Belt, Rope and Chain Drives

.1. Introduction

The belts or ropes are used to transmit power from one shaft to another by means of pulleys which rotate at the same speed or at different speeds. The amount of power transmitted depends upon the following factors :

1. The velocity of the belt.
2. The tension under which the belt is placed on the pulleys.
3. The arc of contact between the belt and the smaller pulley.
4. The conditions under which the belt is used.

It may be noted that

(*a*) The shafts should be properly in line to insure uniform tension across the belt section.

(*b*) The pulleys should not be too close together, in order that the arc of contact on the smaller pulley may be as large as possible.

(*c*) The pulleys should not be so far apart as to cause the belt to weigh heavily on the shafts, thus increasing the friction load on the bearings.

(*d*) A long belt tends to swing from side to side, causing the belt to run out of the pulleys, which in turn develops crooked spots in the belt.

(*e*) The tight side of the belt should be at the bottom, so that whatever sag is present on the loose side will increase the arc of contact at the pulleys.

(*f*) In order to obtain good results with flat belts, the maximum distance between the shafts should not exceed 10 metres and the minimum should not be less than 3.5 times the diameter of the larger pulley.

.2. Selection of a Belt Drive

Following are the various important factors upon which the selection of a belt drive depends:

1. Speed of the driving and driven shafts,
2. Speed reduction ratio,
3. Power to be transmitted,
4. Centre distance between the shafts,
5. Positive drive requirements,
6. Shafts layout,
7. Space available, and
8. Service conditions.

.3. Types of Belt Drives

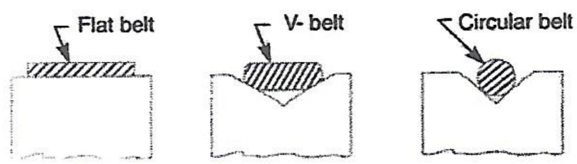
The belt drives are usually classified into the following three groups :

1. ***Light drives.*** These are used to transmit small powers at belt speeds upto about 10 m/s, as in agricultural machines and small machine tools.

2. ***Medium drives.*** These are used to transmit medium power at belt speeds over 10 m/s but up to 22 m/s, as in machine tools.

3. ***Heavy drives.*** These are used to transmit large powers at belt speeds above 22 m/s, as in compressors and generators.

.4. Types of Belts



(*a*) Flat belt. (*b*) V-belt. (*c*) Circular belt.

Fig. 11.1. Types of belts.

Though there are many types of belts used these days, yet the following are important from the subject point of view :

1. ***Flat belt.*** The flat belt, as shown in Fig. 11.1 (*a*), is mostly used in the factories and workshops, where a moderate amount of power is to be transmitted, from one pulley to another when the two pulleys are not more than 8 metres apart.

2. ***V-belt.*** The V-belt, as shown in Fig. 11.1 (*b*), is mostly used in the factories and workshops, where a moderate amount of power is to be transmitted, from one pulley to another, when the two pulleys are very near to each other.

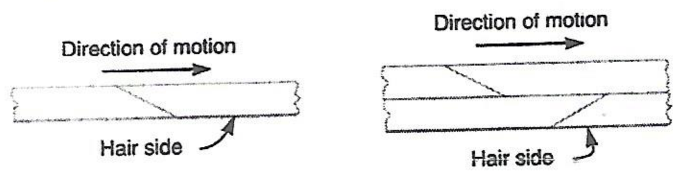
3. ***Circular belt or rope.*** The circular belt or rope, as shown in Fig. 11.1 (*c*), is mostly used in the factories and workshops, where a great amount of power is to be transmitted, from one pulley to another, when the two pulleys are more than 8 meters apart.

If a huge amount of power is to be transmitted, then a single belt may not be sufficient. In such a case, wide pulleys (for V-belts or circular belts) with a number of grooves are used. Then a belt in each groove is provided to transmit the required amount of power from one pulley to another.

11.5. Material used for Belts

The material used for belts and ropes must be strong, flexible, and durable. It must have a high coefficient of friction. The belts, according to the material used, are classified as follows :

1. ***Leather belts.*** The most important material for the belt is leather. The best leather belts are made from 1.2 metres to 1.5 metres long strips cut from either side of the back bone of the top grade steer hides. The hair side of the leather is smoother and harder than the flesh side, but the flesh side is stronger. The fibres on the hair side are perpendicular to the surface, while those on the flesh side are interwoven and parallel to the surface. Therefore for these reasons, the hair side of a belt should be in contact with the pulley surface, as shown in Fig. 11.2. This gives a more intimate contact between the belt and the pulley and places the greatest tensile strength of the belt section on the outside, where the tension is maximum as the belt passes over the pulley.



(*a*) Single layer belt. (*b*) Double layer belt.

Fig. 11.2. Leather belts.

The leather may be either oak-tanned or mineral salt tanned *e.g.* chrome tanned. In order to increase the thickness of belt, the strips are cemented together. The belts are specified according to the number of layers *e.g.* single, double or triple ply and according to the thickness of hides used *e.g.* light, medium or heavy.

The leather belts must be periodically cleaned and dressed or treated with a compound or dressing containing neats foot or other suitable oils so that the belt will remain soft and flexible.

2. ***Cotton or fabric belts.*** Most of the fabric belts are made by folding canvass or cotton duck to three or more layers (depending upon the thickness desired) and stitching together. These belts are woven also into a strip of the desired width and thickness. They are impregnated with some filler like linseed oil in order to make the belts water proof and to prevent injury to the fibres. The cotton belts are cheaper and suitable in warm climates, in damp atmospheres and in exposed positions. Since the cotton belts require little attention, therefore these belts are mostly used in farm machinery, belt conveyor etc.

3. ***Rubber belt.*** The rubber belts are made of layers of fabric impregnated with rubber composition and have a thin layer of rubber on the faces. These belts are very flexible but are quickly destroyed if allowed to come into contact with heat, oil or grease. One of the principal advantage of these belts is that they may be easily made endless. These belts are found suitable for saw mills, paper mills where they are exposed to moisture.

4. ***Balata belts.*** These belts are similar to rubber belts except that balata gum is used in place of rubber. These belts are acid proof and water proof and it is not effected by animal oils or alkalies. The balata belts should not be at temperatures above 40° C because at this temperature the balata begins to soften and becomes sticky. The strength of balata belts is 25 per cent higher than rubber belts.

..6. Types of Flat Belt Drives

The power from one pulley to another may be transmitted by any of the following types of belt drives:

1. ***Open belt drive.*** The open belt drive, as shown in Fig. 11.3, is used with shafts arranged parallel and rotating in the same direction. In this case, the driver *A* pulls the belt from one side (*i.e.* lower side *RQ*) and delivers it to the other side (*i.e.* upper side *LM*). Thus the tension in the lower side belt will be more than that in the upper side belt. The lower side belt (because of more tension) is known as ***tight side*** whereas the upper side belt (because of less tension) is known as ***slack side***, as shown in Fig. 11.3.

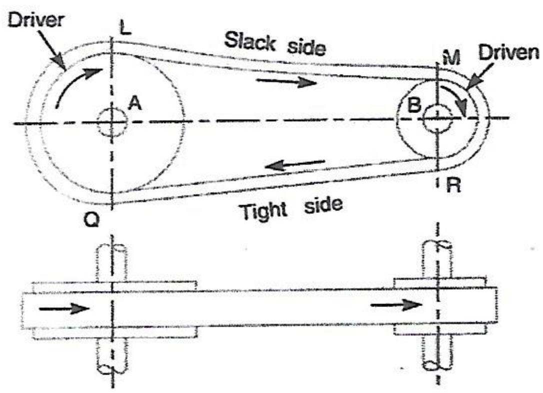


Fig. 11.3. Open belt drive.

2. ***Crossed or twist belt drive.*** The crossed or twist belt drive, as shown in Fig. 11.4, is used with shafts arranged parallel and rotating in the opposite directions.

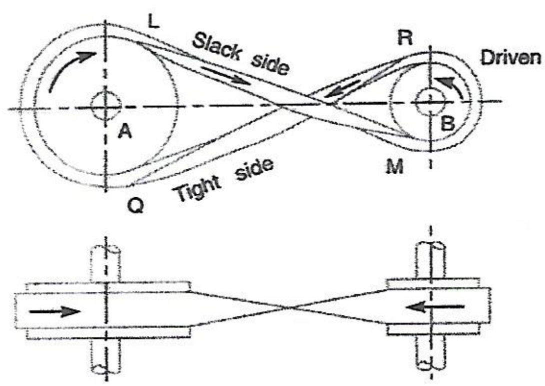


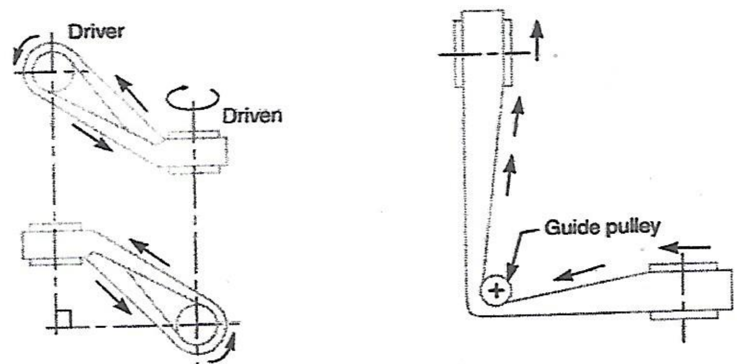
Fig. 11.4. Crossed or twist belt drive.

In this case, the driver pulls the belt from one side (*i.e. RQ*) and delivers it to the other side (*i.e. LM*). Thus the tension in the belt *RQ* will be more than that in the belt *LM*. The belt *RQ* (because of more tension) is known as ***tight side***, whereas the belt *LM* (because of less tension) is known as ***slack side***, as shown in Fig. 11.4.

A little consideration will show that at a point where the belt crosses, it rubs against each other and there will be excessive wear and tear. In order to avoid this, the shafts should be placed at a maximum distance of 20 *b*, where *b* is the width of belt and the speed of the belt should be less than 15 m/s.

3. ***Quarter turn belt drive.*** The quarter turn belt drive also known as right angle belt drive, as shown in Fig. 11.5 (*a*), is used with shafts arranged at right angles and rotating in one definite direction. In order to prevent the belt from leaving the pulley, the width of the face of the pulley should be greater or equal to 1.4 *b*, where *b* is the width of belt.

