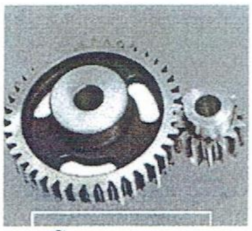
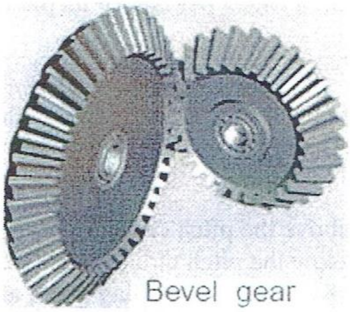
Gear and Gear Trains

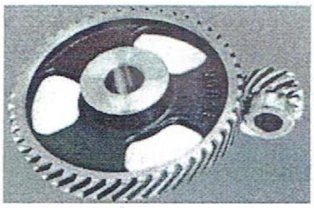
Classification of gears

(1) Depending upon the relative position of their axis

.*(a) parallel (spur gears), (b) intersecting (bevel gears) and (c) non parallel and non intersecting (spiral or cross helical gears).*


Spur gears


Bevel gear


Crossed helical gears.

(2) Depending upon their peripheral speed of the gear.

(a) Low velocity gear $\langle 3\ m/\sec$, *(b) medium velocity gear* $\rangle 3\ m/\sec \langle 15\ m/\sec$ *and (c) high velocity gear* $\rangle 15\ m/\sec$.

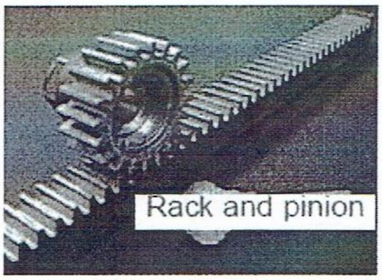
(3) Depending upon type of teeth.

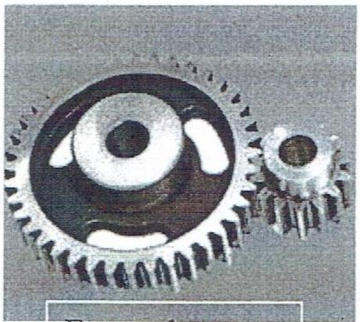
(a) Straight, (b) inclined and (c) curved.

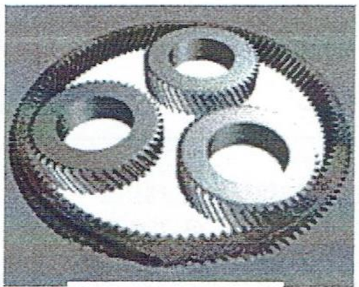

Helical Gears

(4) Depending upon the type of gearing.

(a) Internal gearing. (b) external gearing and (c) rack and pinion.


Rack and pinion


External gears


Internal gears

Spur gears

When two gears are in mesh the larger one is called wheel or spur and the smaller is called pinion. Figure (1) show the terms used in gear.

Pitch circle diameter (D, d): It is the diameter of the pitch circle. The size of the gear is usually specified by the pitch circle diameter. It is also known as ***pitch diameter***.

T, t .... *Number of teeth* Ω, ω .... *Angular velocity of the wheel and pinion*

$$\frac{\Omega}{\omega} = \frac{d}{D} = \frac{t}{T}$$

The pitch circle radius is R and r.

(1-9)

The pitch point is the point of contact of two pitch circles.

Circular pitch (P_c): The distance between a point on one tooth and the corresponding point on an adjacent tooth, measured along the pitch circle. $\left\{P_c = \frac{\pi D}{T} = \frac{\pi d}{t}\right\}$

Dimetral pitch (P): The number of teeth on a wheel per unit of its pitch diameter (No. of teeth per mm of pitch circle diameter). $P = \frac{T}{D} = \frac{t}{d} = \frac{\pi}{P_c}$

Base circle: The circle from which the involute curve forming the tooth profiles are drawn.

