Digital Signal Processing

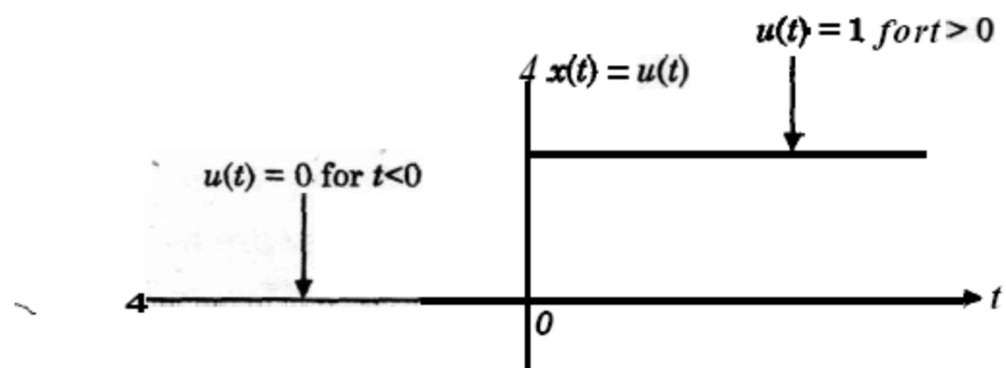
Basic Signals

Elementary signals

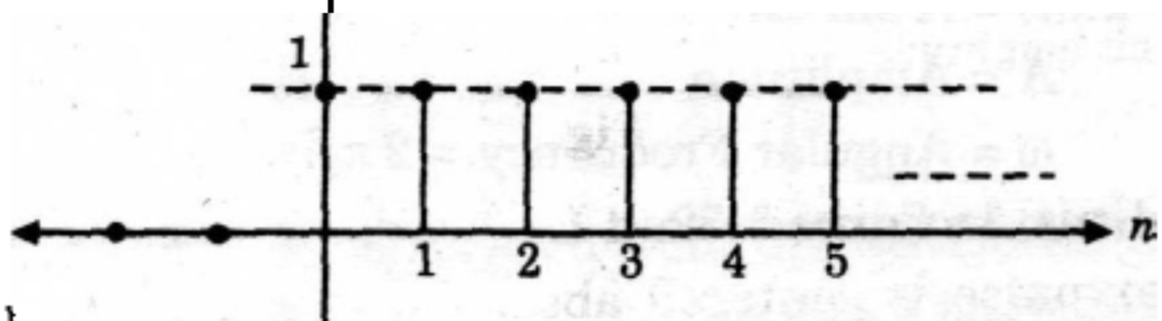
1. Unit step function.

The unit step signal has amplitude of '1' for positive values of independent variable. And it has amplitude of '0' for negative values of independent variable.

Unit step signal : $u(t) = \begin{cases} 1 & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$



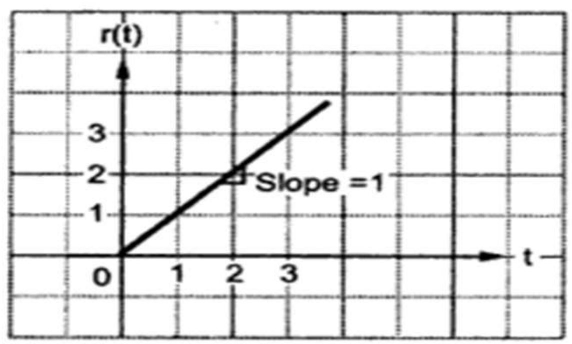
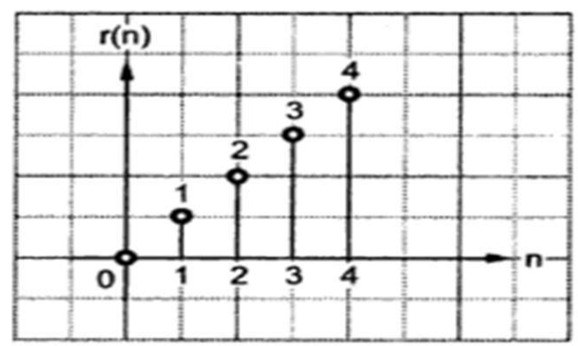
$$u(n) = \begin{cases} 1 & \text{for} \\ 0 & \text{for} \end{cases}$$