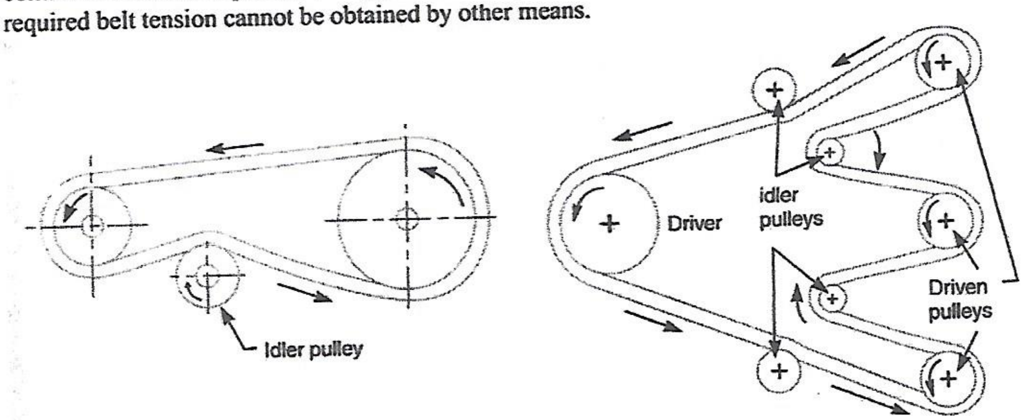
In case the pulleys cannot be arranged, as shown in Fig. 11.5 (*a*), or when the reversible motion is desired, then a ***quarter turn belt drive with guide pulley***, as shown in Fig. 11.5 (*b*), may be used.



(*a*) Quarter turn belt drive. (*b*) Quarter turn belt drive with guide pulley.

Fig. 11.5

4. ***Belt drive with idler pulleys.*** A belt drive with an idler pulley, as shown in Fig. 11.6 (*a*), is used with shafts arranged parallel and when an open belt drive cannot be used due to small angle of contact on the smaller pulley. This type of drive is provided to obtain high velocity ratio and when the required belt tension cannot be obtained by other means.



(*a*) Belt drive with single idler pulley. (*b*) Belt drive with many idler pulleys.

Fig. 11.6

When it is desired to transmit motion from one shaft to several shafts, all arranged in parallel, a belt drive with many idler pulleys, as shown in Fig. 11.6 (*b*), may be employed.

5. ***Compound belt drive.*** A compound belt drive, as shown in Fig. 11.7, is used when power is transmitted from one shaft to another through a number of pulleys.

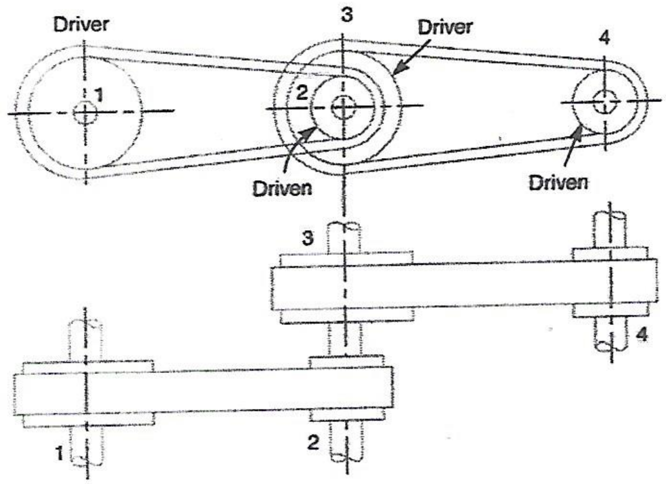


Fig. 11.7. Compound belt brive.

6. ***Stepped or cone pulley drive.*** A stepped or cone pulley drive, as shown in Fig. 11.8, is used for changing the speed of the driven shaft while the main or driving shaft runs at constant speed. This is accomplished by shifting the belt from one part of the steps to the other.

7. ***Fast and loose pulley drive.*** A fast and loose pulley drive, as shown in Fig. 11.9, is used when the driven or machine shaft is to be started or stopped when ever desired without interfering with the driving shaft. A pulley which is keyed to the machine shaft is called ***fast pulley*** and runs at the same speed as that of machine shaft. A loose pulley runs freely over the machine shaft and is incapable of transmitting any power. When the driven shaft is required to be stopped, the belt is pushed on to the loose pulley by means of sliding bar having belt forks.

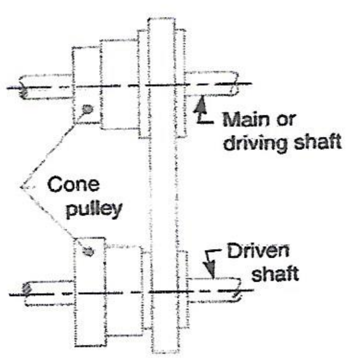


Fig. 11.8. Stepped or cone pulley drive.

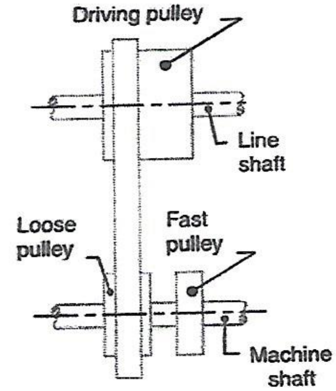


Fig. 11.9. Fast and loose pulley drive.

..7. Velocity Ratio of Belt Drive

It is the **ratio between the velocities of the driver and the follower or driven.** It may be expressed, mathematically, as discussed below :

Let d_1 = Diameter of the driver,

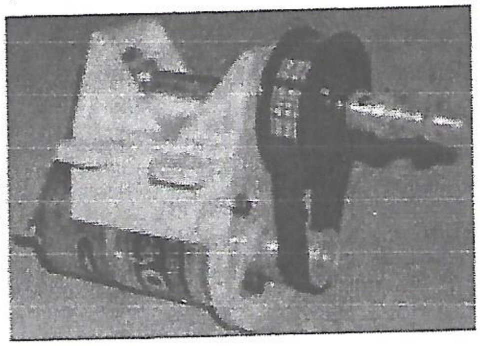
d_2 = Diameter of the follower,

N_1 = Speed of the driver in r.p.m., and

N_2 = Speed of the follower in r.p.m.

$\therefore$ Length of the belt that passes over the driver, in one minute

$$= \pi d_1 . N_1$$



Similarly, length of the belt that passes over the follower, in one minute

$$= \pi d_2 . N_2$$

Since the length of belt that passes over the driver in one minute is equal to the length of belt that passes over the follower in one minute, therefore

$$\pi d_1 . N_1 = \pi d_2 . N_2$$

$\therefore$ Velocity ratio, $\dfrac{N_2}{N_1} = \dfrac{d_1}{d_2}$

When the thickness of the belt (t) is considered, then velocity ratio,

$$\frac{N_2}{N_1} = \frac{d_1 + t}{d_2 + t}$$

Note: The velocity ratio of a belt drive may also be obtained as discussed below :

We know that peripheral velocity of the belt on the driving pulley,

$$v_1 = \frac{\pi d_1 . N_1}{60} \text{ m/s}$$

and peripheral velocity of the belt on the driven or follower pulley,

$$v_2 = \frac{\pi d_2 . N_2}{60} \text{ m/s}$$

When there is no slip, then $v_1 = v_2$.

$$\therefore \quad \frac{\pi d_1 . N_1}{60} = \frac{\pi d_2 . N_2}{60} \quad \text{or} \quad \frac{N_2}{N_1} = \frac{d_1}{d_2}$$

.8. Velocity Ratio of a Compound Belt Drive

Sometimes the power is transmitted from one shaft to another, through a number of pulleys as shown in Fig. 11.7. Consider a pulley 1 driving the pulley 2. Since the pulleys 2 and 3 are keyed to the same shaft, therefore the pulley 1 also drives the pulley 3 which, in turn, drives the pulley 4.

Let d_1 = Diameter of the pulley 1,

N_1 = Speed of the pulley 1 in r.p.m.,

d_2, d_3, d_4, and N_2, N_3, N_4 = Corresponding values for pulleys 2, 3 and 4.

We know that velocity ratio of pulleys 1 and 2,

$$\frac{N_2}{N_1} = \frac{d_1}{d_2} \quad \text{...(i)}$$

Similarly, velocity ratio of pulleys 3 and 4,

$$\frac{N_4}{N_3} = \frac{d_3}{d_4} \quad \text{...(ii)}$$

Multiplying equations (*i*) and (*ii*),

$$\frac{N_2}{N_1} \times \frac{N_4}{N_3} = \frac{d_1}{d_2} \times \frac{d_3}{d_4}$$

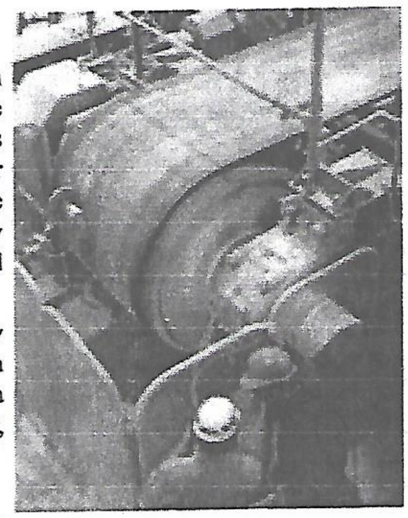
or $$\frac{N_4}{N_1} = \frac{d_1 \times d_3}{d_2 \times d_4} \quad ...(\because N_2 = N_3, \text{ being keyed to the same shaft})$$

A little consideration will show, that if there are six pulleys, then

$$\frac{N_6}{N_1} = \frac{d_1 \times d_3 \times d_5}{d_2 \times d_4 \times d_6}$$

or $$\frac{\text{Speed of last driven}}{\text{Speed of first driver}} = \frac{\text{Product of diameters of drivers}}{\text{Product of diameters of drivens}}$$

.9. Slip of Belt



In the previous articles, we have discussed the motion of belts and shafts assuming a firm frictional grip between the belts and the shafts. But sometimes, the frictional grip becomes insufficient. This may cause some forward motion of the driver without carrying the belt with it. This may also cause some forward motion of the belt without carrying the driven pulley with it. This is called ***slip of the belt*** and is generally expressed as a percentage.

