

2) Tabulate the given dateias follow:

plans (1)	Weight (W) kg - 2 -	radius (r cm) 3	Cent. force ÷ ω^2/g (W.r) kg.cm (4)	Distance from p-L (L) cm	Couple ÷ ω^3/g (W.r.L) kg.cm^2	angles (θ °)	
A	48	1.5	72	-45	-3246		
L (R.P)	W_L	r_L	$W_L r_L$	0	0		
B	56	1.5	84	90	7500		
M	W_M	r_M	$W_M r_M$	180	$180 W_M r_M$		
C	20	1.25	25	225	5625		

a) For determing the angular position of the pulleys Take the system in static balance, as shown in fig(a), and assume the weight W_B is vertical.

Then draw force polygon with suitable scale: assume 2,5 kg = 1 mm from Column (4)

$$\therefore K_{fw} = \frac{W_C \cdot r}{W_C \cdot r \times} = \frac{25}{25 \times \frac{1}{2,5}} = \frac{25}{10} = \boxed{2,5}$$

∴ Take pole o and draw υ_c = 10 mm and in same direction, so the other forces by same way.

∴ $W_C \cdot r$ = 10 mm, $W_A \cdot r$ = 28.8 mm, $W_B \cdot r$ = 33.6 mm

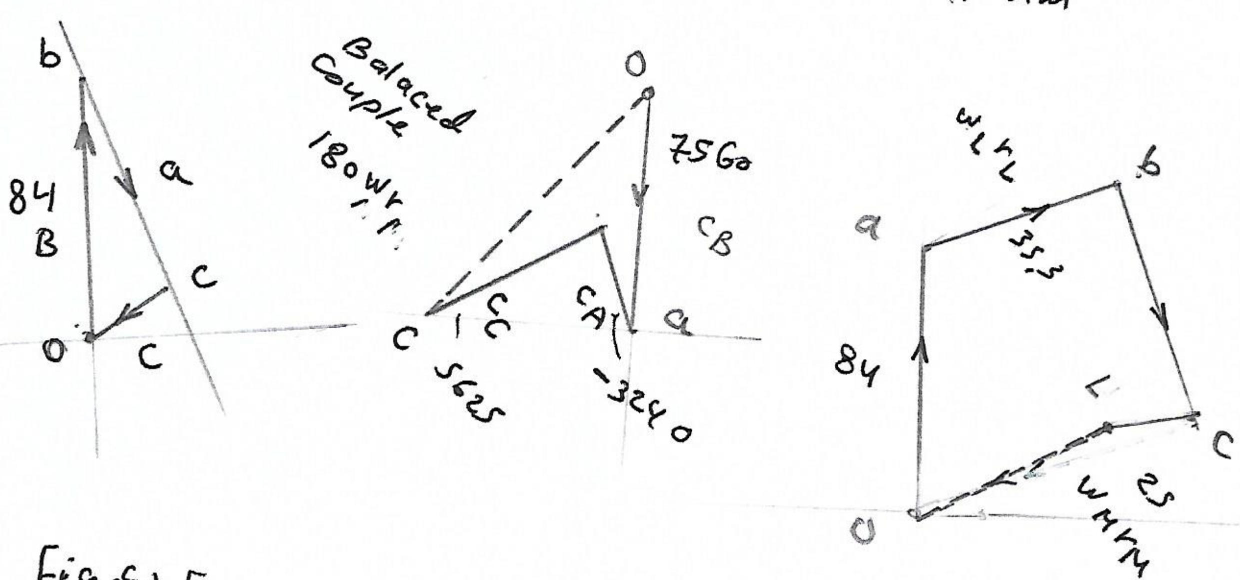


Fig (c) F-polygon.

From fig(c) we get

θ AoB = 161°

θ CoB = 75°

b - for determing the dynamic forces on bearings (reactions), draw the couple polygon from Column (6), with suitable Scale.

where | Coupl A = -3240
| Coupl B = +7560
Coup C = +5625

∴ Take a scal for Couple 224 kg.cm² = 1 m

$$\therefore k_{cupl} = \frac{\text{Coupl } B}{\text{Coupl } \bar{B} \times \frac{1}{224}} = \frac{7560}{7560 * \frac{1}{224}} = \frac{7560}{33{,}8} = \boxed{224}$$

∴ Take pol o and draw CB = 33,8 mm // oA, and the other couples by same way, as in fig(d),

$C_A = 14{,}5$ mm , $C_C = 23$ mm

∴ The balanced couple at bearing M

$180\, W_L r_L = 9710$ kg.cm.

∴ $W_L r_L = 53.3$ kg.cm $= W_M r_M = 53{,}3$ kg.cm $= F_B$

ارتكاز [illegible] Dynamic force at bearing L
قوة (mrω²)

$$F_C = \frac{W_L}{g} \times \omega^2 r_L = \frac{W_L r_L}{g}\left(\frac{2\pi n}{60}\right)^2 = \frac{53.3}{981} \times \left(\frac{2 * 3.14 * 300}{60}\right)^2$$

$= 54{,}34$ kg. → dynamic force on bearing L

* For determing the dynamic force on bearing (M) draw force polygon from Coulmn (4)

Take scale

with suitable scale:
Take scale 2,4 kg, cm = 1 mm

$$\therefore K_{f_c} = \frac{F_{cB}}{f_{cB} \cdot \times \frac{1}{2,4}} = \frac{84}{84 * \frac{1}{2,4}} = \frac{84}{35} = \boxed{2,4}$$

∴ Take pole o, and from o draw $f_{cB} = 35$ mm // WB, then the other force draw by same way as shown in fig(e)

∴ Dynamic force a bearing M

$$\frac{W_M}{g} \omega^2 r_m = \frac{W_M r_m}{g} \left(\frac{2\pi n}{60}\right)^2$$

$$= \frac{53.3}{981} \left(\frac{2 * 3,14 * 300}{60}\right)^2 = 54.34 \text{ kg}$$ → dynamic force on bearing M