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Balancing of rotating masses

1. Balancing of a single rotating mass by a single mass rotating in the same plane

Let

M ….. Rotating mass at ω (rad/sec)

r_1 …... radius of rotating mass

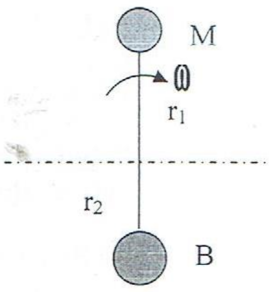
The centrifugal force exerted by M on the shaft

$F_{c1} = M\,\omega^2\,r_1$

This force can be balanced by a single mass attached in the same plane of rotation, such that the centrifugal forces due to the two masses are equal and opposite.

$F_{c2} = B\,\omega^2\,r_2$

$\therefore\; M\,\omega^2\,r_1 = B\,\omega^2\,r_2 \quad\Rightarrow\quad M\,r_1 = B\,r_2$



2. Balancing of a single rotating mass by two masses rotating in different planes

The centrifugal force (F_c) exerted by the mass (M) in the plane A is $F_c = M\,\omega^2\,r$

The centrifugal force (F_{c1}) exerted by the mass (m_1) in the plane K is $F_{c1} = m_1\,\omega^2\,r_1$

The centrifugal force (F_{c2}) exerted by the mass (m_2) in the plane N is $F_{c2} = m_2\,\omega^2\,r_2$

(a)

$\because\; F_c = F_{c1} + F_{c2} \;\Rightarrow\; M\,\omega^2\,r = m_1\,\omega^2\,r_1 + m_2\,\omega^2\,r_2$

$M\,r = m_1\,r_1 + m_2\,r_2$ … (1) (static balance)

Take moment about point (P)

$F_{c1} * L = F_c * L_2$

$m_1 * \omega^2 * r_1 * L = M * \omega^2 * r * L_2$

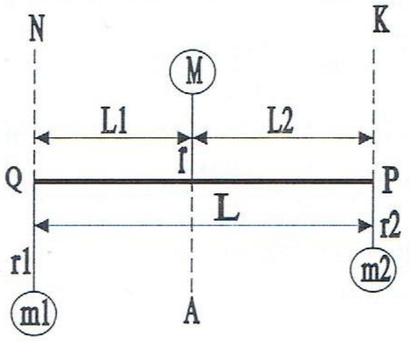
$$\therefore m_1 * r_1 = \frac{M * r * L_2}{L} \quad ....(2)$$

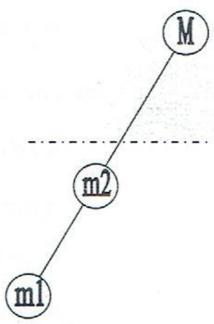
Same way take moment about point (Q)

$F_{c2} * L = F_c * L_1$

$m_2 * \omega^2 * r_2 * L = M * \omega^2 * r * L_1$

$$\therefore m_2 * r_2 = \frac{M * r * L_1}{L} \quad ....(3)$$





For dynamic balance equations (2) and (3) must be satisfied.

(b)

$\because\; F_c + F_{c2} = F_{c1} \;\Rightarrow\; M\,r + m_2\,r_2 = m_1\,r_1$

Take moment about point (P)

$F_{c1} * L = F_c * L_2$

$m_1 * \omega^2 * r_1 * L = M * \omega^2 * r * L_2$

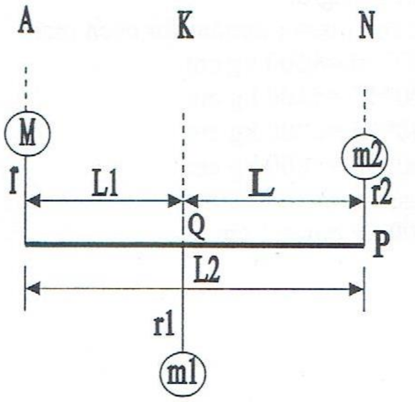
$$\therefore m_1 * r_1 = \frac{M * r * L_2}{L} \quad ....(2)$$

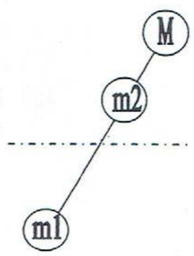
Same way take moment about point (Q)

$F_{c2} * L = F_c * L_1$

$m_2 * \omega^2 * r_2 * L = M * \omega^2 * r * L_1$

$$\therefore m_2 * r_2 = \frac{M * r * L_1}{L} \quad ....(3)$$





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3. Balancing of several masses rotating in the same plane

This type can be solved either **analytically** or **graphically** and the following example will explain the two methods.