Module (m): It is the inverse of dimetral pitch (P). $\left\{m = \frac{D}{T} = \frac{d}{t} = \frac{1}{P}\right\}$

Addendum: The radial height of a tooth above the pitch circle, its standard value equal to one module.
Dedendum: The radial depth of a tooth below the pitch circle, its standard value equal to (1.25) module.
Working depth: The sum of the addendum of the two meshing gears and is equal to (2) module.
Clearance: The difference between the addendum and dedendum, its standard value equal to (0.25) module.
Whole depth: Equal to (addendum + dedendum), also equal to (working depth + clearance).
Pressure angle or angle of obliquity (Φ): The angle between the common normal to the two teeth in contact and the common tangent to the pitch point.

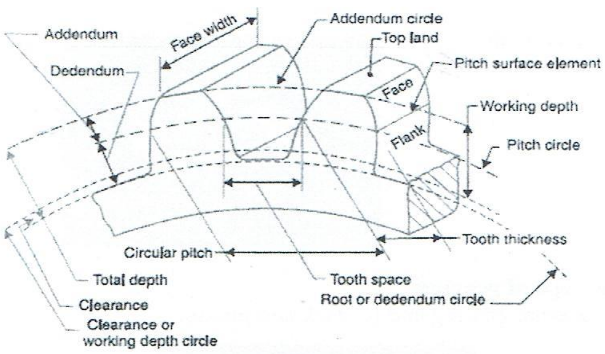


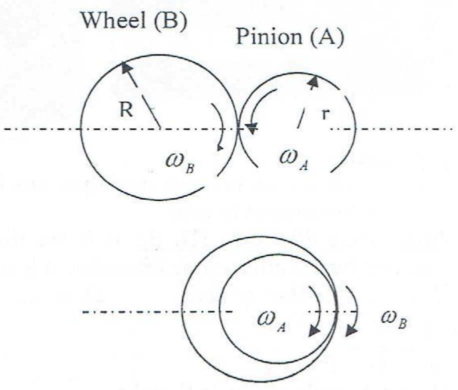
Figure (1) Terms used in gear

Velocity ratio (gear ratio)

Peripheral speed $\omega_A\, r = \omega_B\, R$

Since the two gears have the same pitch so $(\frac{R}{r} = \frac{T_B}{T_A})$

$$\therefore \frac{\omega_A}{\omega_B} = \frac{R}{r} = \frac{T_B}{T_A} = \frac{N_A}{N_B}$$



(2-9)

Center to center distance

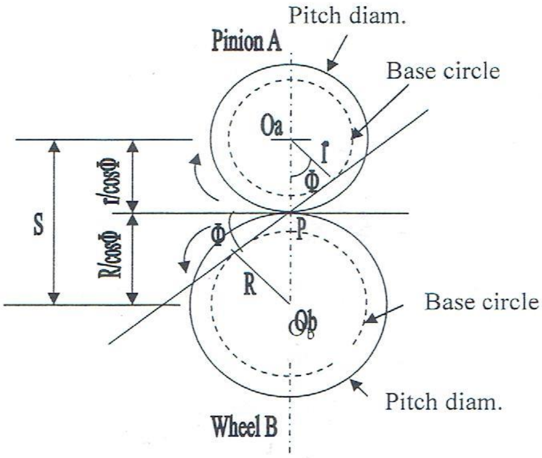
(a) $S = O_aP + O_bP = \dfrac{r}{\cos\phi} + \dfrac{R}{\cos\phi} = \dfrac{(r+R)}{\cos\phi}$

(b)

$\because m = \dfrac{D}{T} \Rightarrow D = m \quad T$

$\therefore D_A = m \quad T_A \ \ and \ \ D_B = m \quad T_B$

$\because S = \dfrac{D_A}{2} + \dfrac{D_B}{2} = \dfrac{m(T_A + T_B)}{2}$



Ex/ Two mating gear A and B. Center to center distance is equal to 375 mm. Gear A rotate at 500 rpm while gear B rotate at 350 rpm (approximately). If m = 5 mm, determine T_A, T_B and N_B (actual).