The result of the belt slipping is to reduce the velocity ratio of the system. As the slipping of the belt is a common phenomenon, thus the belt should never be used where a definite velocity ratio is of importance (as in the case of hour, minute and second arms in a watch).

Let $s_1\,\%$ = Slip between the driver and the belt, and

$s_2\,\%$ = Slip between the belt and the follower.

$\therefore$ Velocity of the belt passing over the driver per second

$$v = \frac{\pi d_1 . N_1}{60} - \frac{\pi d_1 . N_1}{60} \times \frac{s_1}{100} = \frac{\pi d_1 . N_1}{60}\left(1 - \frac{s_1}{100}\right) \quad ...(i)$$

and velocity of the belt passing over the follower per second,

$$\frac{\pi d_2 . N_2}{60} = v - v \times \frac{s_2}{100} = v\left(1 - \frac{s_2}{100}\right)$$

Substituting the value of v from equation (i),

$$\frac{\pi d_2\, N_2}{60} = \frac{\pi d_1\, N_1}{60}\left(1 - \frac{s_1}{100}\right)\left(1 - \frac{s_2}{100}\right)$$

$$\frac{N_2}{N_1} = \frac{d_1}{d_2}\left(1 - \frac{s_1}{100} - \frac{s_2}{100}\right) \quad ...\left(\text{Neglecting } \frac{s_1 \times s_2}{100 \times 100}\right)$$

$$= \frac{d_1}{d_2}\left(1 - \frac{s_1 + s_2}{100}\right) = \frac{d_1}{d_2}\left(1 - \frac{s}{100}\right)$$

... (where $s = s_1 + s_2$, *i.e.* total percentage of slip)

If thickness of the belt (t) is considered, then

$$\frac{N_2}{N_1} = \frac{d_1 + t}{d_2 + t}\left(1 - \frac{s}{100}\right)$$

Example 11.1. *An engine, running at 150 r.p.m., drives a line shaft by means of a belt. The engine pulley is 750 mm diameter and the pulley on the line shaft being 450 mm. A 900 mm diameter pulley on the line shaft drives a 150 mm diameter pulley keyed to a dynamo shaft. Find the speed of the dynamo shaft, when 1. there is no slip, and 2. there is a slip of 2% at each drive.*

Solution. Given : $N_1 = 150$ r.p.m. ; $d_1 = 750$ mm ; $d_2 = 450$ mm ; $d_3 = 900$ mm ; $d_4 = 150$ mm

The arrangement of belt drive is shown in Fig. 11.10.

Let N_4 = Speed of the dynamo shaft.

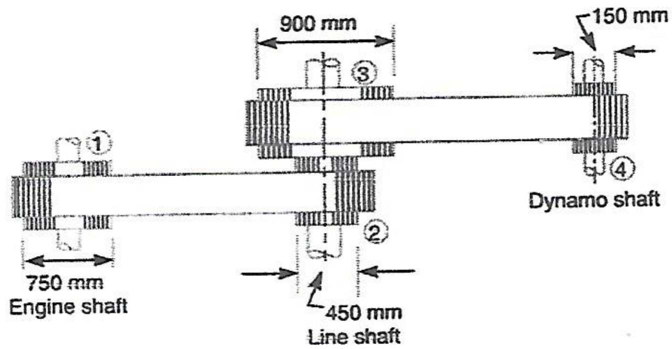


Fig. 11.10

1. ***When there is no slip***

We know that $\frac{N_4}{N_1} = \frac{d_1 \times d_3}{d_2 \times d_4}$ or $\frac{N_4}{150} = \frac{750 \times 900}{450 \times 150} = 10$

$\therefore$ $N_4 = 150 \times 10 = 1500$ r.p.m. **Ans.**

2. ***When there is a slip of 2% at each drive***

We know that $$\frac{N_4}{N_1} = \frac{d_1 \times d_3}{d_2 \times d_4}\left(1 - \frac{s_1}{100}\right)\left(1 - \frac{s_2}{100}\right)$$

$$\frac{N_4}{150} = \frac{750 \times 900}{450 \times 150}\left(1 - \frac{2}{100}\right)\left(1 - \frac{2}{100}\right) = 9.6$$

$\therefore$ $N_4 = 150 \times 9.6 = 1440$ r.p.m. **Ans.**

1 .10. Creep of Belt

When the belt passes from the slack side to the tight side, a certain portion of the belt extends and it contracts again when the belt passes from the tight side to slack side. Due to these changes of length, there is a relative motion between the belt and the pulley surfaces. This relative motion is termed as *creep*. The total effect of creep is to reduce slightly the speed of the driven pulley or follower. Considering creep, the velocity ratio is given by

$$\frac{N_2}{N_1} = \frac{d_1}{d_2} \times \frac{E + \sqrt{\sigma_2}}{E + \sqrt{\sigma_1}}$$

where σ_1 and σ_2 = Stress in the belt on the tight and slack side respectively, and

E = Young's modulus for the material of the belt.

Example 11.2. *The power is transmitted from a pulley 1 m diameter running at 200 r.p.m. to a pulley 2.25 m diameter by means of a belt. Find the speed lost by the driven pulley as a result of creep, if the stress on the tight and slack side of the belt is 1.4 MPa and 0.5 MPa respectively. The Young's modulus for the material of the belt is 100 MPa.*

Solution. Given : $d_1 = 1$ m ; $N_1 = 200$ r.p.m. ; $d_2 = 2.25$ m ; $\sigma_1 = 1.4$ MPa $= 1.4 \times 10^6$ N/m^2; $\sigma_2 = 0.5$ MPa $= 0.5 \times 10^6$ N/m^2 ; $E = 100$ MPa $= 100 \times 10^6$ N/m^2

Let N_2 = Speed of the driven pulley.

Neglecting creep, we know that

$$\frac{N_2}{N_1} = \frac{d_1}{d_2} \quad \text{or} \quad N_2 = N_1 \times \frac{d_1}{d_2} = 200 \times \frac{1}{2.25} = 88.9 \text{ r.p.m.}$$

Considering creep, we know that

$$\frac{N_2}{N_1} = \frac{d_1}{d_2} \times \frac{E + \sqrt{\sigma_2}}{E + \sqrt{\sigma_1}}$$

or

$$N_2 = 200 \times \frac{1}{2.25} \times \frac{100 \times 10^6 + \sqrt{0.5 \times 10^6}}{100 \times 10^6 + \sqrt{1.4 \times 10^6}} = 88.7 \text{ r.p.m.}$$

∴ Speed lost by driven pulley due to creep

$= 88.9 - 88.7 = 0.2$ r.p.m. **Ans.**

.11. Length of an Open Belt Drive

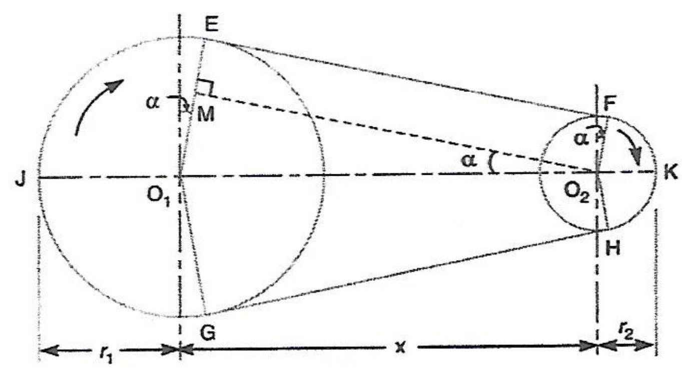


Fig. 11.11. Length of an open belt drive.

We have already discussed in Art. 11.6 that in an open belt drive, both the pulleys rotate in the *same* direction as shown in Fig. 11.11.

Let r_1 and r_2 = Radii of the larger and smaller pulleys,

x = Distance between the centres of two pulleys (*i.e.* $O_1 O_2$), and

L = Total length of the belt.

Let the belt leaves the larger pulley at E and G and the smaller pulley at F and H as shown in Fig. 11.11. Through O_2, draw $O_2 M$ parallel to FE.

From the geometry of the figure, we find that $O_2 M$ will be perpendicular to $O_1 E$.

Let the angle $MO_2 O_1 = \alpha$ radians.

We know that the length of the belt,

$$L = \text{Arc } GJE + EF + \text{Arc } FKH + HG$$

$$= 2\,(\text{Arc } JE + EF + \text{Arc } FK) \quad \ldots(i)$$

From the geometry of the figure, we find that

$$\sin\alpha = \frac{O_1 M}{O_1 O_2} = \frac{O_1 E - EM}{O_1 O_2} = \frac{r_1 - r_2}{x}$$

Since α is very small, therefore putting

$$\sin\alpha = \alpha \text{ (in radians)} = \frac{r_1 - r_2}{x} \quad \ldots(ii)$$

∴ $$\text{Arc } JE = r_1\left(\frac{\pi}{2} + \alpha\right) \quad \ldots(iii)$$

Similarly $$\text{Arc } FK = r_2\left(\frac{\pi}{2} - \alpha\right) \quad \ldots(iv)$$

and $$EF = MO_2 = \sqrt{(O_1 O_2)^2 - (O_1 M)^2} = \sqrt{x^2 - (r_1 - r_2)^2}$$

$$= x\sqrt{1 - \left(\frac{r_1 - r_2}{x}\right)^2}$$

Expanding this equation by binomial theorem,

$$EF = x\left[1 - \frac{1}{2}\left(\frac{r_1 - r_2}{x}\right)^2 + \ldots\right] = x - \frac{(r_1 - r_2)^2}{2x} \quad \ldots(v)$$

Substituting the values of arc *JE* from equation (*iii*), arc *FK* from equation (*iv*) and *EF* from equation (*v*) in equation (*i*), we get

$$L = 2\left[r_1\left(\frac{\pi}{2} + \alpha\right) + x - \frac{(r_1 - r_2)^2}{2x} + r_2\left(\frac{\pi}{2} - \alpha\right)\right]$$

$$= 2\left[r_1 \times \frac{\pi}{2} + r_1.\alpha + x - \frac{(r_1 - r_2)^2}{2x} + r_2 \times \frac{\pi}{2} - r_2.\alpha\right]$$

$$= 2\left[\frac{\pi}{2}(r_1 + r_2) + \alpha(r_1 - r_2) + x - \frac{(r_1 - r_2)^2}{2x}\right]$$

$$= \pi(r_1 + r_2) + 2\alpha(r_1 - r_2) + 2x - \frac{(r_1 - r_2)^2}{x}$$

Substituting the value of $\alpha = \frac{r_1 - r_2}{x}$ from equation (*ii*),

$$L = \pi(r_1 + r_2) + 2 \times \frac{(r_1 - r_2)}{x} \times (r_1 - r_2) + 2x - \frac{(r_1 - r_2)^2}{x}$$

$$= \pi(r_1 + r_2) + \frac{2(r_1 - r_2)^2}{x} + 2x - \frac{(r_1 - r_2)^2}{x}$$

$$= \pi(r_1 + r_2) + 2x + \frac{(r_1 - r_2)^2}{x} \quad \text{...(In terms of pulley radii)}$$

$$= \frac{\pi}{2}(d_1 + d_2) + 2x + \frac{(d_1 - d_2)^2}{4x} \quad \text{...(In terms of pulley diameters)}$$

12. Length of a Cross Belt Drive

We have already discussed in Art. 11.6 that in a cross belt drive, both the pulleys rotate in ***opposite*** directions as shown in Fig. 11.12.

