Class: Third Class
Subject: Theory of Machines
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CH/1: Velocity and acceleration السرعه والتعجيل

1- السرعه بالاجهزه

Velocity in Mechanisms

Motion of a link

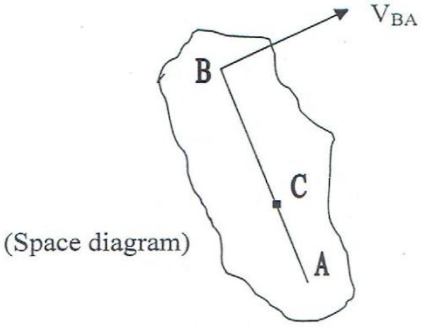
AB is a rigid link (distance between A and B is constant). The velocity of any point on a link with respect to another point on the same link is always perpendicular to the line joining these points.

The relative velocity of B with respect to A (V_{BA}) is represented by the vector $\overline{ab}$ and is perpendicular to the link AB

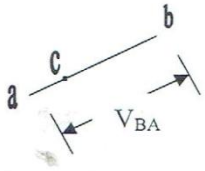
$$V_{BA} = \overline{ab} = \omega * AB$$

$$V_{CA} = \overline{ac} = \omega * CA$$

$$\frac{V_{CA}}{V_{BA}} = \frac{AC}{AB} = \frac{\overline{ac}}{\overline{ab}}$$



(Space diagram)



(Vector diagram)

Velocity of a point on a link by relative velocity method

Consider two points A and B on a link AB

Let $V_A \rightarrow$ known in magnitude and direction

$V_B \rightarrow$ known in direction

Find magnitude of V_B.

By drawing the velocity diagram

(a) Take point o

(b) From o draw oa // V_A and equal (to some suitable scale)

(c) From a draw line ⊥ AB to represent V_{BA}

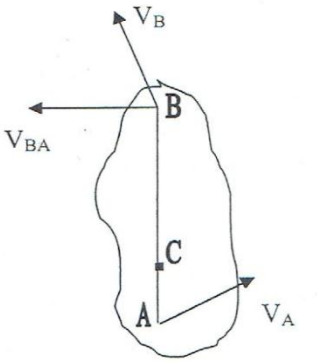
(d) From o draw line // V_B , these two lines intersect at b

(e) Measure V_B which give the required velocity of B to the scale.

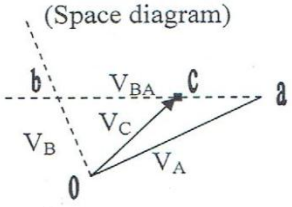
To find the velocity of any point on the AB (say point C)

$$\frac{ac}{ab} = \frac{AC}{AB} \Rightarrow \therefore ac = ab * \frac{AC}{AB}$$

Join (oc) which represent (V_C) and the vector (ac) represent (V_{CA}).



(Space diagram)



(Velocity diagram)

Velocities in slider crank mechanism

Let (A) … slider, (AB) … connecting rod, (OB) … crank and (r) … radius of crank.

To determine the velocity of A (V_A),

(1) Draw space diagram to a scale.

(2) To draw velocity diagram

(a) From point (o) draw vector (ob) ⊥ (OB) to represent V_B.

$\overline{ob} = V_B = \omega\, r$ *(to some suitable scale)*

(b) From point (b) draw vector (ba) ⊥ (BA) to represent V_{AB}.

(c) From point (o) draw vector (oa) // path of motion of the slider A.

So that vectors (ba) and (oa) intersect at point (a).

$\therefore \overline{oa}$ *represent* V_A (to the scale).

Angular velocity of AB $(\omega_{AB}) = \frac{V_{AB}}{AB} = \frac{vector\,(ba)}{AB}$

(d) The absolute velocity of any point (say E) on the AB may found

$$\frac{be}{ba} = \frac{BE}{BA} \Rightarrow \therefore be = ba * \frac{BE}{BA} \quad (which\ represent\ V_{EB})$$

Join point (o) and point (e), so that vector $(\overline{oe})$ *represent* (V_E)

Ex/ Given OB = 0.5 m, PB = 2 m and N_{BO} = 180 rpm (clockwise) find

(a) Velocity of piston, (b) angular velocity of (PB), (c) velocity of point (E) 1.5 m from gudgeon pin and (d) linear velocity of point (G) which has the least velocity relative to crankshaft.