Solution:

$S = \dfrac{m(T_A + T_B)}{2} \Rightarrow \quad 750 = 5\,(T_A + T_B) \ \Rightarrow \ (T_A + T_B) = 150 \ ......(1)$

$\dfrac{N_A}{N_B} = \dfrac{T_B}{T_A} \Rightarrow \ \dfrac{500}{350} = \dfrac{T_B}{T_A} \ .....(2) \quad from\ equation\ (1)\ and\ (2) \ \Rightarrow \ T_A = 61.76\ and\ T_B = 88.24$

The selection is $T_A = 60$ and $T_B = 90$

The actual speed of gear B is $\Rightarrow \dfrac{500}{N_B} = \dfrac{90}{60} \Rightarrow N_B = 333.3\ rpm$

Ex/ Two parallel shafts about 60 cm apart, are to be connected by spur wheels. One shaft is to be run at 360 rpm and the other at 120 rpm. Design the wheels, if the circular pitch is to be 25 mm.

Solution:

$\dfrac{N_1}{N_2} = \dfrac{d_2}{d_1} = \dfrac{T_2}{T_1} = \dfrac{360}{120} = 3 \Rightarrow d_2 = 3 * d_1 \ ....(1)$

$\because S = \dfrac{d_1 + d_2}{2} \Rightarrow 60 = \dfrac{d_1 + d_2}{2} \Rightarrow d_1 + d_2 = 120 \ ......(2) \quad from\ equations\ (1)\ and\ (2)$

$d_1 = 30\ cm\ and\ \ d_2 = 90\ cm$

$\therefore T_1 = \dfrac{\pi\, d_1}{P_c} = \dfrac{\pi * 30}{2.5} = 37.7 \quad and \quad T_2 = \dfrac{\pi\, d_2}{P_c} = \dfrac{\pi * 90}{2.5} = 113.1$

Since the number of teeth on both the wheels are to be in complete number. Then the selection is
$T_1 = 38$ and $T_2 = 3T_1 = 3 * 38 = 114$
The exact diameter for both wheels is

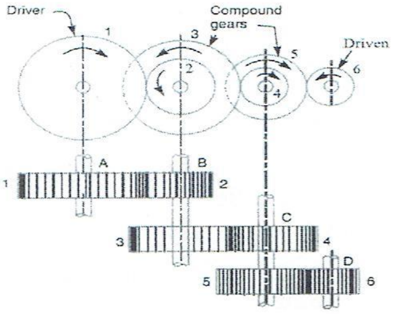
$d_1' = \dfrac{T_1 * P_c}{\pi} = \dfrac{38 * 2.5}{\pi} = 30.24\ cm \quad and \quad d_2' = \dfrac{T_2 * P_c}{\pi} = \dfrac{114 * 2.5}{\pi} = 90.72\ cm$

$\therefore The\ exact\ distance\ between\ the\ two\ shaft\ (S') = \dfrac{d_1' + d_2'}{2} = \dfrac{30.24 + 90.72}{2} = 60.48\ cm$

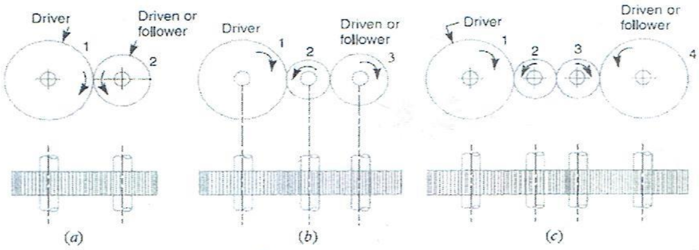
(3-9)

Gear trains

(1) Simple gear train --- each shaft carries a single gear.
(2) Compound gear train --- each shaft carries two wheels.
(3) Epicyclic gear train.



Compound gear train.



Simple gear train.