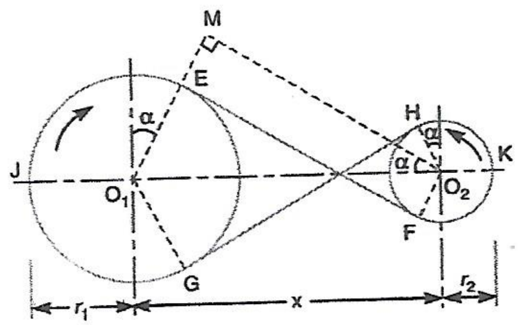


Fig. 11.12. Length of a cross belt drive.

Let r_1 and r_2 = Radii of the larger and smaller pulleys,

x = Distance between the centres of two pulleys (*i.e.* $O_1 O_2$), and

L = Total length of the belt.

Let the belt leaves the larger pulley at E and G and the smaller pulley at F and H, as shown in Fig. 11.12. Through O_2, draw O_2M parallel to FE.

From the geometry of the figure, we find that O_2M will be perpendicular to O_1E.

Let the angle $MO_2 O_1 = \alpha$ radians.

We know that the length of the belt,

$$L = \text{Arc } GJE + EF + \text{Arc } FKH + HG$$
$$= 2\,(\text{Arc } JE + EF + \text{Arc } FK) \quad \text{...(i)}$$

From the geometry of the figure, we find that

$$\sin\alpha = \frac{O_1 M}{O_1 O_2} = \frac{O_1 E + EM}{O_1 O_2} = \frac{r_1 + r_2}{x}$$

Since α is very small, therefore putting

$$\sin\alpha = \alpha \text{ (in radians)} = \frac{r_1 + r_2}{x} \quad \text{...(ii)}$$

$$\therefore \quad \text{Arc } JE = r_1\left(\frac{\pi}{2} + \alpha\right) \quad \text{...(iii)}$$

Similarly $$\text{Arc } FK = r_2\left(\frac{\pi}{2} + \alpha\right) \quad \text{...(iv)}$$

and $$EF = MO_2 = \sqrt{(O_1O_2)^2 - (O_1 M)^2} = \sqrt{x^2 - (r_1 + r_2)^2}$$
$$= x\sqrt{1 - \left(\frac{r_1 + r_2}{x}\right)^2}$$

Expanding this equation by binomial theorem,

$$EF = x\left[1 - \frac{1}{2}\left(\frac{r_1 + r_2}{x}\right)^2 + \ldots\right] = x - \frac{(r_1 + r_2)^2}{2\,x} \qquad \ldots(v)$$

Substituting the values of arc *JE* from equation (*iii*), arc *FK* from equation (*iv*) and *EF* from equation (*v*) in equation (*i*), we get

$$L = 2\left[r_1\left(\frac{\pi}{2} + \alpha\right) + x - \frac{(r_1 + r_2)^2}{2x} + r_2\left(\frac{\pi}{2} + \alpha\right)\right]$$

$$= 2\left[r_1 \times \frac{\pi}{2} + r_1.\alpha + x - \frac{(r_1 + r_2)^2}{2x} + r_2 \times \frac{\pi}{2} + r_2.\alpha\right]$$

$$= 2\left[\frac{\pi}{2}(r_1 + r_2) + \alpha(r_1 + r_2) + x - \frac{(r_1 + r_2)^2}{2x}\right]$$

$$= \pi(r_1 + r_2) + 2\alpha(r_1 + r_2) + 2x - \frac{(r_1 + r_2)^2}{x}$$

Substituting the value of $\alpha = \dfrac{r_1 + r_2}{x}$ from equation (*ii*),

$$L = \pi(r_1 + r_2) + \frac{2(r_1 + r_2)}{x} \times (r_1 + r_2) + 2x - \frac{(r_1 + r_2)^2}{x}$$

$$= \pi(r_1 + r_2) + \frac{2(r_1 + r_2)^2}{x} + 2x - \frac{(r_1 + r_2)^2}{x}$$

$$= \pi(r_1 + r_2) + 2x + \frac{(r_1 + r_2)^2}{x} \qquad \ldots\text{(In terms of pulley radii)}$$

$$= \frac{\pi}{2}(d_1 + d_2) + 2x + \frac{(d_1 + d_2)^2}{4x} \qquad \ldots\text{(In terms of pulley diameters)}$$

It may be noted that the above expression is a function of $(r_1 + r_2)$. It is thus obvious that if sum of the radii of the two pulleys be constant, then length of the belt required will also remain constant, provided the distance between centres of the pulleys remain unchanged.

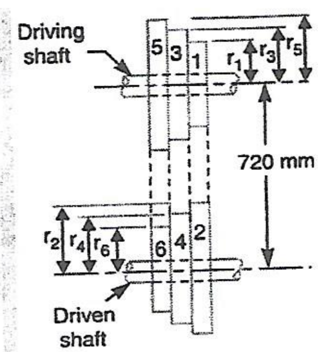


Fig. 11.13.

Example 11.3. *A shaft which rotates at a constant speed of 160 r.p.m. is connected by belting to a parallel shaft 720 mm apart, which has to run at 60, 80 and 100 r.p.m. The smallest pulley on the driving shaft is 40 mm in radius. Determine the remaining radii of the two stepped pulleys for 1. a crossed belt, and 2. an open belt. Neglect belt thickness and slip.*

Solution. Given : $N_1 = N_3 = N_5 = 160$ r.p.m. ; $x = 720$ mm ; $N_2 = 60$ r.p.m.; $N_4 = 80$ r.p.m.; $N_6 = 100$ r.p.m. ; $r_1 = 40$ mm

Let r_2, r_3, r_4, r_5 and r_6 be the radii of the pulleys 2, 3, 4, 5, and 6 respectively, as shown in Fig. 11.13.

1. ***For a crossed belt***

We know that for pulleys 1 and 2,

$$\frac{N_2}{N_1} = \frac{r_1}{r_2}$$

or

$$r_2 = r_1 \times \frac{N_1}{N_2} = 40 \times \frac{160}{60} = 106.7 \text{ mm } \textbf{Ans.}$$

and for pulleys 3 and 4,

$$\frac{N_4}{N_3} = \frac{r_3}{r_4} \text{ or } r_4 = r_3 \times \frac{N_3}{N_4} = r_3 \times \frac{160}{80} = 2\,r_3$$

We know that for a crossed belt drive,

$$r_1 + r_2 = r_3 + r_4 = r_5 + r_6 = 40 + 106.7 = 146.7 \text{ mm} \quad ...(i)$$

$\therefore$

$$r_3 + 2\,r_3 = 146.7 \text{ or } r_3 = 146.7/3 = 48.9 \text{ mm } \textbf{Ans.}$$

and

$$r_4 = 2\,r_3 = 2 \times 48.9 = 97.8 \text{ mm } \textbf{Ans.}$$

Now for pulleys 5 and 6,

$$\frac{N_6}{N_5} = \frac{r_5}{r_6} \text{ or } r_6 = r_5 \times \frac{N_5}{N_6} = r_5 \times \frac{160}{100} = 1.6\,r_5$$

From equation (*i*),

$$r_5 + 1.6\,r_5 = 146.7 \text{ or } r_5 = 146.7/2.6 = 56.4 \text{ mm } \textbf{Ans.}$$

and

$$r_6 = 1.6\,r_5 = 1.6 \times 56.4 = 90.2 \text{ mm } \textbf{Ans.}$$

2. ***For an open belt***

We know that for pulleys 1 and 2,

$$\frac{N_2}{N_1} = \frac{r_1}{r_2} \text{ or } r_2 = r_1 \times \frac{N_1}{N_2} = 40 \times \frac{160}{60} = 106.7 \text{ mm } \textbf{Ans.}$$

and for pulleys 3 and 4,

$$\frac{N_4}{N_3} = \frac{r_3}{r_4} \text{ or } r_4 = r_3 \times \frac{N_3}{N_4} = r_3 \times \frac{160}{80} = 2\,r_3$$

We know that length of belt for an open belt drive,

$$L = \pi(r_1 + r_2) + \frac{(r_2 - r_1)^2}{x} + 2x$$

$$= \pi(40 + 106.7) + \frac{(106.7 - 40)^2}{720} + 2 \times 720 = 1907 \text{ mm}$$