Ex1/ The weight of four masses m_1, m_2, m_3 and m_4 are 200 kg, 300 kg, 240 kg and 260 kg respectively. The corresponding radii of rotation are 20 cm, 15 cm, 25 cm and 30 cm respectively and the angles between successive masses are 45°, 75° and 135°. Find the position and magnitude of the balance weight required if its radius of rotation is 20 cm.

Solution

(1) **Analytical method**

(a) By resolving centrifugal force (F_c) for each mass horizontally and vertically.

$$\sum H = m_1 r_1 \cos\theta_1 + m_2 r_2 \cos\theta_2 + m_3 r_3 \cos\theta_3 + m_4 r_4 \cos\theta_4$$

$$= 200*20*\cos 0 + 300*15*\cos 45 + 240*25*\cos 120 + 260*30*\cos 255 = 2122\ kg.cm$$

$$\sum V = m_1 r_1 \sin\theta_1 + m_2 r_2 \sin\theta_2 + m_3 r_3 \sin\theta_3 + m_4 r_4 \sin\theta_4$$

$$= 200*20*\sin 0 + 300*15*\sin 45 + 240*25*\sin 120 + 260*30*\sin 255 = 811\ kg.cm$$

(b) Find out the resultant force (R) for all masses.

$$R = \sqrt{\left(\sum H\right)^2 + \left(\sum V\right)^2} = \sqrt{(2122)^2 + (811)^2} = 2200\ kg.cm$$

The angle which the resultant force makes with horizontal

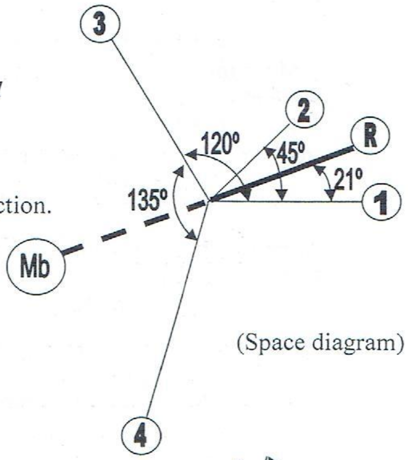
$$\theta = \tan^{-1}\frac{\sum V}{\sum H} = \tan^{-1}\frac{811}{2122} = 21^\circ$$

(c) The balancing force = resultant force (R) but in opposite direction.

$$\because M_b r = R \Rightarrow M_b = \frac{R}{r} = \frac{2200}{20} = 110\ kg$$

$\therefore$ *the angle of balancing weight*

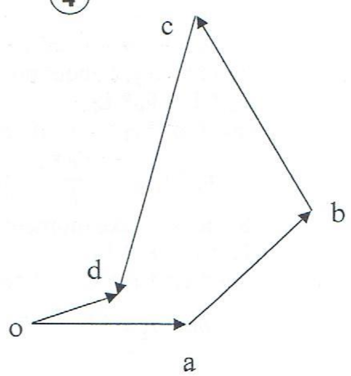
$$\theta_b = 180 + 21 = 201^\circ$$



(Space diagram)

(2) **Graphical method**

(a) Draw space diagram
(b) Calculate the mass moment for each mass
$m_1 r_1$ = 200*20 =4000 kg cm
$m_2 r_2$ = 300*15 =4500 kg cm
$m_3 r_3$ = 240*25 =6000 kg cm
$m_4 r_4$ = 260*30 =7800 kg cm
(c) Draw mass moment diagram
Scale 2000 kg cm = 1 cm



(Mass moment diagram)

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$\overline{oa} = 4000/2000 = 2\ cm\ \angle 0^\circ$

$\overline{ab} = 4500/2000 = 2.25\ \ cm\ \angle 45^\circ$

$\overline{bc} = 6000/2000 = 3\ cm\ \angle 120^\circ$

$\overline{cd} = 7800/2000 = 3.9\ cm\ \angle 255^\circ$

$\therefore \overline{od}$ *represent the resultant force* (R)

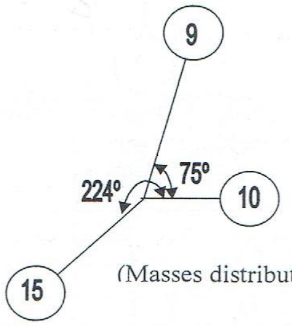
By measurement vector od = 1.1 cm
Resultant force (od) = 1.1*2000 = 2200 kg cm
Balancing force = Resultant force but in opposite direction

$$M_b r = R \Rightarrow M_b = \frac{R}{r} = \frac{2200}{20} = 110\ kg$$

By measurement $\theta_b = 201^\circ$ *from* m_1

Ex2/ Three masses are attached to a shaft as follows: 10 kg at 90 mm radius, 15 kg at 120 mm radius and 9 kg at 150 mm radius. The masses to be arranged so that the shaft is in complete balance. Determine the angular position of masses relative to 10 kg mass. All the masses are in the same plane.

