CH/1: الفصل الاول

مقدمه عن مخططات الازاحة والحركة

Introduction and graphical representation of motions

Definition:

The subject, theory of machine, is also named as dynamics of machinery or mechanics of machines. It comprises of study of relative motion of different parts of machines and the various forces acting on them. It may be classified as: (1) Kinematics, (2) Dynamics, (3) Kinetics and (4) Static's.

Kinematics: Is a study of relative motion of machine parts only.

Dynamics: Is a study of the forces acting on machine parts only.

Kinetics: Is a study of inertia forces which arise from the combined effect of the mass and motion of the machine parts.

Static's: Is a study of forces and their effect while the machine parts are at rest.

We shall discuss the relative motion of bodies without considering of the forces causing the motion. In other words kinematics deals with the geometry of motion and concepts like displacement, velocity and acceleration considered as a function of time.

Linear displacement: Change in the position of a body with respect to a certain fixed point. It may be along a straight or curved path.

The displacement of a body is a vector quantity (magnitude and direction). Linear displacement represented graphically by a straight line.

Linear velocity: It may be defined as the rate of change of linear displacement of a body with respect to the time (in a particular direction). It is a vector quantity (magnitude and direction).

$$V \text{ or } u = \frac{dS}{dt}$$

Linear acceleration: It may be defined as the rate of change of linear velocity of a body with respect to time. It is a vector quantity.

$$a = \frac{dV}{dt} = \frac{d}{dt}\left(\frac{dS}{dt}\right) = \frac{d^2S}{dt^2}$$

Equations of linear motion

(1) $V = u + at$ (2) $S = ut + \frac{1}{2}at^2$ (3) $V^2 = u^2 + 2aS$ (4) $S = \frac{(u+V)}{2}t$

Where u .... *Initial velocity of body*

V .... *Final velocity*

a .... *Acceleration*

S .... *Displacement of the body in t second.*

Notes:

(1) In case of vertical motion the body is subjected to gravity, then g should substituted for (a) in the above equations.

(2) When a body falls freely from a height (h), its velocity (V) with which it will hit the ground is given by

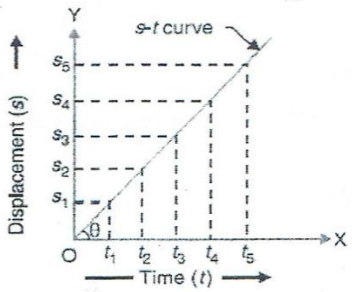
$V = \sqrt{2gh}$ Where $g = 9.81$ m/sec^2 or $g = 981$ cm/sec^2

(1) Graphical representation of displacement with respect to time (Displ. – Time curve)

(a) Uniform velocity

When the body moves with uniform velocity, equal distances are covered in equal interval of time (straight line).

The motion of the body is governed by equation **(S = ut)**



(8-1)

Velocity at instant 1 (u_1) $= \frac{S_1}{t_1}$

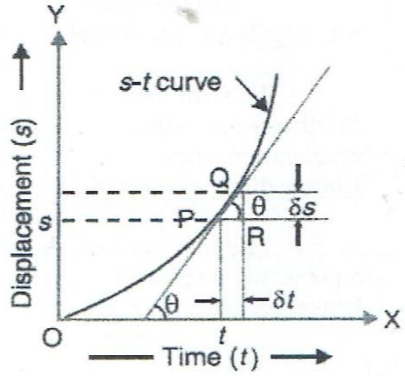
Velocity at instant 2 (u_2) $= \frac{S_2}{t_2}$

Since velocity is uniform

$\frac{S_1}{t_1} = \frac{S_2}{t_2} = ... = \tan\theta$ (The slope of the (S-t) curve at any instant gives the velocity).

(b) Variable velocity

When the body moves with variable velocity, unequal distance are covered in equal intervals of time or equal distance are covered in unequal interval of time (curve).