Solution $\omega_{BO} = \frac{2\pi N}{60} = \frac{2\pi * 180}{60} = 18.85\ rad/\sec$

$V_{BO} = V_B = \omega_{BO} * OB = 18.85 * 0.5 = 9.42\ m/\sec$ (*and it is perpendicular to OB*)

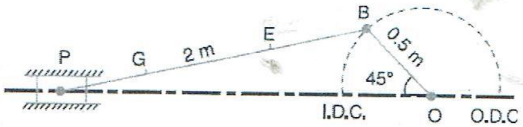
(1) Draw space diagram by taking scale 0.5 m = 1 cm. So that OB = 1 cm and BP = 4 cm

(2) Draw velocity diagram, by taking scale 3 m/sec = 1 cm.

(a)

(1) Draw ob ⊥ OB (to the scale) to represent V_B (or V_{BO}).

$\therefore vector\ (ob) = V_{BO} = V_B = 9.42\ m/\sec$



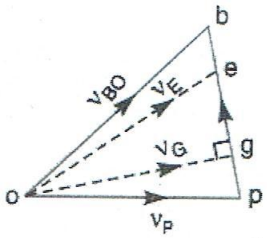
Space diagram.

(2) From point (b) draw (pb) perpendicular to (BP) to represent (V_{PB}).

(3) From point (o) draw (op) parallel to (PO) to represent (V_{PO} or V_P).

(4) The two vectors (pb) and (op) intersect at point (p).

$\therefore$ *By measurement and from velocity diagram* $(V_P) = vector\ (op) = 8.15\ m/\sec$



Velocity diagram.

(b) By measurement V_{PB} = 6.8 m/sec

$\therefore \omega_{PB} = \frac{V_{PB}}{PB} = \frac{6.8}{2} = 3.4\ rad/\sec$

(c) By dividing the vector (bp) at point (e) in the same ratio as point (E) divide (PB).

$\frac{be}{bp} = \frac{BE}{BP} \Rightarrow be = bp * \frac{BE}{BP}$

Join points (o) and (e) so, that vector (oe) represent (V_E).

$\therefore V_E = 8.5\ m/\sec$ (*by measurement*).

(d) From point (o) draw perpendicular line to the vector (bp) since the length of (og) will be the least.

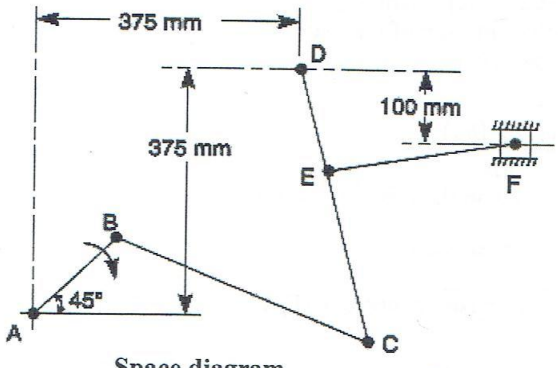
$\frac{bg}{bp} = \frac{BG}{BP} \Rightarrow BG = BP * \frac{bg}{bp} = 1.47\ m$ (*by measurement*).

$\therefore V_G = vector\ (og) = 8\ m/\sec$ (*by measurement*).

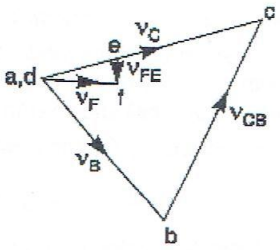
Ex/ Given AB =DE = 150 mm, BC = CD = 450 mm, EF = 375 mm and N_{BA} = 120 rpm. Find (a) velocity of F and (b) angular velocity of DC.

Solution $\omega_{BA} = \frac{2\pi N_{BA}}{60} = \frac{2\pi * 120}{60} = 4\pi\ rad/\sec$

$V_{BA} = V_B = \omega_{BA} * AB = 4\pi * 150 = 1880\ mm/\sec$



Space diagram



Velocity diagram

(1) Draw space diagram by taking scale 50 mm = 1 cm. So that AB = DE = 3 cm, BC = CD = 9 cm and EF = 7.5 cm.

(2) Draw velocity diagram, by taking scale 400 m/sec = 1 cm.

(1) Points (a) and (d) are a fixed point.

(2) From point (a) draw (ab) perpendicular to (AB) to represent (V_{BA} or V_B).

Vector ab = V_{BA} = V_B = 1880 mm/sec to some suitable scale.

(3) From point (b) draw (bc) perpendicular to (BC) to represent (V_{CB}) (direction only).

(4) From point (d) draw (dc) perpendicular to (DC) to represent (V_{CD} or V_D) (direction only).

(5) The two lines (bc) and (dc) intersect at point (c).