$$u(n) = \{\underset{\uparrow}{1}, 1, 1, 1, \ldots\}$$

Elementary signals

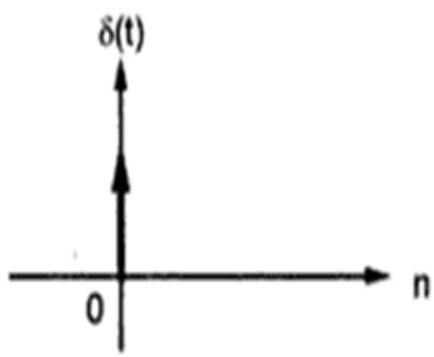
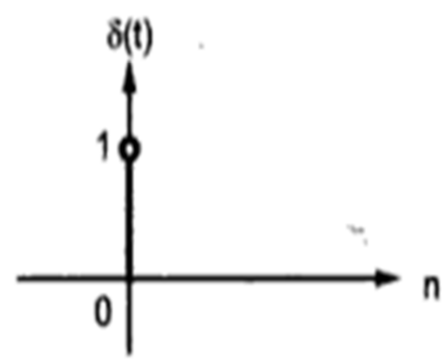
2. Unit Ramp Function.

Parameter	CT unit impulse signal $r(t)$	DT unit ramp signal $r(n)$
Definition	It is linearly growing function for positive values of independent variable.	The amplitude of every sample increases linearly with its number(n) for positive values of 'n'
Mathematical representation	$r(t) = \begin{cases} t & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$ $= t\,u(t)$ Since $u(t) = 1$ for $t \geq 0$ and $u(t) = 0$ for $t < 0$	$r(n) = \begin{cases} n & \text{for } n \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$ $= n\,u(n)$ Since $u(n) = 1$ for $n \geq 0$ and $u(n) = 0$ for $n < 0$ $r(n) = \{\underset{\uparrow}{0}, 1, 2, 3, 4, 5, ...\}$
Waveform		

- The unit ramp function is the integral of the unit step function.
- It is called the unit ramp function because for positive t, its slope is one amplitude unit per time.

Elementary signals

3. Unit Impulse or Delta Function

Parameter	Unit impulse signal $\delta(t)$	Unit sample signal $\delta(n)$
Definition	Area under unit impulse approaches '1' as its width approaches zero. Thus it has zero value everywhere except t = 0.	Amplitude of unit sample is '1' at n = 0 and it has zero value at all other values of n.
Mathematical representation	$\int_{-\infty}^{\infty} \delta(t)dt = 1$ and $t \to 0$ $\delta(t) = 0$ for $t \neq 0$	$\delta(n) = \begin{cases} 1 & for\ n = 0 \\ 0 & for\ n \neq 0 \end{cases}$ or $\delta(n) = \{0, 0, 0, \underset{\uparrow}{1}, 0, 0, 0\}$
Waveform		

Elementary signals

Properties of the Impulse Function

The Shifting Property

$$\int_{-\infty}^{\infty} \mathrm{g}(t)\,\delta(t - t_0)\,dt = \mathrm{g}(t_0)$$

The Replication Property

$$g(t) \otimes \delta(t) = g\ (t)$$

Elementary signals

4. Sinusoidal & Exponential Signals

Sinusoids and exponentials are important in signal and system analysis because they arise naturally in the solutions of the differential equations.

- Sinusoidal Signals can expressed in either of two ways :

cyclic frequency *form*- $A \sin 2\Pi f_o t = A \sin(2\Pi/T_o)t$

radian frequency form- $A \sin \omega_o t$

$\omega_o = 2\Pi f_o = 2\Pi/T_o$

T_o = Time Period of the Sinusoidal Wave

Sinusoidal & Exponential Signals

$x(t) = A \sin(2\Pi f_o t + \theta)$

$= A \sin(\omega_o t + \theta)$

$x(t) = Ae^{at}$ Real Exponential

$= Ae^{j\omega t} = A[\cos(\omega_o t) + j \sin(\omega_o t)]$ Complex Exponential