Solution



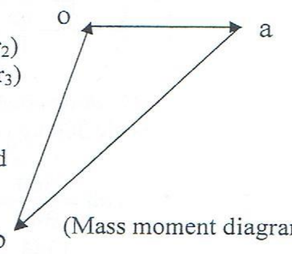
(Masses distribution)

(1) Find out the mass moment for each mass
$m_1r_1 = 10*90 = 900$ kg mm
$m_2r_2 = 15*120 = 1800$ kg mm
$m_3r_3 = 9*150 = 1350$ kg mm

(2) To draw mass moment diagram
Scale 450 kg mm = 1 cm
oa = 900/450 = 2 cm (at 0°)
ab = 1800/450 = 4 cm
bo = 1350/450 = 3 cm

(a) from (o) draw (vector oa) to represent (m_1r_1)
(b) from (a) draw arc with radius of 4 cm (vector ab) to represent (m_2r_2)
(c) from (o) draw arc with radius of 3 cm (vector bo) to represent (m_3r_3)
(d) vectors ab and bo intersect at point b
(e) join ab and bo
(f) from the diagram the angular position of masses can be determined

By measurement
Angle between 10 kg and 15 kg = 224° Anticlockwise
Angle between 10 kg and 9 kg = 75° Anticlockwise



(Mass moment diagram)

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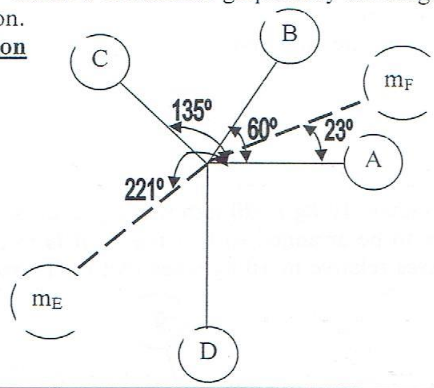
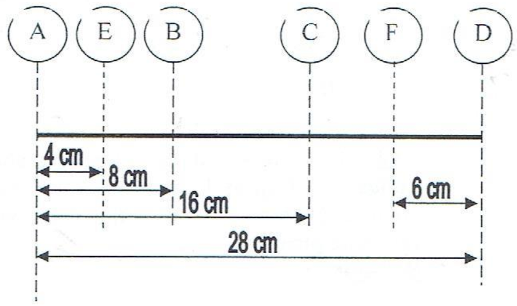
4. Balancing of several masses rotating in different planes

The following example will explain the procedure.

Ex3/ A rotating shaft carry four unbalanced masses 18 kg, 14 kg, 16 kg and 12 kg at radii 5 cm, 6 cm, 7 cm and 6 cm respectively. The 2nd, 3rd and 4th masses revolve in plane 8 cm, 16 cm and 28 cm respectively measured from the plane of the first mass and are angularly located at 60°, 135° and 270° respectively measured anticlockwise from the first mass locating from this mass end of the shaft. The shaft dynamically balanced by two masses both located at 5 cm radii and revolving in plane mid way between there of 1st and 2nd masses and mid way between these of 3rd and 4th masses. Determine graphically the magnitude of the masses and their respective angular position.