(6) Since point (E) lie on (DC)

$$\therefore \frac{ce}{cd} = \frac{CE}{CD} \Rightarrow ce = 2250 * \frac{300}{450} = 1500 \ mm/\sec \ (when \ cd = 2250 \ mm/\sec \ by \ measurement)$$

(7) From point (e) draw (ef) perpendicular to (EF) to represent (V_{FE}) (direction only).

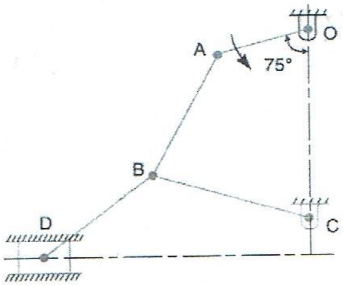
(8) From point (d) draw (df) parallel to the path of (F) to represent (V_F) (direction only).

(9) The two lines (ef) and (df) intersect at point (f).

$$\therefore V_F = vector \ fd = 690 \ mm/\sec \ (by \ measurement)$$

$$\omega_{CD} = \frac{V_{CD}}{CD} = \frac{vector \ dc}{CD} = \frac{2250}{450} = 5 \ rad/\sec$$

Ex/ In figure shown, the angular velocity of the crank OA is 600 rpm. Determine the linear velocity of the slider D and the angular velocity of the link BD, when the crank is inclined at an angle of 75° to the vertical. The dimensions of various links are: OA = 28 mm; AB = 44 mm; BC 49 mm; and BD = 46 mm. The center distance between the centers of rotation O and C is 65 mm. The path of travel of the slider is 11 mm below the fixed point C. The slider moves along a horizontal path and OC is vertical.



Space diagram

Solution

$\omega_{OA} = 2\pi \times 600/60 = 62.84$ rad/sec

Since OA = 28 mm = 0.028 m, therefore velocity of A with respect to O or velocity of A (because O is a fixed point),

$V_{AO} = V_A = \omega_{OA} * OA = 62.84 * 0.028 = 1.76$ m/sec ($\perp$ OA)

First of all draw the space diagram, to some suitable scale.

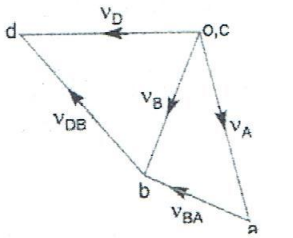
1. Since the points *O* and *C* are fixed, therefore these points are marked as one point, in the velocity diagram. Now from point *o*, draw vector *oa* perpendicular to *OA*, to some suitable scale, to represent the velocity of *A* with respect to *O* or simply velocity of *A* such that

vector oa = V_{AO} = V_A = 1.76 m/s

2. From point a, draw vector ab perpendicular to AB to represent the velocity of B with respect A (i.e. V_{BA}) and from point c, draw vector cb perpendicular to CB to represent the velocity of B with respect to C or simply velocity of B (i.e. V_{BC} or V_B). The vectors ab and cb intersect at b.

3. From point b, draw vector bd perpendicular to BD to represent the velocity of D with respect to B (i.e. V_{DB}) and from point o, draw vector od parallel to the path of motion of the slider D which is horizontal, to represent the velocity of D (i.e. V_D). The vectors bd and od intersect at d. By measurement, we find that velocity of the slider D,

V_D = vector od = 1.6 m/s



Velocity diagram

By measurement from velocity diagram, we find that velocity of *D* with respect to *B*,

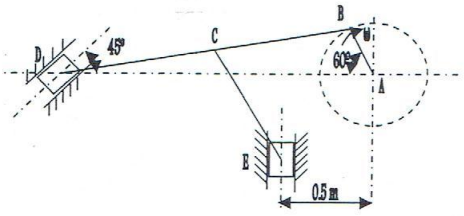
V_{DB} = vector bd = 1.7 m/s

Since the length of link *BD* = 46 mm = 0.046 m, therefore angular velocity of the link *BD*,

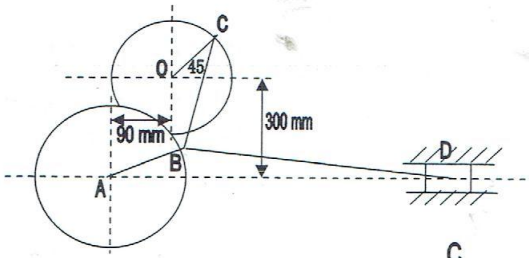
$\omega_{BD} = V_{DB} / BD = 1.7 / 0.046 = 36.96$ rad/sec (Clockwise about *B*).