Consider point (P) on the (S-t) curve and let this point travel to (Q) by a small distance (δS) in a small interval of time (δt). Let the chord joining the points (P) and (Q) makes an angle (θ) with the horizontal. The average velocity of the moving point during the interval (PQ) is

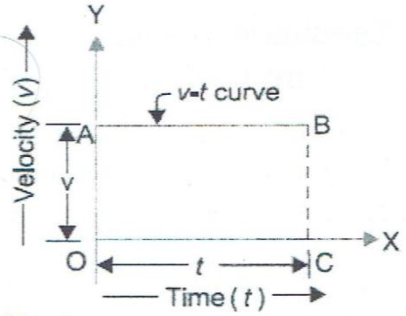
$\tan\theta = \frac{\delta S}{\delta t} \Rightarrow$ *in the limit when* $\delta t \to 0$, Q will tend to approach (P) and (PQ) become tangent to the curve at (P). Thus the velocity at (P)

$V_P = \tan\theta = \frac{dS}{dt} \Rightarrow \therefore$ The slop of the tangent at any instant on (S-t) curve gives the velocity at that instant.

(2) Graphical representation of velocity with respect to time (Vel. – Time curve)

(a) Uniform velocity

When the body moves with constant velocity and the acceleration is zero (straight line).



Distance covered by a body in time (t) second = V * t = area under the (V – t) curve
= area of rectangular OABC

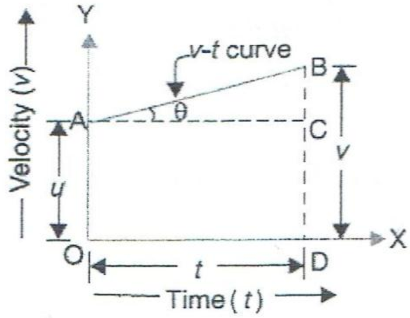
(b) Variable velocity

When the body moves with constant acceleration, in such a case, there is equal variation of velocity in equal intervals of time and (V – t) curve will be a straight line (AB) inclined at an angle (θ).

The equation of motion

$V = u + at$ and $S = ut + \frac{1}{2}at^2$

then $\tan\theta = \frac{BC}{AC} = \frac{V-u}{t} = \frac{change\ in\ velocity}{t} =$ *acceleration (a)*



∴ Slope of (V – t) curve represent the acceleration of the moving body

$a = \tan\theta = \frac{V-u}{t} \Rightarrow V = u + at$

(8-2)

∵ The distance moved by a body is given by the area under the (V – t) curve

∴ The distance (S) moved in time (t) = area OABD

S = area OACD + area ABC

$$S = ut + \frac{1}{2}t(V-u) = ut + \frac{1}{2}at^2$$

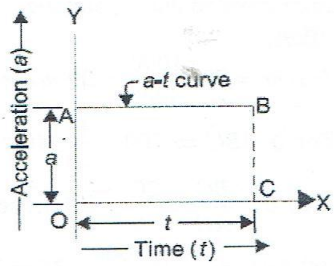
(3) Graphical representation of acceleration with respect to time (Acc. – Time curve)

(a) Uniform acceleration

When the body moves with uniform acceleration, the (a – t) curve is straight line.

∵ Change in velocity = a * t

∴ Area under (a – t) curve represent the change in velocity.



(b) Variable acceleration

The (a – t) curve may be having any shape depending upon the value of acceleration at various instance.

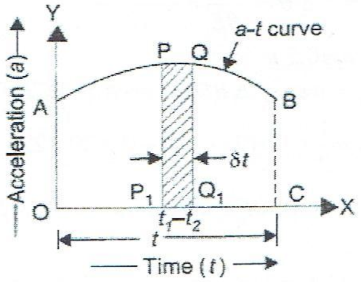
$$\because a = \frac{dV}{dt} \Rightarrow dV = a\,dt \quad \text{integrating both side}$$

$$\int_{V_1}^{V_2} dV = \int_{t_1}^{t_2} a\,dt \Rightarrow V_1 - V_2 = \int_{t_1}^{t_2} a\,dt$$

Where $(V_1 - V_2)$ …. Represent the area PQQ_1P_1 under (a – t) curve between intervals t_1 and t_2.