(1) Velocity ratio of simple gear trains

Peripheral speed of A = Peripheral speed of B

$$\pi D_a N_a = \pi D_B N_B \Rightarrow \frac{N_b}{N_a} = \frac{D_a}{D_b}$$

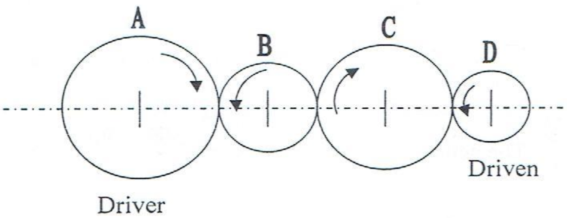
Diametral pitch of A = Diametral pitch of B

$$\frac{T_a}{D_a} = \frac{T_b}{D_b}$$

$$\therefore \frac{N_b}{N_a} = \frac{T_a}{T_b} \Rightarrow N_b = N_a \frac{T_a}{T_b}$$

$$Similarly \;\; N_c = N_b \frac{T_b}{T_c} = N_a \frac{T_a}{T_b} * \frac{T_b}{T_c} = N_a \frac{T_a}{T_c}$$

$$also \;\; \frac{N_d}{N_c} = \frac{T_c}{T_d} \Rightarrow N_d = N_c \frac{T_c}{T_d} = N_a \frac{T_a}{T_c} * \frac{T_c}{T_d} = N_a \frac{T_a}{T_d}$$



$$\therefore \frac{N_d}{N_a} = \frac{T_a}{T_d}$$

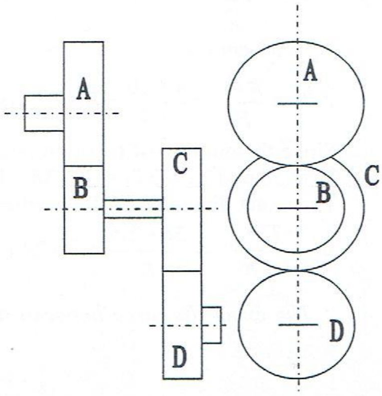
(2) Velocity ratio of compound gear trains

$$\frac{N_b}{N_a} = \frac{T_a}{T_b} \Rightarrow N_b = N_a \frac{T_a}{T_b} = N_c \;\; (because\ B\ and\ C\ on\ the\ same\ shaft)$$

$$\frac{N_d}{N_c} = \frac{T_c}{T_d} \Rightarrow N_d = N_c \frac{T_c}{T_d} = N_a \frac{T_a}{T_b} * \frac{T_c}{T_d}$$

$$\frac{N_d}{N_a} = \frac{T_a * T_c}{T_b * T_d}$$

$$\frac{Speed\ of\ driven}{Speed\ of\ driver} = \frac{Product\ of\ teeth\ on\ driver}{Product\ of\ teeth\ on\ driven} \quad (4\text{-}9)$$



(3) Epicyclic gear trains

It could be simple or compound. It is used for transmitting very high velocity ratio.

(a) Simple epicyclic gear trains

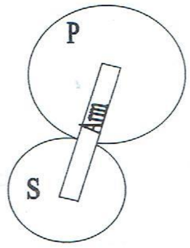
Two wheels (P) and (S) connected by arm.

- Let us suppose that the wheel (S) is fixed.

(1) Suppose (P) and (S) do not mesh. For one revolution of the arm, (P) will rotate one revolution.

(2) Now if (P) mesh with (S). So for one revolution to the arm (N_{arm}), (P) will rotate

$$N_P = N_S(N_{arm} + \frac{T_S}{T_P}) = N_S(1 + \frac{T_S}{T_P})$$



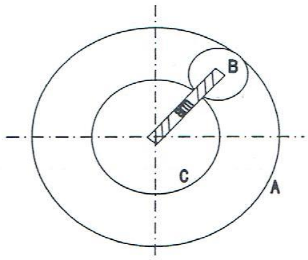
Tabular method

Step No.	Condition of motion	Revolution of element		
		Arm	P	S
1	Arm fixed and P rotate (+x) rev. (clockwise)	0	+ x	$-x\frac{T_P}{T_S}$
2	Add (+ y) to all elements	+ y	+ y	+ y
3	Total motion	+ y	x + y	$y - x\frac{T_P}{T_S}$

Ex/ An epicyclic gear consists of three wheels A, B and C as shown below. When A has 72 internal teeth, C has 32 external teeth. The wheel B gears with both A and C and is carried on an arm which rotate about the center A at 18 rpm. If the wheel A is fixed, determine the speed of wheels B and C.