(Home works)

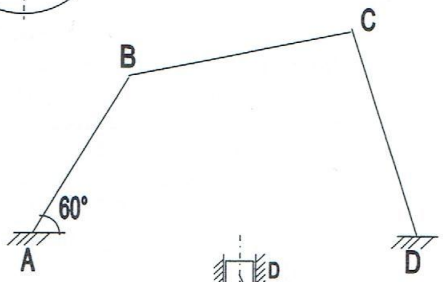
Q1/ The dimensions of the mechanism shown are as follows:
AB = 0.45 m; BD = 1.5 m; BC = CE = 0.9 m. The crank AB turns uniformly at 180 rpm in the clockwise direction and the blocks at D and E are working in frictionless guides. Draw the velocity diagram for the mechanism and find the velocities of the sliders D and E in their guides.



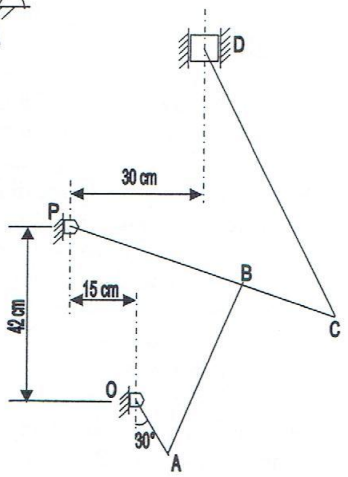
Q2/ In the toggle mechanism shown in figure below, D is constrained to move on a horizontal path. The dimensions of various links are as follows:
AB = 200 mm; BC = 300 mm; OC = 150 mm; and BD = 450 mm.
The crank OC is rotating in a counter clockwise direction at a speed of 180 rpm. Find the velocity of D, and angular velocity of BD.



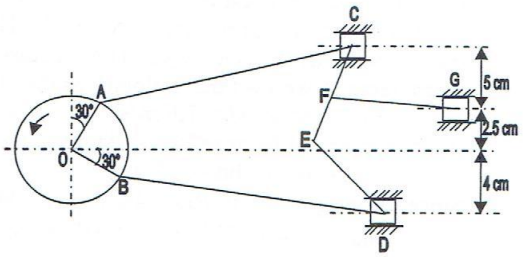
Q3/ The dimensions and configuration of the four bar mechanism shown in figure are as follows:
AB = 30 cm, CD = 36 cm, BC = 36 cm and AD = 60 cm. The crank AB has an angular velocity of 10 rad/sec clockwise. Determine the angular velocity of DB and BC and velocity of the joint C.



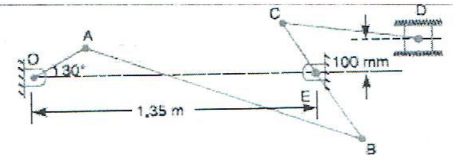
Q4/ Find out the velocity of the slider D and the angular velocity of link CD for the engine mechanism shown in figure. The dimensions of the various links are
OA = 15 cm; AB = 45 cm; PB = 24 cm; BC = 21 cm; CD = 66 cm and N_{OA} = 180 rpm.



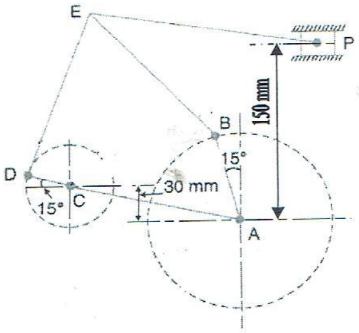
Q5/ In the mechanism shown in figure OA and OB are two equal cranks at right angle rotating about O at a speed of 40 rpm anticlockwise. The dimensions of the various links as follows:
OA = OB = 5 cm; AC = BD = 17.5 cm; DE = CE = 7.5 cm; FG = 11.5 cm and EF = FC. Find the velocity of the slider G.



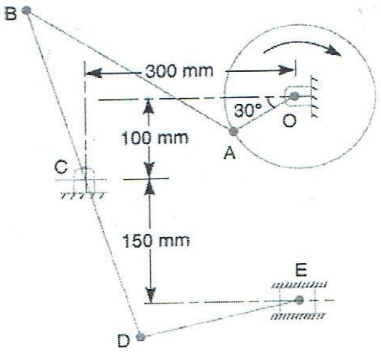
Q6/A mechanism, as shown in figure, have the following dimensions:
OA = 200 mm; AB = 1.5 m; BC = 600 mm; CD = 500 mm and BE = 400 mm. If crank OA rotates uniformly at 120 rpm clockwise, find (1) the velocity of B, C and D (2) the angular velocity of the links AB, BC and CD.