Since the length of the belt in an open belt drive is constant, therefore for pulleys 3 and 4, length of the belt (L),

$$1907 = \pi(r_3 + r_4) + \frac{(r_4 - r_3)^2}{x} + 2x$$

$$= \pi(r_3 + 2r_3) + \frac{(2r_3 - r_3)^2}{720} + 2 \times 720$$

$$= 9.426\, r_3 + 0.0014\, (r_3)^2 + 1440$$

or $\quad 0.0014\,(r_3)^2 + 9.426\, r_3 - 467 = 0$

$$\therefore \quad r_3 = \frac{-9.426 \pm \sqrt{(9.426)^2 + 4 \times 0.0014 \times 467}}{2 \times 0.0014}$$

$$= \frac{-9.426 \pm 9.564}{0.0028} = 49.3 \text{ mm} \quad \textbf{Ans.} \qquad \text{...(Taking +ve sign)}$$

and $\quad r_4 = 2\, r_3 = 2 \times 49.3 = 98.6$ mm **Ans.**

Now for pulleys 5 and 6,

$$\frac{N_6}{N_5} = \frac{r_5}{r_6} \quad \text{or}$$

$$r_6 = \frac{N_5}{N_6} \times r_5 = \frac{160}{100} \times r_5 = 1.6\, r_5$$

and length of the belt (L),

$$1907 = \pi(r_5 + r_6) + \frac{(r_6 - r_5)^2}{x} + 2x$$

$$= \pi(r_5 + 1.6 r_5) + \frac{(1.6 r_5 - r_5)^2}{720} + 2 \times 720$$

$$= 8.17\, r_5 + 0.0005\,(r_5)^2 + 1440$$

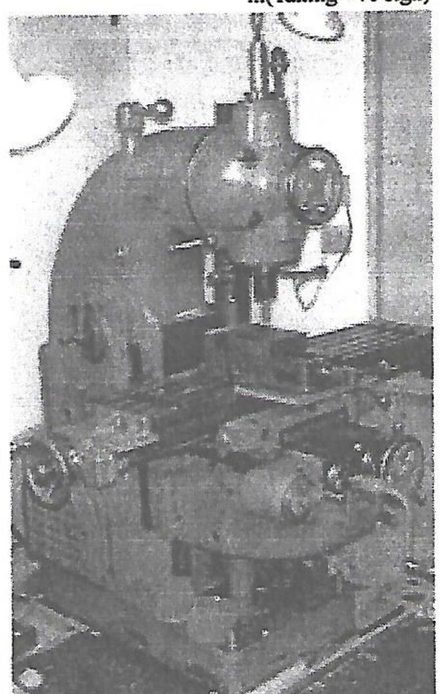
or $\quad 0.0005\,(r_5)^2 + 8.17\, r_5 - 467 = 0$

$$\therefore \quad r_5 = \frac{-8.17 \pm \sqrt{(8.17)^2 + 4 \times 0.0005 \times 467}}{2 \times 0.0005}$$

$$= \frac{-8.17 \pm 8.23}{0.001} = 60 \text{ mm} \quad \textbf{Ans.}$$

...(Taking +ve sign)

and $\quad r_6 = 1.6\, r_5 = 1.6 \times 60 = 96$ mm **Ans.**



Milling machine is used for dressing surfaces by rotary cutters.

Note : This picture is given as additional information and is not a direct example of the current chapter.

.13. Power Transmitted by a Belt

Fig. 11.14 shows the driving pulley (or driver) A and the driven pulley (or follower) B. We have already discussed that the driving pulley pulls the belt from one side and delivers the same to the other side. It is thus obvious that the tension on the former side (*i.e.* tight side) will be greater than the latter side (*i.e.* slack side) as shown in Fig. 11.14.

Let $\quad T_1$ and T_2 = Tensions in the tight and slack side of the belt respectively in newtons,

r_1 and r_2 = Radii of the driver and follower respectively, and
v = Velocity of the belt in m/s.

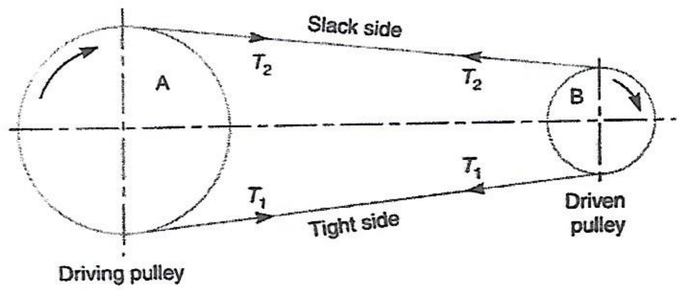


Fig. 11.14. Power transmitted by a belt.

The effective turning (driving) force at the circumference of the follower is the difference between the two tensions (*i.e.* $T_1 - T_2$).

$\therefore$ Work done per second $= (T_1 - T_2)\, v$ N-m/s

and power transmitted, $P = (T_1 - T_2)\, v$ W ...($\because$ 1 N-m/s = 1 W)

A little consideration will show that the torque exerted on the driving pulley is $(T_1 - T_2)\, r_1$. Similarly, the torque exerted on the driven pulley *i.e.* follower is $(T_1 - T_2)\, r_2$.

.14. Ratio of Driving Tensions For Flat Belt Drive

Consider a driven pulley rotating in the clockwise direction as shown in Fig. 11.15.

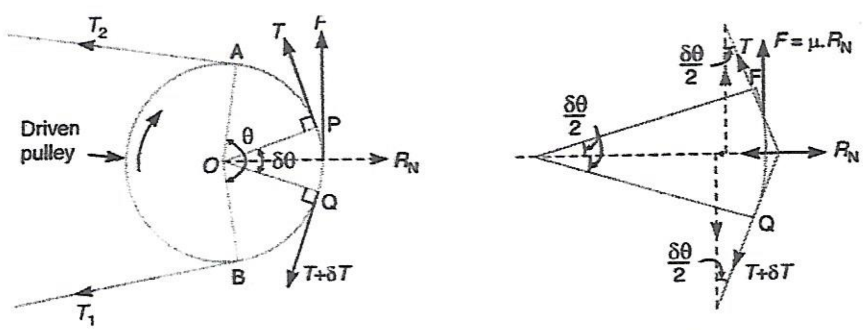


Fig. 11.15. Ratio of driving tensions for flat belt.

Let T_1 = Tension in the belt on the tight side,

T_2 = Tension in the belt on the slack side, and

θ = Angle of contact in radians (*i.e.* angle subtended by the arc AB, along which the belt touches the pulley at the centre).

Now consider a small portion of the belt PQ, subtending an angle $\delta\theta$ at the centre of the pulley as shown in Fig. 11.15. The belt PQ is in equilibrium under the following forces :

1. Tension T in the belt at P,
2. Tension $(T + \delta T)$ in the belt at Q,
3. Normal reaction R_N, and
4. Frictional force, $F = \mu \times R_N$, where μ is the coefficient of friction between the belt and pulley.

Resolving all the forces horizontally and equating the same,

$$R_N = (T + \delta T)\sin\frac{\delta\theta}{2} + T\sin\frac{\delta\theta}{2} \quad ...(i)$$

Since the angle $\delta\theta$ is very small, therefore putting $\sin\delta\theta/2 = \delta\theta/2$ in equation (*i*),

$$R_N = (T + \delta T)\frac{\delta\theta}{2} + T \times \frac{\delta\theta}{2} = \frac{T.\delta\theta}{2} + \frac{\delta T.\delta\theta}{2} + \frac{T.\delta\theta}{2} = T.\delta\theta \quad ...(ii)$$

$$...\left(\text{Neglecting } \frac{\delta T.\delta\theta}{2}\right)$$

Now resolving the forces vertically, we have

$$\mu \times R_N = (T + \delta T)\cos\frac{\delta\theta}{2} - T\cos\frac{\delta\theta}{2} \quad ...(iii)$$

Since the angle $\delta\theta$ is very small, therefore putting $\cos\delta\theta/2 = 1$ in equation (*iii*),

$$\mu \times R_N = T + \delta T - T = \delta T \text{ or } R_N = \frac{\delta T}{\mu} \quad ...(iv)$$

Equating the values of R_N from equations (*ii*) and (*iv*),

$$T.\delta\theta = \frac{\delta T}{\mu} \quad \text{or} \quad \frac{\delta T}{T} = \mu.\delta\theta$$

Integrating both sides between the limits T_2 and T_1 and from 0 to θ respectively,

i.e.

$$\int_{T_2}^{T_1}\frac{\delta T}{T} = \mu\int_0^{\theta}\delta\theta \quad \text{or} \quad \log_e\left(\frac{T_1}{T_2}\right) = \mu.\theta \text{ or } \frac{T_1}{T_2} = e^{\mu.\theta} \quad ...(v)$$

Equation (*v*) can be expressed in terms of corresponding logarithm to the base 10, *i.e.*

$$2.3\log\left(\frac{T_1}{T_2}\right) = \mu.\theta$$

The above expression gives the relation between the tight side and slack side tensions, in terms of coefficient of friction and the angle of contact.

11.15. Determination of Angle of Contact

When the two pulleys of different diameters are connected by means of an open belt as shown in Fig. 11.16 (*a*), then the angle of contact or lap (θ) at the smaller pulley must be taken into consideration.

Let

r_1 = Radius of larger pulley,

r_2 = Radius of smaller pulley, and

x = Distance between centres of two pulleys (*i.e.* $O_1 O_2$).

From Fig. 11.16 (*a*),

$$\sin\alpha = \frac{O_1 M}{O_1 O_2} = \frac{O_1 E - ME}{O_1 O_2} = \frac{r_1 - r_2}{x} \quad ...(\because ME = O_2 F = r_2)$$

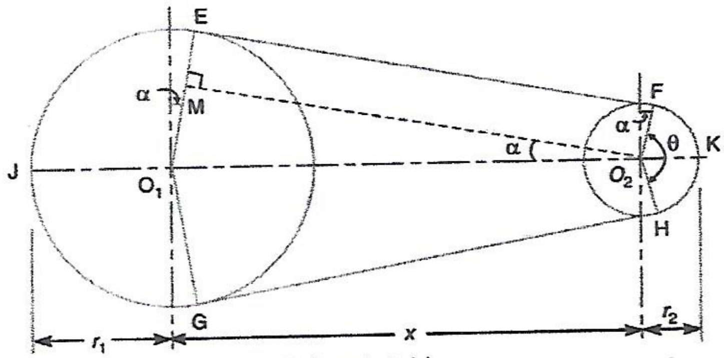
$\therefore$ Angle of contact or lap,

$$\theta = (180° - 2\alpha)\frac{\pi}{180} \text{ rad}$$

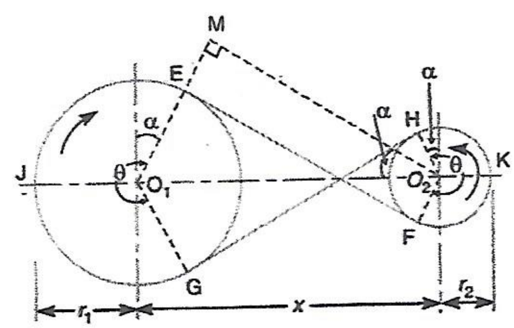
A little consideration will show that when the two pulleys are connected by means of a crossed belt as shown in Fig. 11.16 (*b*), then the angle of contact or lap (θ) on both the pulleys is same. From Fig. 11.16 (*b*),

$$\sin\alpha = \frac{O_1 M}{O_1 O_2} = \frac{O_1 E + ME}{O_1 O_2} = \frac{r_1 + r_2}{x}$$

∴ Angle of contact or lap, $\theta = (180° + 2\alpha)\frac{\pi}{180}$ rad



(*a*) Open belt drive.