Solution:

Step No.	Condition of motion	Revolution of element			
		Arm	C	B	A
1	Arm fixed , C rotate (+ x) revolution	0	+ x	$-x\frac{T_C}{T_B}$	$-x\frac{T_C}{T_A}$
2	Add (+ y) rev. to all element	+ y	+ y	+ y	+ y
3	Total motion	$+y$	$x+y$	$y-x\frac{T_C}{T_B}$	$y-x\frac{T_C}{T_A}$



$\because y = 18\ rpm \quad and \quad y - x\frac{T_C}{T_A} = 0 \Rightarrow 18 - x * \frac{32}{72} = 0 \Rightarrow x = 40.5\ rpm$

$\therefore N_C = x + y = 40.5 + 18 = 58.5\ rpm$

Let (d_A), (d_B) and (d_C) pitch diameter.

$\therefore d_B + \frac{d_C}{2} = \frac{d_A}{2}$ (Since the number of teeth are proportional to their pitch diameter)

$\therefore T_B + \frac{T_C}{2} = \frac{T_A}{2} \Rightarrow T_B = \frac{72}{2} - \frac{32}{2} = 20$

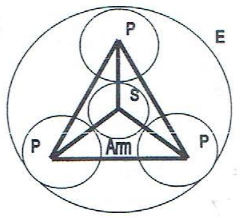
$\therefore N_B = y - x\frac{T_C}{T_B} = 18 - 40.5 * \frac{32}{20} = 46.8\ rpm$

(5-9)

Ex/ An epicyclic train of gear as shown in figure, the size of different toothed gears are such that the arm rotates at 1/5 of the speed of gear S. The number of teeth of gear S is 16. Determine (a) number of teeth on different gears of train (b) velocity ratio of arm with respect to gear S and gear P if gear E is fixed and the arm make +1 revolution.

Solution:

Step No.	Condition of motion	Revolution of element			
		Arm	S	P	E
1	Arm fixed , S rotate (+ x) revolution (anticlockwise)	0	+ x	$-x\frac{T_S}{T_P}$	$-x\frac{T_S}{T_E}$
2	Add (+ y) rev. to all element	+ y	+ y	+ y	+ y
3	Total motion	$+y$	$x+y$	$y-x\frac{T_S}{T_P}$	$y-x\frac{T_S}{T_E}$



(a) Since (S) makes 5 rev. and the arm makes 1 rev.

$y = 1\ rpm, \quad and\ x + y = 5 \Rightarrow \therefore x = 4\ rpm$

$\because gear\ (E)\ fixed \Rightarrow \therefore y \quad x\frac{T_S}{T_E} = 0 \Rightarrow 1 \quad 4\frac{16}{T_E} = 0 \Rightarrow \therefore T_E = 64$

$\because \frac{d_S}{2} + d_P = \frac{d_E}{2} \Rightarrow d_S + 2d_P = d_E \Rightarrow \therefore T_S + 2T_P = T_E \Rightarrow 16 + 2T_P = 64 \Rightarrow \therefore T_P = 24$

(b) $\because y = 1\ rpm \quad and\ y \quad x\frac{T_S}{T_E} = 0 \quad \Rightarrow 1 \quad x \quad \frac{16}{24} = 0 \Rightarrow \therefore x = 4\ rpm$

$\therefore N_S = x + y = 4 + 1 = 5\ rpm, \qquad N_P = y - x\frac{T_S}{T_P} = 1 - 4 * \frac{16}{24} = -1.66\ rpm \quad and \quad N_{arm} = y = 1\ rpm$

$\therefore \frac{N_{arm}}{N_S} = \frac{1}{5} = 0.2 \qquad and \qquad \frac{N_{arm}}{N_P} = \frac{1}{-1.66} = -0.6$

(6-9)

(b) Compund epicyclic gear trains

Ex/ An epicyclic gear train shown in figure, with compound planets B - C. Gear B has 20 teeth and meshes with an annulus gear A which has 60 teeth. C has 16 teeth and meshes with the gear D. If gear A is fixed and the arm rotates at 200 rpm, find the speed of gear D.