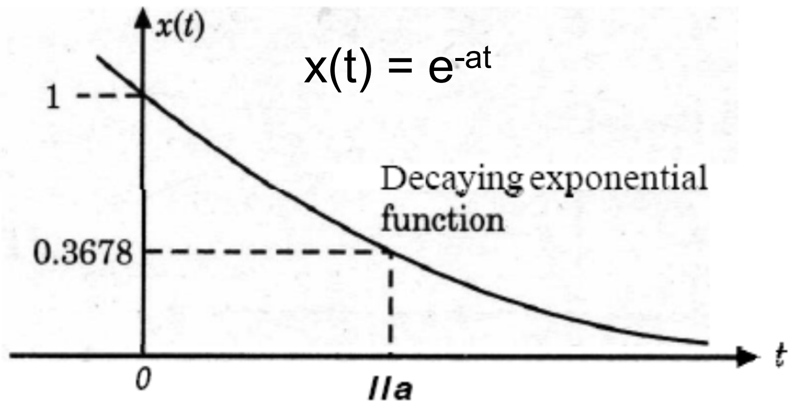
θ = Phase of sinusoidal wave

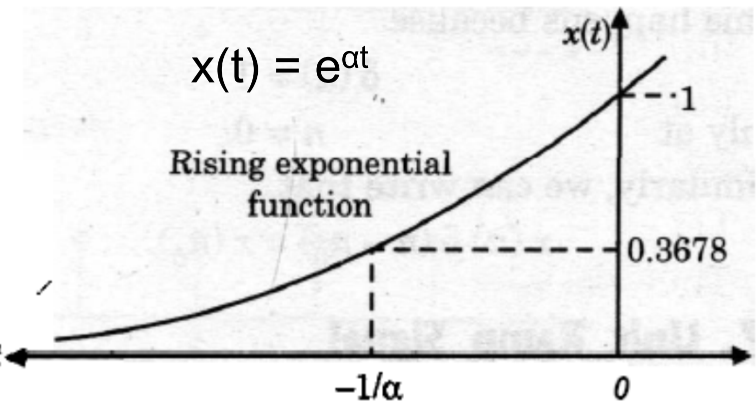
A = amplitude of a sinusoidal or exponential signal

f_o = fundamental cyclic frequency of sinusoidal signal

ω_o = radian frequency

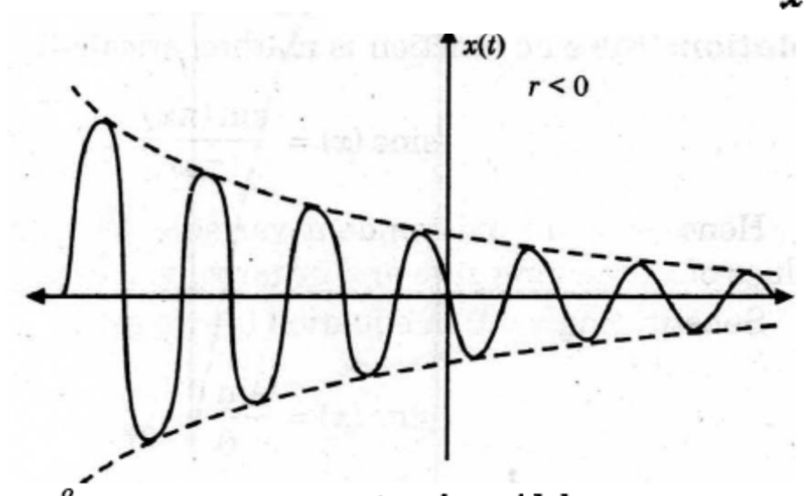
Real Exponential Signals and damped Sinusoidal