If the initial and final velocities of body are (u) and (V)

$$V - u = \int_0^t a\,dt = area\ OABC$$



Equation of angular motion

Note: If a body rotating at the rate of (N) rpm then its angular velocity is $\omega = \frac{2\pi N}{60}\ (rad/\sec)$

Relation between linear and angular quantities of motion

$$S = r\theta \quad \Rightarrow V = \frac{dS}{dt} = \frac{d(r\theta)}{dt} = r\frac{d\theta}{dt} = r\omega \quad \Rightarrow a = \frac{dV}{dt} = \frac{d(r\omega)}{dt} = r\frac{d\omega}{dt} = r\alpha$$

Acceleration of a particle along circular path

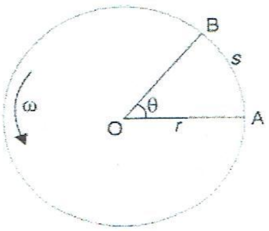
When a particle moves on a circular path with **variable velocity**, it is subjected to two types of acceleration

(1) Tangential acceleration $a_t = \alpha r$

(2) Normal or centripetal acceleration $a_n = \omega^2 r$, both are perpendicular to each other,

(3) Total acceleration or resultant acceleration of the moving body is $a = \sqrt{(a_t)^2 + (a_n)^2}$

(4) Its angle of inclination with the tangential acceleration is given by $\left[\theta = \tan^{-1}\left(\frac{a_n}{a_t}\right)\right]$



And when the particle moves with a **constant velocity**, then $a_t = 0$ and the body subjected to a_n only.

(8-3)

Introduction and graphical representation of motions
(Solved problems)

Ex1/

(a) A car starts from rest and accelerate uniformly to a speed of 72 km/hr over a distance of 500 m. Calculate the acceleration and time taken to attain the speed.

(b) If further acceleration raises the speed to 90 km/hr in 10 second. Find this acceleration and further distance moved.

(c) The brakes are now applied to bring the car to rest under uniform retardation in 5 second. Find the distance traveled during braking.

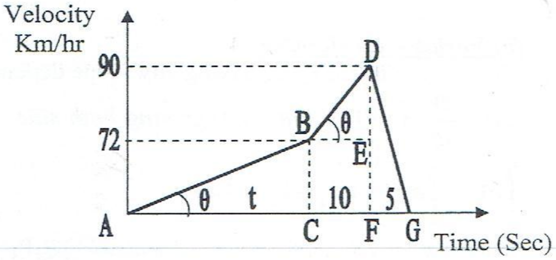
Solution:

$72\ km/hr = 72 * \frac{1000}{3600} = 20\ m/\sec$

(a) For $\Delta\ ABC \Rightarrow 500 = \frac{1}{2} * AC * BC = \frac{1}{2} * t * 20 \Rightarrow \therefore t = 50\ Sec$

$a = \tan\theta = \frac{BC}{AC} = \frac{20}{50} = 0.4\ m/\sec^2$

(b) $90\ km/hr = 90 * \frac{1000}{3600} = 25\ m/\sec$



For $\Delta\ BDE$

$a = \tan\theta = \frac{DE}{BE} = \frac{25-20}{10}$

$\therefore a = 0.5\ m/\sec^2$

$S_1 = area\ of\ \Delta\ BDE + area\ of\ BCEF$

$S_1 = \frac{1}{2} * 10 * (25-20) + 10 * 20 = 225\ m$

(c) For $\Delta\ DFG \Rightarrow S_2 = area\ of\ \Delta\ DFG = \frac{1}{2} * 5 * 25 = 62.5\ m$

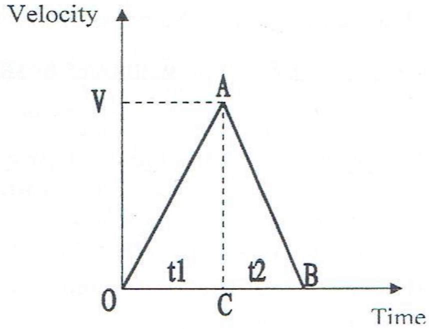
Ex2/ A railway train travel between two station 15 km a part in 18 min, if the train accelerates for apart of journey uniformly followed by uniform retardation. Find graphically the maximum speed attained during the journey.