Solution





Plane	Mass (m) kg	Radius (r) Cm	Mass moment (mr) kg cm	Distance (L) cm	Couple (mrL) kg cm^2
A	18	5	90	-4	-360
E	m_E	5	$5m_E$	0	0
B	14	6	84	4	336
C	16	7	112	12	1344
F	m_F	5	$5m_F$	18	$90m_F$
D	12	6	72	24	1728

(1) Draw couple diagram

Scale 300 kg cm^2 = 1 cm

$$\therefore oa = \frac{360}{300} = 1.2\ cm\angle 0^\circ, \quad ab = \frac{336}{300} = 1.1\ cm\angle 60^\circ,$$

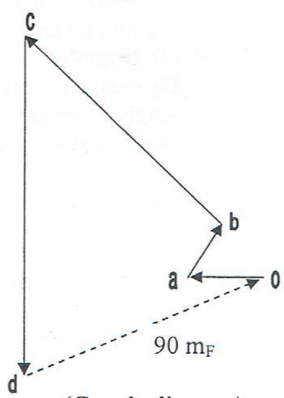
$$bc = \frac{1344}{300} = 4.48\ cm\angle 135^\circ\ \ and$$

$$cd = \frac{1728}{300} = 5.76\ cm\angle 270^\circ$$

From couple diagram

90 m_F = vector do = 4.2 cm

90 m_F = 4.2 * 300 = 1260 kg cm^2



(Couple diagram)

$$\therefore m_F = \frac{1260}{90} = 14\ kg$$

And by measurement the angular position of $m_F = 23°$ from A

(2) Draw mass moment diagram

Scale 30 kg cm = 1 cm

$$\therefore oa = \frac{90}{30} = 3\ cm\ \angle 0^\circ,\ ab = \frac{84}{30} = 2.8\ cm \angle 60^\circ,\ bc = \frac{112}{30} = 3.7\ cm \angle 135^\circ,$$

$$cf = \frac{5*14}{30} = 2.3\ cm \angle 23^\circ \quad and \quad fd = \frac{72}{30} = 2.4\ cm \angle 270^\circ$$

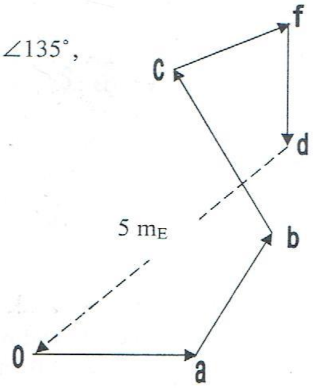
From mass moment diagram

$5m_E$ = vector do = 5.3 cm

$5m_E = 5.4 * 30 = 159$ kg

$$\therefore m_E = \frac{159}{5} = 31.8\ kg$$

And by measurement the angular position of $m_E = 221°$ from A



(Mass moment diagram)

Balancing of rotating masses

(Homework's)

Q1/ A disturbing mass 600 kg are attached to a shaft. The shaft is rotating at a uniform angular velocity ω rad/sec and the radius of rotation of the disturbing mass is 270 mm. The disturbing mass is to be balanced by two masses in two different planes. The radius of rotation of the balancing masses is 450 mm each. The distances between the two planes of the balancing masses is 1.5 m and the distance between the plane of the disturbing mass and one of the planes of the balancing masses is 300 mm. determine

(a) The distance between the plane of disturbing mass and the plane of the other balancing mass.

(b) Magnitude of balancing masses when:

(i) The planes of balancing masses are on the same side of the plane of the disturbing mass

(ii) The planes of the balancing masses are on either side of the plane of the disturbing mass.

Q2/ Four masses are attached to a shaft at planes A, B, C and D at equal radii. The distance of the planes B, C and D from A are 40 cm, 50 cm and 120 cm respectively. The masses at A, B, and C are 60 kg, 45 kg and 70 kg respectively. If the system is in complete balance, determine the mass at D and position of masses B, C and D with respect to A.

Q3/ Three masses of magnitudes 8 kg, 6 kg and 5 kg are attached rigidly to the shaft. The masses are rotating in the same plane. The corresponding radii of rotation are 150 mm, 170 mm and 100 mm respectively. The angles made by these masses with horizontal are 45°, 135° and 240° respectively. Find the:

(a) Magnitude of the balancing mass.

(b) Position of the balancing mass if its radius of rotation is 150 mm.

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Q4/ Four masses A, B, C and D are attached to a rotating shaft with radii 50 mm, 62.5 mm, 100 mm and 75 mm respectively. The distance between planes A and B; between planes B and C and between planes C and D are 600 mm each. The masses B, C and D are 20 kg, 10 kg and 8 kg respectively. If the shaft is in complete balance, then find the:
(a) Magnitude of the mass A.
(b) Angular positions of the four masses.

Q5/ A shaft carry four rotating masses A, B, C and D. The mass A concentrated at a radius of 12 cm, B at 15 cm, C at 14 cm and D at 18 cm. The masses A, C and D are 15 kg, 10 kg and 8 kg respectively. The planes of rotation of A and B are 15 cm apart and of B and C are 18 cm apart. The angle between the masses A and C is 90°. If the shaft is in complete dynamic balance, determine the:
(a) Angles between the masses A, B and D.
(b) Distance between the planes of revolution of C and D.
(c) Mass B.