Solution:

Step No.	Condition of motion	Revolution of element			
		Arm	D	B-C	A
1	Arm fixed, D rotate (+ x) revolution (anticlockwise)	0	+ x	$-x\frac{T_D}{T_C}$	$-x\frac{T_D * T_B}{T_C * T_A}$
2	Add (+ y) rev. to all element	+ y	+ y	+ y	+ y
3	Total motion	$+y$	$x+y$	$y-x\frac{T_D}{T_C}$	$y-x\frac{T_D}{T_C}\frac{T_B}{T_A}$

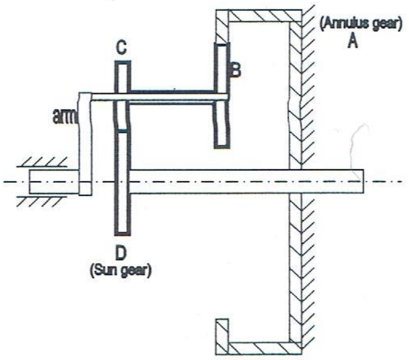
$\because N_{arm} = y = 200\ rpm$

$and\ \ y - x\frac{T_D * T_B}{T_C * T_A} = 0 \Rightarrow 200 - x\frac{T_D * 20}{16 * 60} = 0 \Rightarrow x * T_D = 9600.....(1)$

$\because \frac{d_D}{2} + \frac{d_C}{2} + \frac{d_B}{2} = \frac{d_A}{2} \Rightarrow T_D + T_C + T_B = T_A \Rightarrow T_D = 60 \quad 16 \quad 20 = 24$

$Sub.\ in\ equation(1) \quad \therefore x \quad 24 = 9600 \Rightarrow x = \frac{9600}{24} = 400\ rpm$

$\therefore N_D = x + y = 400 + 200 = 600\ rpm$



(Homework's)

Q1/ Two shafts which are 300 mm apart are to be connected by spur gearing. The driving shaft rotates at 1400 rpm and the driven is to be rotate at an approximate speed of 400 rpm. If the module pitch of the gears is 8 mm, determine the number of teeth on the gears and the actual speed of the driven shaft. Assume that the gears available are of teeth rising in steps of 3 and the smallest gear dose not has teeth less than 15.

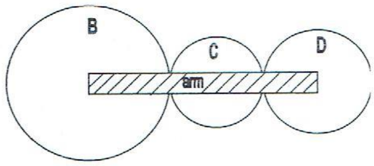
Q2/ Find the pitch diameter, diametral pitch and module of a toothed gear having 36 teeth and circular pitch of 13 mm.

Q3/ Two wheels of 18 and 45 teeth respectively are geared together. Determine the pitch circle radius of the two wheels, if the circular pitch is 50 cm.

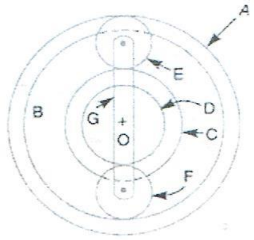
Q4/ In a compound train of gears spur gear 1, with 40 teeth and rotating at 15 rpm, drives a pinion 6 with 15 teeth at 300 rpm across two intermediate spindles. The first intermediate has gears 2 and 3 and the second carries gears 4 and 5. Gear 2 gearing with gear 1 while 3 gearing with gear 4 and 5 gearing with gear 6. If gears 2, 4 and 5 have 20, 15 and 30 teeth respectively, calculate the number of teeth on gear 3.

(7-9)

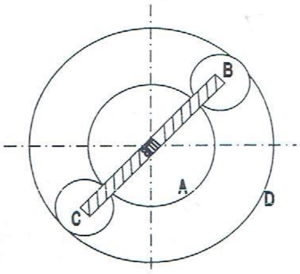
Q5/ An epicyclic gear train shown gear B has 80 teeth and is rigidly fixed to the motor shaft. Gear C has 16 teeth and gears with B and also with gear D which has 20 teeth. The arm rotates about the same shaft on which B is fixed and carries gear C and D. If the arm runs at 1000 rpm, find the speed of gear C and D.