Solution:

$\because$ Total distance = 15 km

Total time = $t_1 + t_2$ = 18 min

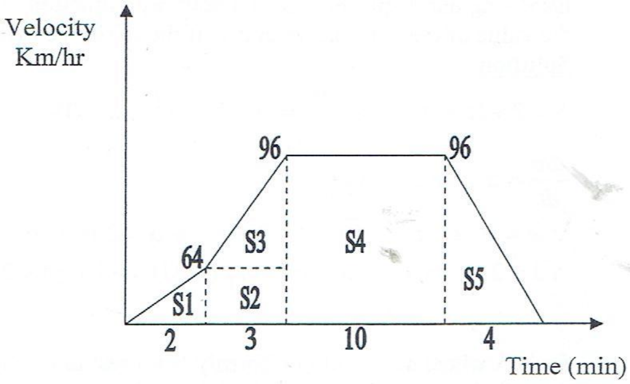
$\because S = \frac{1}{2} * OB * AC \Rightarrow 15 = \frac{1}{2} * \frac{18}{60} * V \Rightarrow V = 100\ km/hr$



(8-4)

Ex3/ The (V – t) diagram for a vehicle is shown in figure below. Determine the total distance traveled and the average velocity.

Solution:



Distance traveled = area under (V – t) curve ∵
S = S1 + S2 + S3 + S4 + S5

$$S1=\frac{1}{2}\ \ \frac{2}{60}\ \ 64=1.06\ km$$

$$S2=\frac{3}{60}\ \ 64=3.2\ km$$

$$S3=\frac{1}{2}\ \ \frac{3}{60}\ \ (96\ \ 64)=0.8\ km$$

$$S4=\frac{10}{60}\ \ 96=15.16\ km$$

$$S5=\frac{1}{2}\ \ \frac{4}{60}\ \ 96=3.2\ km$$

$$\therefore S=1.06+3.2+0.8+15.16+3.2=23.42\ km$$

$$average\ velocity=\frac{total\ distance}{total\ time}=\frac{23.42}{\frac{19}{60}}=73.95\ km/hr$$

Ex4/ The motion of a particle are given by ($a=t^3-3t^2+5$), where a is the acceleration in m/sec^2 and t is the time in seconds. The velocity of the particle at t = 1 sec is 6.25 m/sec and displacement is 8.3 m. Calculate the displacement and velocity at t = 2 sec.

Solution:

$$\frac{dV}{dt}=t^3-3t^2+5\Rightarrow\int dV=\int(t^3-3t^2+5)\ dt$$

$$\therefore V=\frac{t^4}{4}-3*\frac{t^3}{3}+5t+C_1 \qquad sub. \qquad V=6.25\ m/\sec\ at\ t=1\sec$$

$$6.25=\frac{1}{4}-1+5+C_1\Rightarrow C_1=2\ \Rightarrow\ \therefore V=\frac{t^4}{4}-t^3+5t+2$$

$$\therefore velocity\ at\ t=2\sec\ \Rightarrow\ V=\frac{2^4}{4}-2^3+(5*2)+2=8\ m/\sec$$

$$\because\frac{dS}{dt}=\frac{t^4}{4}-t^3+5t+2\ \Rightarrow\ \int dS=\int(\frac{t^4}{4}-t^3+5t+2)\ dt$$

$$\therefore S=\frac{t^5}{(4*5)}-\frac{t^4}{4}+5*\frac{t^2}{2}+2t+C_2 \qquad sub.\ \ S=8.3\ m\ at\ t=1\sec$$

$$8.3=\frac{1}{20}-\frac{1}{4}+\frac{5}{2}+2+C_2\ \Rightarrow\ C_2=4 \qquad \therefore S=\frac{t^5}{20}-\frac{t^4}{4}+\frac{5}{2}t^2+2t+4$$

$$\therefore displacement\ at\ t=2\sec\ \Rightarrow\ S=\frac{2^5}{20}-\frac{2^4}{4}+\frac{(5*2^2)}{2}+2*2+4=15.6\ m$$

(8-5)

Ex5/ Angular displacement of a body as a function of time is given by equation $\theta = 2 + bt + ct^2$. Determine the value of the constant b and c if the initial velocity is 4 rad/sec and 2 sec later it is 10 rad/sec.

Solution:

$\theta = 2 + bt + ct^2 \Rightarrow \frac{d\theta}{dt} = \omega = b + 2ct$ ....(1)

$\frac{d\omega}{dt} = \alpha = 2c$ ....(2)

$\because \omega = \omega_o + \alpha t \Rightarrow 10 = 4 + 2 * \alpha \Rightarrow \alpha = 3\ rad/\sec^2$ *sub. in equ.* (2)

$\therefore 3 = 2 * c \Rightarrow c = 1.5$ *sub. in equ.* (1) $\Rightarrow 10 = b + 2 * 1.5 * 2 \Rightarrow b = 4$

Ex6/ A wheel accelerate uniformly from rest to a speed of 300 rpm in 0.5 sec. It then rotates at that speed for 2 sec before retarding to rest in 0.3 sec. How many revolution dose it make during the entire time interval.