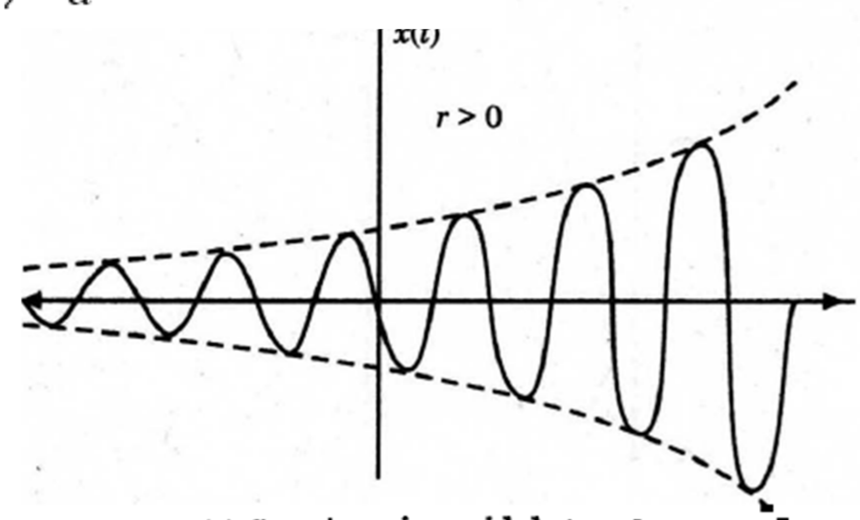


A discrete time exponential signal is expressed as

$$x(n) = a^n$$



(b) Decaying sinusoidal signal



(a) Growing sinusoidal signal

$$e^{rt} \cos(\omega_0 t + \theta)$$

Discrete Time Exponential and Sinusoidal Signals

- DT signals can be defined in a manner analogous to their continuous-time counter part