(*b*) Crossed belt drive.

Fig. 11.16

Example 11.4. *Find the power transmitted by a belt running over a pulley of 600 mm diameter at 200 r.p.m. The coefficient of friction between the belt and the pulley is 0.25, angle of lap 160° and maximum tension in the belt is 2500 N.*

Solution. Given : $d = 600$ mm $= 0.6$ m ; $N = 200$ r.p.m. ; $\mu = 0.25$; $\theta = 160° = 160 \times \pi / 180 = 2.793$ rad ; $T_1 = 2500$ N

We know that velocity of the belt,

$$v = \frac{\pi d . N}{60} = \frac{\pi \times 0.6 \times 200}{60} = 6.284 \text{ m/s}$$

Let T_2 = Tension in the slack side of the belt.

We know that $2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.25 \times 2.793 = 0.6982$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{0.6982}{2.3} = 0.3036$$

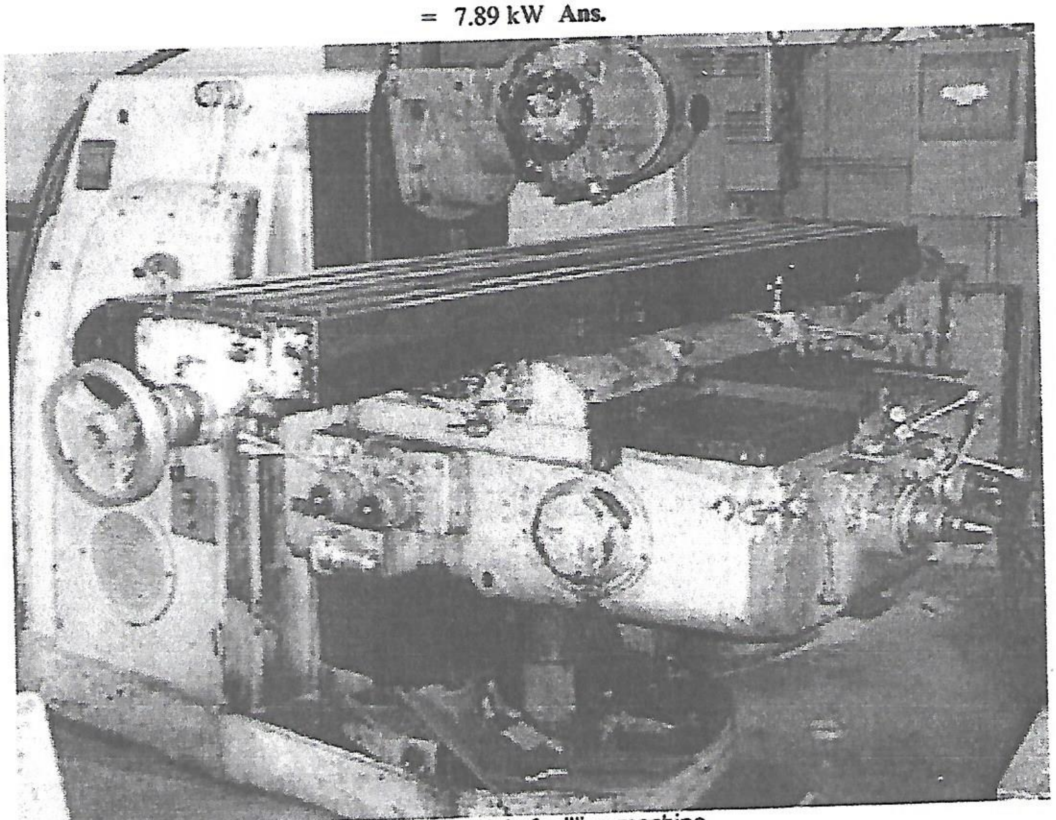
$\therefore$ $$\frac{T_1}{T_2} = 2.01 \quad \text{...(Taking antilog of 0.3036)}$$

and $$T_2 = \frac{T_1}{2.01} = \frac{2500}{2.01} = 1244 \text{ N}$$

We know that power transmitted by the belt,

$$P = (T_1 - T_2)\, v = (2500 - 1244)\, 6.284 = 7890 \text{ W}$$

$$= 7.89 \text{ kW} \quad \textbf{Ans.}$$



Another model of milling machine.

Note : This picture is given as additional information and is not a direct example of the current chapter.

Example 11.5. *A casting weighing 9 kN hangs freely from a rope which makes 2.5 turns round a drum of 300 mm diameter revolving at 20 r.p.m. The other end of the rope is pulled by a man. The coefficient of friction is 0.25. Determine 1. The force required by the man, and 2. The power to raise the casting.*

Solution. Given : $W = T_1 = 9$ kN $= 9000$ N ; $d = 300$ mm $= 0.3$ m ; $N = 20$ r.p.m. ; $\mu = 0.25$

1. *Force required by the man*

Let T_2 = Force required by the man.

Since the rope makes 2.5 turns round the drum, therefore angle of contact,

$$\theta = 2.5 \times 2\pi = 5\pi \text{ rad}$$

We know that $2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.25 \times 5\pi = 3.9275$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{3.9275}{2.3} = 1.71 \text{ or } \frac{T_1}{T_2} = 51$$

...(Taking antilog of 1.71)

$$\therefore \quad T_2 = \frac{T_1}{51} = \frac{9000}{51} = 176.47 \text{ N} \textbf{ Ans.}$$

2. ***Power to raise the casting***

We know that velocity of the rope,

$$v = \frac{\pi d.N}{60} = \frac{\pi \times 0.3 \times 20}{60} = 0.3142 \text{ m/s}$$

∴ Power to raise the casting,

$$P = (T_1 - T_2)\, v = (9000 - 176.47)\, 0.3142 = 2772 \text{ W}$$
$$= 2.772 \text{ kW} \textbf{ Ans.}$$

Example 11.6. *Two pulleys, one 450 mm diameter and the other 200 mm diameter are on parallel shafts 1.95 m apart and are connected by a crossed belt. Find the length of the belt required and the angle of contact between the belt and each pulley.*

What power can be transmitted by the belt when the larger pulley rotates at 200 rev/min, if the maximum permissible tension in the belt is 1 kN, and the coefficient of friction between the belt and pulley is 0.25 ?

Solution. Given : d_1 = 450 mm = 0.45 m or r_1 = 0.225 m ; d_2 = 200 mm = 0.2 m or r_2 = 0.1 m ; x = 1.95 m ; N_1 = 200 r.p.m. ; T_1 = 1 kN = 1000 N ; μ = 0.25

We know that speed of the belt,

$$v = \frac{\pi d_1.N_1}{60} = \frac{\pi \times 0.45 \times 200}{60} = 4.714 \text{ m/s}$$

Length of the belt

We know that length of the crossed belt,

$$L = \pi(r_1 + r_2) + 2x + \frac{(r_1 + r_2)^2}{x}$$

$$= \pi(0.225 + 0.1) + 2 \times 1.95 + \frac{(0.225 + 0.1)^2}{1.95} = 4.975 \text{ m} \textbf{ Ans.}$$

Angle of contact between the belt and each pulley

Let θ = Angle of contact between the belt and each pulley.

We know that for a crossed belt drive,

$$\sin\alpha = \frac{r_1 + r_2}{x} = \frac{0.225 + 0.1}{1.95} = 0.1667 \text{ or } \alpha = 9.6°$$

$$\therefore \quad \theta = 180° + 2\alpha = 180° + 2 \times 9.6° = 199.2°$$

$$= 199.2 \times \frac{\pi}{180} = 3.477 \text{ rad} \textbf{ Ans.}$$

Power transmitted

Let T_2 = Tension in the slack side of the belt.

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.25 \times 3.477 = 0.8692$$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{0.8692}{2.3} = 0.378 \text{ or } \frac{T_1}{T_2} = 2.387 \quad \text{...(Taking antilog of 0.378)}$$

$$\therefore \quad T_2 = \frac{T_1}{2.387} = \frac{1000}{2.387} = 419 \text{ N}$$

We know that power transmitted,

$$P = (T_1 - T_2)\, v = (1000 - 419)\, 4.714 = 2740 \text{ W} = 2.74 \text{ kW} \quad \textbf{Ans.}$$

16. Centrifugal Tension

Since the belt continuously runs over the pulleys, therefore, some centrifugal force is caused, whose effect is to increase the tension on both, tight as well as the slack sides. The tension caused by centrifugal force is called ***centrifugal tension.*** At lower belt speeds (less than 10 m/s), the centrifugal tension is very small, but at higher belt speeds (more than 10 m/s), its effect is considerable and thus should be taken into account.

Consider a small portion *PQ* of the belt subtending an angle $d\theta$ the centre of the pulley as shown in Fig. 11.17.

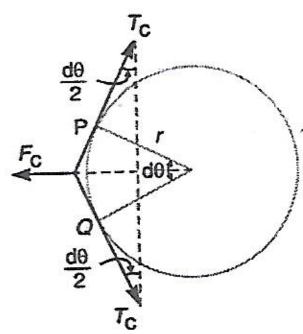


Fig. 11.17. Centrifugal tension.

Let m = Mass of the belt per unit length in kg,

v = Linear velocity of the belt in m/s,

r = Radius of the pulley over which the belt runs in metres, and

T_C = Centrifugal tension acting tangentially at *P* and *Q* in newtons.

We know that length of the belt *PQ*

$$= r.\, d\theta$$

and mass of the belt *PQ* $= m.\, r.\, d\theta$

$\therefore$ Centrifugal force acting on the belt *PQ*,

$$F_C = (m.r.d\theta)\frac{v^2}{r} = m.d\theta.v^2$$

The centrifugal tension T_C acting tangentially at *P* and *Q* keeps the belt in equilibrium.