Solution:

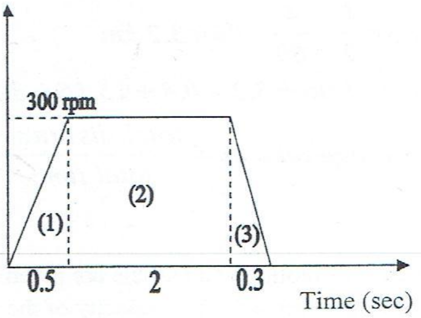
$\theta_1 = \frac{1}{2} * 0.5 * \frac{300}{60} = 1.25\ rev.$

$\theta_2 = 2 * \frac{300}{60} = 10\ rev.$

$\theta_3 = \frac{1}{2} * 0.3 * \frac{300}{60} = 0.83\ rev.$

$Total\ number\ of\ revolutions = \theta_1 + \theta_2 + \theta_3$

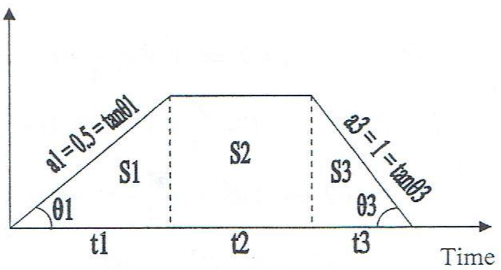
$= 1.25 + 10 + 0.83 = 12.03\ rev.$



Ex7/ A car starting from rest increasing it speed to 108 km/hr with a constant acceleration of 0.5 m/sec^2. At this speed it runs for some time and their after come to rest with constant retardation of 1 m/sec^2. If the total distance covered by the car is 3 km, determine graphically what the total time is spending for the journey.

Solution:

<u>*Acceleration period*</u>



$\because V = \frac{108000}{3600} = 30\ m/\sec$

$a_1 = 0.5 = \tan\theta_1 = \frac{V}{t_1} \Rightarrow t = \frac{30}{0.5} = 60\ \sec$

$S_1 = \frac{1}{2} V t_1 = \frac{1}{2} * 30 * 60 = 900\ m$

<u>*Retarding period*</u>

$a_3 = 1 = \tan\theta_3 = \frac{V}{t_3} \Rightarrow t_3 = 30\ \sec$

$S_3 = \frac{1}{2} V t_3 = \frac{1}{2} * 30 * 30 = 450\ m$

$\because S = S_1 + S_2 + S_3 \Rightarrow 3000 = 900 + S_2 + 450 \Rightarrow S_2 = 1650\ m$

$\because S_2 = V t_2 \Rightarrow 1650 = 30 * t_2 \Rightarrow t_2 = 55\ \sec$

$\therefore total\ time = t_1 + t_2 + t_3 = 60 + 55 + 30 = 145\ \sec$

(8-6)

Ex8/ The cutting stroke of a planning machine are 50 cm and it is completed in 1 sec. The planning table accelerate uniformly during the first 12.5 cm of the stroke, the speed remain constant during the next 25 cm of the stroke and retards uniformly during the last 12.5 cm of the stroke. Find the maximum cutting speed.

Solution:

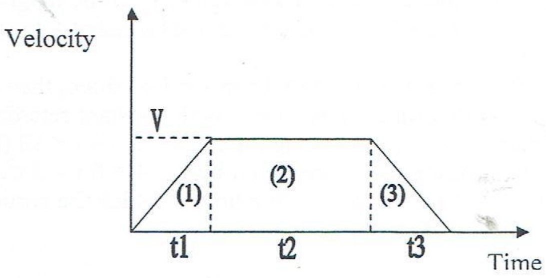
$$S_1 = \frac{1}{2}Vt_1 = 12.5 \Rightarrow t_1 = \frac{25}{V}$$

$$S_2 = Vt_2 = 25 \Rightarrow t_2 = \frac{25}{V}$$

$$S_3 = \frac{1}{2}Vt_3 = 12.5 \Rightarrow t_3 = \frac{25}{V}$$

$$\because t_1 + t_2 + t_3 = 1$$

$$\therefore \frac{25}{V} + \frac{25}{V} + \frac{25}{V} = 1 \Rightarrow V = 75\ cm/\sec$$



Introduction and graphical representation of motions
(Home works)

Q1/ The angular displacement of a body is a function of time and is given by the equation $\theta = 10 + 3t + 6t^2$ where t is in sec. Determine the angular velocity, displacement and acceleration when $t = 5$ sec.