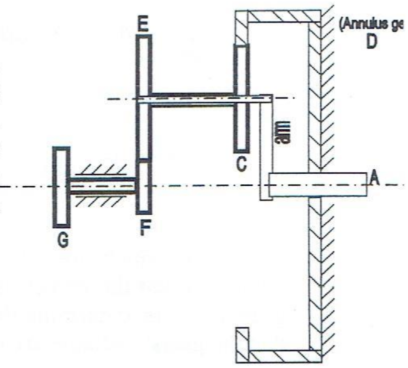
Q6/ An epicyclic gear train shown, the gears A and B are internal gears and the gear C-D is a compound gear. The gears E and F have the same pitch and number of teeth ($T_e = T_f = 18$), $T_c = 28$ and $T_d = 26$. (1) Find the number of teeth on A and B; (2) If the arm G makes 100 rpm clockwise and A is fixed, find the speed of B; and (3) If the arm G makes 100 rpm clockwise and wheel A makes 10 rpm counter clockwise ; find the speed of wheel B. **(Note A and B not a compound gears)**



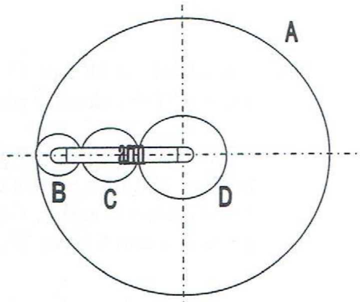
Q7/ An epicyclic train of gear as shown in figure, how many revolution does the arm, to which the pinions B and C are attached, make: (a) When A makes one revolution clockwise and D makes half a revolution anticlockwise, and (b) When A makes one revolution clockwise and D is fixed. The number of teeth in the gear A and D are 40 and 90 respectively.



Q8/ An epicyclic reduction gear shown in figure, has a shaft A fixed to arm. The arm has a pin fixed to its outer end and two gears C and E which are rigidly fixed (C-E compound gear) revolve on this pin. Gear C meshes with annular gear D and gear E with pinion F. G is the driver gear and D is kept stationary. The number of the teeth is as follows: $T_D = 80$, $T_C = 10$, $T_E = 24$ and $T_F = 10$. The gear G runs at 200 rpm, find the speed of shaft A.

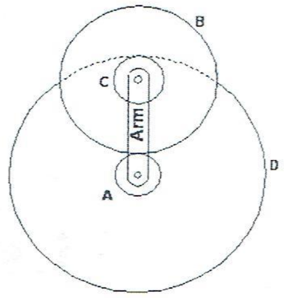


Q9/ An epicyclic train as shown in figure below is composed of a fixed annular wheel **A** having 150 teeth. Meshing with **A** is a wheel **B** which drives wheel **D** through an idle wheel **C**, **D** being concentric with **A**. Wheels **B** and **C** are carried on an arm which revolves clockwise at 100 rpm about the axis of **A** or **D**. If the wheels **B** and **D** are having 25 teeth and 40 teeth respectively, find the number of teeth of **C** and the speed and direction of rotation of **C**.



(8-9)

Q10/ An epicyclic gear train shown in figure, the arm is keyed to the same shaft as the load drum and the wheel A is keyed to the second shaft which caries a chain wheel, the chain being operated by hand. The two shafts have common axis but can rotate independently. The wheels B and C are compound and rotate together on a pin carried at the end of arm. The wheel D has internal teeth and is fixed to the outer casing of the block so it does not rotate. The wheels A and B have 16 and 36 teeth respectively and the gears have equal module of 3 mm. The speed of A is 100 rpm when the speed of arm is 10 rpm. Find the number of teeth on wheels C and D.



Q11/ An epicyclic gear train shown in figure, the arm is keyed to the same shaft as the load drum and the wheel A is keyed to the second shaft which caries a chain wheel, the chain being operated by hand. The two shafts have common axis but can rotate independently. The wheels B and C are compound and rotate together on a pin carried at the end of arm. The wheel D has internal teeth and is fixed to the outer casing of the block so it does not rotate. The wheels A and B have 16 and 36 teeth respectively with a module of 3 mm. The wheels C and D have a module of 4 mm. Find (1) the number of teeth on wheels C and D when the speed of A is ten times the speed of arm, and (2) the speed of wheel D when the wheel A is fixed and the arm rotates at 450 rpm anticlockwise.