Now resolving the forces (*i.e.* centrifugal force and centrifugal tension) horizontally and equating the same, we have

$$T_C \sin\left(\frac{d\theta}{2}\right) + T_C \sin\left(\frac{d\theta}{2}\right) = F_C = m.d\theta.v^2$$

Since the angle $d\theta$ is very small, therefore, putting $\sin\left(\frac{d\theta}{2}\right) = \frac{d\theta}{2}$, in the above expression,

$$2T_C\left(\frac{d\theta}{2}\right) = m.d\theta.v^2 \quad \text{or} \quad T_C = m.v^2$$

Notes : 1. When the centrifugal tension is taken into account, then total tension in the tight side,

$$T_{t1} = T_1 + T_C$$

and total tension in the slack side,

$$T_{t2} = T_2 + T_C$$

2. Power transmitted,

$$P = (T_{t1} - T_{t2})\, v \quad \text{...(in watts)}$$

$$= [(T_1 + T_C) - (T_2 + T_C)]\, v = (T_1 - T_2)\, v \quad \text{...(same as before)}$$

Thus we see that centrifugal tension has no effect on the power transmitted.

3. The ratio of driving tensions may also be written as

$$2.3\log\left(\frac{T_{t1} - T_C}{T_{t2} - T_C}\right) = \mu.\theta$$

where T_{t1} = Maximum or total tension in the belt.

..17. Maximum Tension in the Belt

A little consideration will show that the maximum tension in the belt (T) is equal to the total tension in the tight side of the belt (T_{t1}).

Let
σ = Maximum safe stress in N/mm^2,
b = Width of the belt in mm, and
t = Thickness of the belt in mm.

We know that maximum tension in the belt,

$$T = \text{Maximum stress} \times \text{cross-sectional area of belt} = \sigma.\, b.\, t$$

When centrifugal tension is neglected, then

$T(\text{or } T_{t1}) = T_1$, *i.e.* Tension in the tight side of the belt

and when centrifugal tension is considered, then

$$T(\text{or } T_{t1}) = T_1 + T_C$$

.18. Condition For the Transmission of Maximum Power

We know that power transmitted by a belt,

$$P = (T_1 - T_2)\, v \quad \text{...}(i)$$

where
T_1 = Tension in the tight side of the belt in newtons,
T_2 = Tension in the slack side of the belt in newtons, and
v = Velocity of the belt in m/s.

From Art. 11.14, we have also seen that the ratio of driving tensions is

$$\frac{T_1}{T_2} = e^{\mu.\theta} \quad \text{or} \quad T_2 = \frac{T_1}{e^{\mu.\theta}} \quad \text{...}(ii)$$

Substituting the value of T_2 in equation (*i*),

$$P = \left(T_1 - \frac{T_1}{e^{\mu.\theta}}\right)v = T_1\left(1 - \frac{1}{e^{\mu.\theta}}\right)v = T_1.v.C \quad \text{...}(iii)$$

where $$C = 1 - \frac{1}{e^{\mu.\theta}}$$

We know that $T_1 = T - T_C$

where T = Maximum tension to which the belt can be subjected in newtons, and

T_C = Centrifugal tension in newtons.

Substituting the value of T_1 in equation (*iii*),

$$P = (T - T_C)\, v.C$$

$$= (T - m.v^2)\, v.C = (T.v - m\, v^3)\, C \quad \text{... (Substituting } T_C = m.\, v^2\text{)}$$

For maximum power, differentiate the above expression with respect to v and equate to zero, *i.e.*

$$\frac{dP}{dv} = 0 \quad \text{or} \quad \frac{d}{dv}(T.v - mv^3)\, C = 0$$

$$\therefore \quad T - 3\, m\,.\, v^2 = 0$$

or $$T - 3\, T_C = 0 \text{ or } T = 3\, T_C \quad \text{...}(iv)$$

It shows that when the power transmitted is maximum, 1/3rd of the maximum tension is absorbed as centrifugal tension.

Notes : 1. We know that $T_1 = T - T_C$ and for maximum power, $T_C = \frac{T}{3}$.

$$\therefore \quad T_1 = T - \frac{T}{3} = \frac{2T}{3}$$

2. From equation (*iv*), the velocity of the belt for the maximum power,

$$v = \sqrt{\frac{T}{3\,m}}$$

Example. 11.7. *A shaft rotating at 200 r.p.m. drives another shaft at 300 r.p.m. and transmits 6 kW through a belt. The belt is 100 mm wide and 10 mm thick. The distance between the shafts is 4m. The smaller pulley is 0.5 m in diameter. Calculate the stress in the belt, if it is 1. an open belt drive, and 2. a cross belt drive. Take $\mu = 0.3$.*

Solution. Given : $N_1 = 200$ r.p.m. ; $N_2 = 300$ r.p.m. ; $P = 6$ kW $= 6 \times 10^3$ W ; $b = 100$ mm ; $t = 10$ mm ; $x = 4$ m ; $d_2 = 0.5$ m ; $\mu = 0.3$

Let σ = Stress in the belt.

1. ***Stress in the belt for an open belt drive***

First of all, let us find out the diameter of larger pulley (d_1). We know that

$$\frac{N_2}{N_1} = \frac{d_1}{d_2} \quad \text{or} \quad d_1 = \frac{N_2.d_2}{N_1} = \frac{300 \times 0.5}{200} = 0.75\,\text{m}$$

and velocity of the belt, $$v = \frac{\pi\, d_2.N_2}{60} = \frac{\pi \times 0.5 \times 300}{60} = 7.855 \text{ m/s}$$

Now let us find the angle of contact on the smaller pulley. We know that, for an open belt drive,

$$\sin \alpha = \frac{r_1 - r_2}{x} = \frac{d_1 - d_2}{2x} = \frac{0.75 - 0.5}{2 \times 4} = 0.03125 \quad \text{or} \quad \alpha = 1.8°$$

∴ Angle of contact, $\theta = 180° - 2\alpha = 180 - 2 \times 1.8 = 176.4°$

$= 176.4 \times \pi / 180 = 3.08$ rad

Let T_1 = Tension in the tight side of the belt, and

T_2 = Tension in the slack side of the belt.

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.3 \times 3.08 = 0.924$$

$$\therefore \quad \log\left(\frac{T_1}{T_2}\right) = \frac{0.924}{2.3} = 0.4017 \text{ or } \frac{T_1}{T_2} = 2.52 \quad \text{...(i)}$$

...(Taking antilog of 0.4017)

We also know that power transmitted (P),

$$6 \times 10^3 = (T_1 - T_2)\, v = (T_1 - T_2)\, 7.855$$

$$\therefore \quad T_1 - T_2 = 6 \times 10^3 / 7.855 = 764 \text{ N} \quad \text{...(ii)}$$

From equations (*i*) and (*ii*),

$$T_1 = 1267 \text{ N, and } T_2 = 503 \text{ N}$$

We know that maximum tension in the belt (T_1),

$$1267 = \sigma . b . t = \sigma \times 100 \times 10 = 1000\,\sigma$$

$$\therefore \quad \sigma = 1267 / 1000 = 1.267 \text{ N/mm}^2 = 1.267 \text{ MPa} \quad \textbf{Ans.}$$

...[∵ 1 MPa = 1 MN/m² = 1 N/mm²]

Stress in the belt for a cross belt drive

We know that for a cross belt drive,

$$\sin \alpha = \frac{r_1 + r_2}{x} = \frac{d_1 + d_2}{2x} = \frac{0.75 + 0.5}{2 \times 4} = 0.1562 \text{ or } \alpha = 9°$$

∴ Angle of contact, $\theta = 180° + 2\alpha = 180 + 2 \times 9 = 198°$

$= 198 \times \pi / 180 = 3.456$ rad

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.3 \times 3.456 = 1.0368$$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{1.0368}{2.3} = 0.4508 \text{ or } \frac{T_1}{T_2} = 2.82 \quad \text{...(iii)}$$

...(Taking antilog of 0.4508)

From equations (*ii*) and (*iii*),

$$T_1 = 1184 \text{ N and } T_2 = 420 \text{ N}$$

We know that maximum tension in the belt (T_1),

$$1184 = \sigma . b . t = \sigma \times 100 \times 10 = 1000\,\sigma$$

$$\therefore \quad \sigma = 1184 / 1000 = 1.184 \text{ N/mm}^2 = 1.184 \text{ MPa} \quad \textbf{Ans.}$$

Example 11.8. *A leather belt is required to transmit 7.5 kW from a pulley 1.2 m in diameter, running at 250 r.p.m. The angle embraced is 165° and the coefficient of friction between the belt and the pulley is 0.3. If the safe working stress for the leather belt is 1.5 MPa, density of leather 1 Mg/m³ and thickness of belt 10 mm, determine the width of the belt taking centrifugal tension into account.*

Solution. Given : $P = 7.5$ kW = 7500 W ; $d = 1.2$ m ; $N = 250$ r.p.m. ; $\theta = 165° = 165 \times \pi / 180 = 2.88$ rad ; $\mu = 0.3$; $\sigma = 1.5$ MPa = 1.5×10^6 * N/m² ; $\rho = 1$ Mg/m³ = 1×10^6 g/m³ = 1000 kg/m³; $t = 10$ mm = 0.01 m

Let b = Width of belt in metres,

T_1 = Tension in the tight side of the belt in N, and

T_2 = Tension in the slack side of the belt in N.