$x[n] = A \sin(2\Pi n/N_o+\theta)$ Discrete Time Sinusoidal Signal

$= A \sin(2\Pi F_o n+ \theta)$

Discrete Time Exponential Signal

$x[n] = a^n$

n = the discrete time

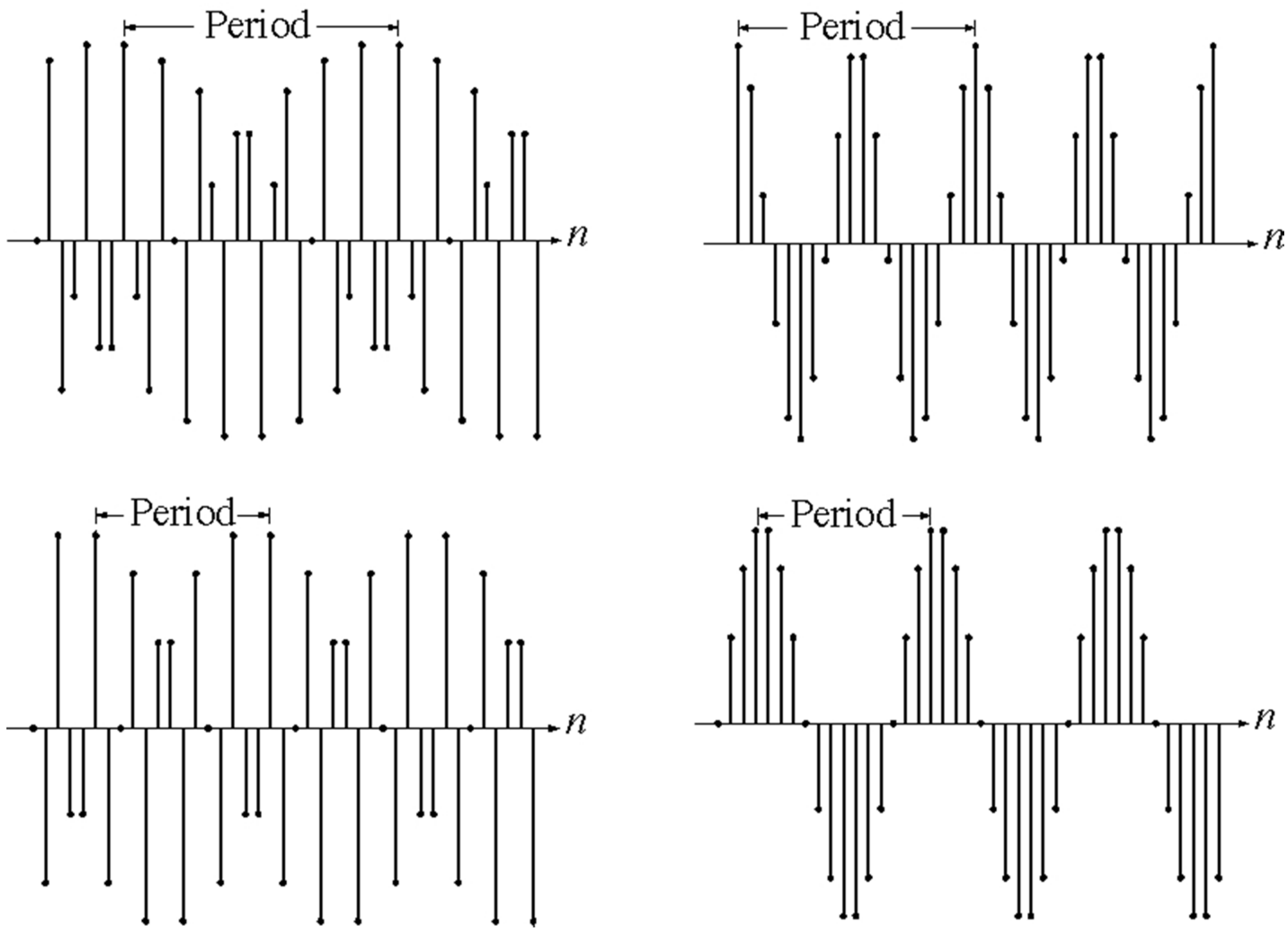
A = amplitude

θ = phase shifting radians,

N_o = Discrete Period of the wave

$1/N_0 = F_o = \Omega_o/2\Pi$ = Discrete Frequency

Discrete Time Sinusoidal Signals

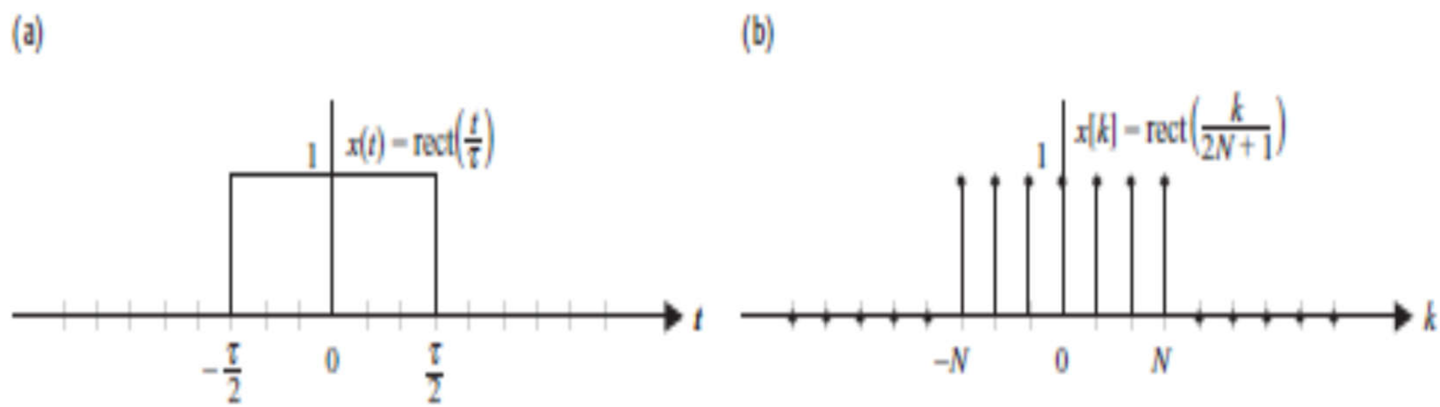


Elementary signals

5. Rectangular Pulse or Gate Function

$$\mathrm{rect}\left(\frac{t}{\tau}\right) = \begin{cases} 1 & |t| \leq \tau/2 \\ 0 & |t| > \tau/2 \end{cases}$$

$$\mathrm{rect}\left(\frac{k}{2N+1}\right) = \begin{cases} 1 & |k| \leq N \\ 0 & |k| > N \end{cases}$$



Elementary signals

6. Sinc Function

$$\operatorname{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$$