Q6/ A shaft carries four masses A, B, C and D of magnitude 200 kg, 300 kg, 400 kg and 200 kg respectively and revolving at radii 80 mm, 70 mm, 60 mm and 80 mm in planes measured from A at 300 mm, 400 mm and 700 mm. The angles between the cranks measured anticlockwise are A to B 45°, B to C 70° and C to D 120°. The balancing masses are to be placed in planes X and Y. The distance between the planes A and X is 100 mm, between X and Y is 400 mm and between Y and D is 200 mm. If the balancing masses revolve at a radius of 100 mm, find their magnitudes and angular positions.

Q7/ Four masses A, B, C and D as shown below are to be completely balance.

	A	B	C	D
Mass (kg)	-	30	50	40
Radius (mm)	180	240	120	150

The planes containing masses B and C are 300 mm apart. The angle between planes containing B and C is 90°. B and C make angles of 210° and 120° respectively with D in the same sense. Find:
1. The magnitude and the angular position of mass A; and
2. The position of planes A and D.

Q8/ A, B, C and D are four masses carried by a rotating shaft at radii 100, 125, 200 and 150 mm respectively. The planes in which the masses revolve are spaced 600 mm apart and the mass of B, C and D are 10 kg, 5 kg, and 4 kg respectively. Find the required mass A and the relative angular settings of the four masses so that the shaft shall be in complete balance.

Q9/ A shaft carries four masses in parallel planes A, B, C and D in this order along its length. The masses at B and C are 18 kg and 12.5 kg respectively, and each has an eccentricity of 60 mm. The masses at A and D have an eccentricity of 80 mm. The angle between the masses at B and C is 100° and that between the masses at B and A is 190°, both being measured in the same direction. The axial distance between the planes A and B is 100 mm and that between B and C is 200 mm. If the shaft is in complete dynamic balance, determine:
1. The magnitude of the masses at A and D; **2.** The distance between planes A and D; and
3. The angular position of the mass at D.

Q10/ A shaft have three eccentrics, each 75 mm diameter and 25 mm thick, machined in one piece with the shaft. The central planes of the eccentric are 60 mm apart. The distance of the centers from the axis of rotation are 12 mm, 18 mm and 12 mm and their angular positions are 120° apart. The density of metal is 7000 kg/m^3. Find the amount of out-of-balance force and couple at 600 rpm. If the shaft is balanced by adding two masses at a radius 75 mm and at distances of 100 mm from the central plane of the middle eccentric, find the amount of the masses and their angular positions.

Q11/ Four masses A, B, C and D as shown below are to be completely balanced.

	A	B	C	D
Mass (kg)	--	30	50	40
Radius (mm)	180	240	120	150

The angle between masses B and C is 90° and between C and D is 120° in the same sense. Find the magnitude and the angular position of mass A. All masses rotating in the same plane.

Q12/ A shaft carries four masses A, B, C and D of magnitude 200 kg, 300 kg, 400 kg and 200 kg respectively and revolving at radii 80 mm, 70 mm, 60 mm and 80 mm. The angles between these masses measured anticlockwise are A to B 45°, B to C 70° and C to D 120°. If the balancing masses revolve at a radius of 100 mm, find their magnitudes and angular positions. All masses rotating in the same plane.

Q13/ Four masses *A*, *B*, *C* and *D* are attached to a shaft and revolve in the same plane. The masses are 12 kg, 10 kg, 18 kg and 15 kg respectively and their radii of rotations are 40 mm, 50 mm, 60 mm and 30 mm. The angular position of the masses *B*, *C* and *D* are 60°, 135° and 270° from the mass *A*. Find the magnitude and position of the balancing mass at a radius of 100 mm.

Q14/ Four masses 150 kg, 200 kg, 100 kg and 250 kg are attached to a shaft revolving at radii 150 mm, 200 mm, 100 mm and 250 mm; in planes A, B, C and D respectively. The planes B, C and D are at distance 350 mm, 500 mm and 800 mm from plane A. The masses in plane B, C and D are at an angle 150°, 200° and 300° measured anticlockwise from mass in plane A. It is required to balance the system by placing the balancing masses in planes P and Q which are midway between the planes A and B, and between C and D respectively. If the balancing masses revolve at radius 180 mm, find the magnitude and angular position of the balance masses.