We know that velocity of the belt,

$$v = \pi d . N / 60 = \pi \times 1.2 \times 250/60 = 15.71 \text{ m/s}$$

and power transmitted (P),

$$7500 = (T_1 - T_2)\, v = (T_1 - T_2)\, 15.71$$

$$\therefore \quad T_1 - T_2 = 7500 / 15.71 = 477.4 \text{ N} \quad \text{...}(i)$$

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu . \theta = 0.3 \times 2.88 = 0.864$$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{0.864}{2.3} = 0.3756 \text{ or } \frac{T_1}{T_2} = 2.375 \quad \text{...}(ii)$$

...(Taking antilog of 0.3756)

From equations (*i*) and (*ii*),

$$T_1 = 824.6 \text{ N, and } T_2 = 347.2 \text{ N}$$

We know that mass of the belt per metre length,

$$m = \text{Area} \times \text{length} \times \text{density} = b.t.l.\rho$$

$$= b \times 0.01 \times 1 \times 1000 = 10\, b \text{ kg}$$

$\therefore$ Centrifugal tension,

$$T_C = m . v^2 = 10\, b\, (15.71)^2 = 2468\, b \text{ N}$$

and maximum tension in the belt,

$$T = \sigma . b . t = 1.5 \times 10^6 \times b \times 0.01 = 15\,000\, b \text{ N}$$

We know that $T = T_1 + T_C$ or $15000\, b = 824.6 + 2468\, b$

$$15\,000\, b - 2468\, b = 824.6 \text{ or } 12\,532\, b = 824.6$$

$$\therefore \quad b = 824.6 / 12532 = 0.0658 \text{ m} = 65.8 \text{ mm} \quad \textbf{Ans.}$$

Example. 11.9. *Determine the width of a 9.75 mm thick leather belt required to transmit 15 kW from a motor running at 900 r.p.m. The diameter of the driving pulley of the motor is 300 mm. The driven pulley runs at 300 r.p.m. and the distance between the centre of two pulleys is 3 metres. The density of the leather is 1000 kg/m³. The maximum allowable stress in the leather is 2.5 MPa. The coefficient of friction between the leather and pulley is 0.3. Assume open belt drive and neglect the sag and slip of the belt.*

Solution. Given : $t = 9.75$ mm = 9.75×10^{-3} m ; $P = 15$ kW = 15×10^3 W ; $N_1 = 900$ r.p.m. ; $d_1 = 300$ mm = 0.3 m ; $N_2 = 300$ r.p.m. ; $x = 3$m ; $\rho = 1000$ kg/m³ ; $\sigma = 2.5$ MPa = 2.5×10^6 N/m² ; $\mu = 0.3$

* 1 MPa = 1×10^6 N/m²

First of all, let us find out the diameter of the driven pulley (d_2). We know that

$$\frac{N_2}{N_1} = \frac{d_1}{d_2} \text{ or } d_2 = \frac{N_1 \times d_1}{N_2} = \frac{900 \times 0.3}{300} = 0.9 \text{ m}$$

and velocity of the belt,
$$v = \frac{\pi d_1 N_1}{60} = \frac{\pi \times 0.3 \times 900}{60} = 14.14 \text{ m/s}$$

For an open belt drive,

$$\sin \alpha = \frac{r_2 - r_1}{x} = \frac{d_2 - d_1}{2x} = \frac{0.9 - 0.3}{2 \times 3} = 0.1 \quad \text{...}(\because d_2 > d_1)$$

or
$$\alpha = 5.74°$$

∴ Angle of lap,
$$\theta = 180° - 2\alpha = 180 - 2 \times 5.74 = 168.52°$$
$$= 168.52 \times \pi / 180 = 2.94 \text{ rad}$$

Let T_1 = Tension in the tight side of the belt, and

T_2 = Tension in the slack side of the belt.

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.3 \times 2.94 = 0.882$$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{0.882}{2.3} = 0.3835 \text{ or } \frac{T_1}{T_2} = 2.42 \quad \text{...}(i)$$

... (Taking antilog of 0.3835)

We also know that power transmitted (P),

$$15 \times 10^3 = (T_1 - T_2)\, v = (T_1 - T_2)\, 14.14$$

∴
$$T_1 - T_2 = 15 \times 10^3 / 14.14 = 1060 \text{ N} \quad \text{....}(ii)$$

From equations (*i*) and (*ii*),

$$T_1 = 1806 \text{ N}$$

Let b = Width of the belt in metres.

We know that mass of the belt per metre length,

$$m = \text{Area} \times \text{length} \times \text{density} = b.t.l.\rho$$
$$= b \times 9.75 \times 10^{-3} \times 1 \times 1000 = 9.75\, b \text{ kg}$$

∴ Centrifugal tension,

$$T_C = m.v^2 = 9.75\, b\, (14.14)^2 = 1950\, b \text{ N}$$

Maximum tension in the belt,

$$T = \sigma.\, b.\, t = 2.5 \times 10^6 \times b \times 9.75 \times 10^{-3} = 24\,400\, b \text{ N}$$

We know that
$$T = T_1 + T_C \text{ or } T - T_C = T_1$$

$$24\,400\, b - 1950\, b = 1806 \text{ or } 22\,450\, b = 1806$$

∴
$$b = 1806 / 22\,450 = 0.080 \text{ m} = 80 \text{ mm} \quad \textbf{Ans.}$$

Example. 11.10. *A pulley is driven by a flat belt, the angle of lap being 120°. The belt is 100 mm wide by 6 mm thick and density1000 kg/m³. If the coefficient of friction is 0.3 and the maximum stress in the belt is not to exceed 2 MPa, find the greatest power which the belt can transmit and the corresponding speed of the belt.*

Solution. Given : $\theta = 120° = 120 \times \pi / 180 = 2.1$ rad ; $b = 100$ mm $= 0.1$ m ; $t = 6$ mm $= 0.006$ m ; $\rho = 1000$ kg / m^3 ; $\mu = 0.3$; $\sigma = 2$ MPa $= 2 \times 10^6$ N/m^2

Speed of the belt for greatest power

We know that maximum tension in the belt,

$$T = \sigma .\ b.\ t = 2 \times 10^6 \times 0.1 \times 0.006 = 1200 \text{ N}$$

and mass of the belt per metre length,

$$m = \text{Area} \times \text{length} \times \text{density} = b.\ t.\ l.\ \rho$$
$$= 0.1 \times 0.006 \times 1 \times 1000 = 0.6 \text{ kg/m}$$

∴ Speed of the belt for greatest power,

$$v = \sqrt{\frac{T}{3m}} = \sqrt{\frac{1200}{3 \times 0.6}} = 25.82 \text{ m/s} \qquad \textbf{Ans.}$$

Greatest power which the belt can transmit

We know that for maximum power to be transmitted, centrifugal tension,

$$T_C = T/3 = 1200/3 = 400 \text{ N}$$

and tension in the tight side of the belt,

$$T_1 = T - T_C = 1200 - 400 = 800 \text{ N}$$

Let T_2 = Tension in the slack side of the belt.

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.3 \times 2.1 = 0.63$$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{0.63}{2.3} = 0.2739 \quad \text{or} \quad \frac{T_1}{T_2} = 1.88 \qquad \text{...(Taking antilog of 0.2739)}$$

and

$$T_2 = \frac{T_1}{1.88} = \frac{800}{1.88} = 425.5 \text{ N}$$

∴ Greatest power which the belt can transmit,

$$P = (T_1 - T_2)\ v = (800 - 425.5)\ 25.82 = 9670 \text{ W} = 9.67 \text{ kW} \quad \textbf{Ans.}$$

Example 11.11. *An open belt drive connects two pulleys 1.2 m and 0.5 m diameter, on parallel shafts 4 metres apart. The mass of the belt is 0.9 kg per metre length and the maximum tension is not to exceed 2000 N. The coefficient of friction is 0.3. The 1.2 m pulley, which is the driver, runs at 200 r.p.m. Due to belt slip on one of the pulleys, the velocity of the driven shaft is only 450 r.p.m. Calculate the torque on each of the two shafts, the power transmitted, and power lost in friction. What is the efficiency of the drive ?*

Solution. Given : $d_1 = 1.2$ m or $r_1 = 0.6$ m ; $d_2 = 0.5$ m or $r_2 = 0.25$ m ; $x = 4$ m ; $m = 0.9$ kg/m; $T = 2000$ N ; $\mu = 0.3$; $N_1 = 200$ r.p.m. ; $N_2 = 450$ r.p.m.

We know that velocity of the belt,

$$v = \frac{\pi d_1 . N_1}{60} = \frac{\pi \times 1.2 \times 200}{60} = 12.57 \text{ m/s}$$

and centrifugal tension, $\quad T_C = m.v^2 = 0.9\ (12.57)^2 = 142$ N

∴ Tension in the tight side of the belt,

$$T_1 = T - T_C = 2000 - 142 = 1858 \text{ N}$$

We know that for an open belt drive,

$$\sin\alpha = \frac{r_1 - r_2}{x} = \frac{0.6 - 0.25}{4} = 0.0875 \text{ or } \alpha = 5.02°$$

∴ Angle of lap on the smaller pulley,

$$\theta = 180° - 2\alpha = 180° - 2 \times 5.02° = 169.96°$$

$$= 169.96 \times \pi / 180 = 2.967 \text{ rad}$$

Let T_2 = Tension in the slack side of the belt.

We know that

$$2.3\log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.3 \times 2.967 = 0.8901$$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{0.8901}{2.3} = 0.387 \text{ or } \frac{T_1}{T_2} = 2.438$$

...(Taking antilog of 0.387)

$$\therefore \quad T_2 = \frac{T_1}{2.438} = \frac{1858}{2.438} = 762 \text{ N}$$

Torque on the shaft of larger pulley

We know that torque on the shaft of larger pulley,

$$T_L = (T_1 - T_2)\, r_1 = (1858 - 762)\, 0.6 = 657.6 \text{ N-m}$$ **Ans.**

Torque on the shaft of smaller pulley

We know that torque on the shaft of smaller pulley,

$$T_S = (T_1 - T_2)\, r_2 = (1858 - 762)\, 0.25 = 274 \text{ N-m}$$ **Ans.**

Power transmitted

We know that the power transmitted,

$$P = (T_1 - T_2)\, v = (1858 - 762)\, 12.57 = 13780 \text{ W}$$

$$= 13.78 \text{ kW}$$ **Ans.**

Power lost in friction

We know that input power,

$$P_1 = \frac{T_L \times 2\pi N_1}{60} = \frac{657.6 \times 2\pi \times 200}{60} = 13\,780 \text{ W} = 13.78 \text{kW}$$

and output power,

$$P_2 = \frac{T_S \times 2\pi N_2}{60} = \frac{274 \times 2\pi \times 450}{60} = 12\,910 \text{ W} = 12.91 \text{kW}$$

∴ Power lost in friction $= P_1 - P_2 = 13.78 - 12.91 = 0.87$ kW **Ans.**

Efficiency of the drive

We know that efficiency of the drive,

$$\eta = \frac{\text{Output power}}{\text{Input power}} = \frac{12.91}{13.78} = 0.937 \text{ or } 93.7\%$$ **Ans.**