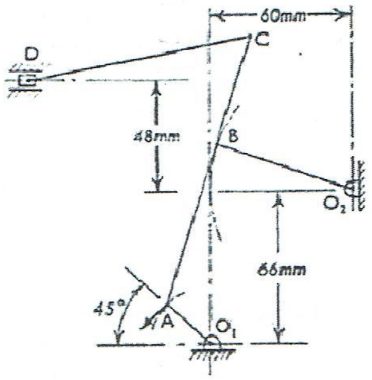
Q7/ In a mechanism shown in figure, the dimensions are as follows: AB = 80 mm; CD = 40 mm; BE = DE = 150 mm; AC =150 mm; and EP = 200 mm. The speed of the crank CD is 1140 rpm and of the crank AB is 570 rpm clockwise. Determine the velocity of the piston P for the given configuration.



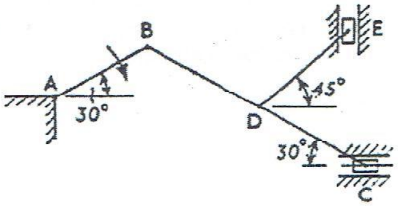
Q8/ In a mechanism as shown in figure, the crank OA is 100 mm long and rotates in a clockwise direction at a speed of 100 rpm. The straight rod BCD rocks on a fixed point at C. The links BC and CD are each 200 mm long and the link AB is 300 mm long. The slider E, which is driven by the rod DE is 250 mm long. Find the velocity of E.



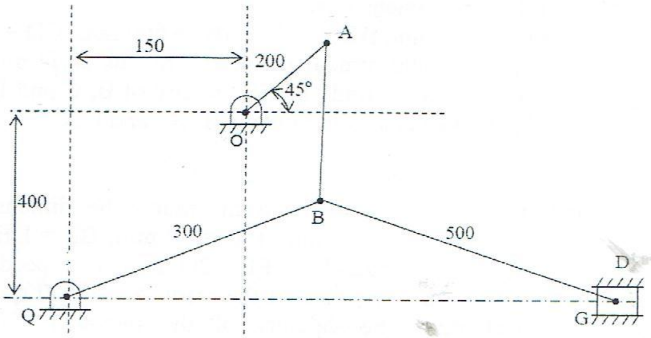
Q9/ The diagram of a linkage is given in Fig. below. Find the velocity of the slider D and the angular velocity of DC, when the crank O1A is in the given position and the speed of rotation is 90 rev/min anticlockwise. O1A = 24 mm, O2B = 60 mm, CD = 96 mm, AB = 72 mm, CB = 48 mm.



Q10/ In the mechanism shown in Fig., the crank AB is 75 mm long and rotate uniformly clockwise at 8 rad/sec. Given that BD = DC = DE; BC = 300 mm, draw the velocity diagrams, and find the velocity of the pistons at C and E.

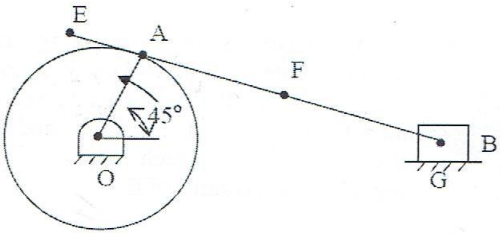


Q11/ In a toggle mechanism shown in figure the crank OA rotates at 210 rpm counter clockwise. Determine the velocity of slider D and angular velocity of link BD.

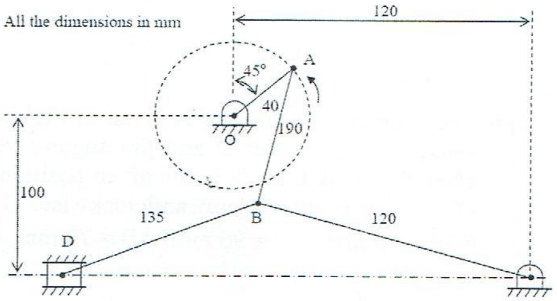


Q12/ In a slider crank mechanism the crank OA is 200 mm long and rotates at 40 rad/sec in a counter clockwise direction. The length of the connecting rod AB is 800 mm. Determine,

i) The velocity of the slider B.
ii) Velocity of point E located at a distance of 200 mm on the connecting rod extended.
iii) The position and velocity of point F on the connecting rod having the least absolute velocity.
iv) The angular velocity of connecting rod.



Q13/ Figure below shows a toggle mechanisms in which the crank OA rotates at 120 rpm. Find the velocity of the slider D.



Q14/ A toggle mechanism is shown in figure along with the diagrams of the links in mm. find the velocities of the points B and C and the angular velocities of links AB. The crank rotates at 50 rpm in the clockwise direction.