Q2/ A wheel accelerate uniformly from rest to 2000 rpm in 20 sec. What is its angular acceleration, how many revolutions dose the wheel make in attaining the speed of 2000 rpm.

Q3/ A steam turbine are running at 2400 rpm. On shutting off steam it slows down and comes to rest in 4 min. If the angular retardation is uniform, find its magnitude in rad/sec^2 and also the number of revolutions made by the turbine before coming to rest.

Q4/ Express in rad/sec (a) 5 rev/sec (b) 270 rpm.

Q5/ The displacement of a body as a function of time is given by the equation $x = (t - B)^3$ where t is in sec. Determine the value of constant B when the acceleration of the body is 30 m/sec^2 at time t equal 10 sec.

Q6/ The motion of a particle are given by $a = \sin t$, where a is the acceleration in m/sec^2 and t are the time in seconds. The velocity of the particle at $t = 1$ sec is 3.5 m/sec, and the displacement is 5 m. calculate the displacement and velocity at $t = 4$ sec.

Q7/

(a) A car moving at a constant speed of 54 km/hr over a distance of 300 m. Calculate the time taken to attain this distance.

(b) If the car accelerate uniformly raises the speed to 90 km/hr in 15 second. Find this acceleration and the distance moved.

(c) The brakes are now applied to bring the car to rest under uniform retardation in 10 second. Find the distance traveled during braking.

Q8/ A train traveling at a velocity of 100 km/h are decreases to 90 km/h in the first 40 sec after application of the brakes. Calculate the velocity at the end of a further 80 sec assuming that, during the whole period of 120 sec the retardation was constant.

Q9/ A car moving at a speed of 40 km/h accelerate uniformly to a speed of 100 km/h within 5 sec. At this speed, it runs for 3 sec, and then after accelerates uniformly to a speed of 120 km/hr within 2 sec, and at that time come to rest with constant retardation within 6 sec. (a) Draw the V-t curve for the journey and (b) Find the average speed of the car.

Q10/ A body move on a straight line with acceleration of 18 m/s^2, its speed become 82 m/s after 4 sec of its start moving. Find the equation of its speed as a function of time.

(8-7)

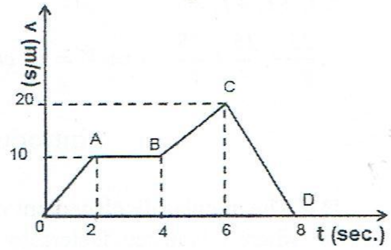
Q11/ The displacement of a particle along a straight line at time t is given by $s = b_0 + b_1 t + b_2 t^2$. Find the acceleration of the particle.

Q12/ A car accelerate uniformly from a speed of 3 m/sec to 8 m/sec in 10 sec and then travel with constant speed for another 5 sec. (a) Draw velocity – time diagram, (b) what is its acceleration, (c) what is the total distance travelled, (d) draw the displacement – time diagram, and (e) draw the acceleration – time diagram.

Q13/ A particle moves at a speed of 15 m/sec for 10 sec, then accelerate uniformly to a speed of 25 m/sec in 5 sec and at that time come to rest with constant retardation within 15 sec. Calculate the acceleration of the particle for the following intervals, (a) $0 < t < 10$ (b) $10 < t < 15$ (c) $15 < t < 30$.

Q14/ The motion of a particle are given by $s = 4 + 5t - 2t^2$. Calculate (a) the velocity and acceleration of the particle after 2 sec and (b) the time at which the particle is instantaneously at rest.

Q15/ Velocity-time graph of a particle moving in a straight line is shown in figure. Plot the corresponding displacement time graph of a particle if at t = 0 displacement s = 0.



Q16/ A truck moving with constant acceleration cover the distance between two points 180 m apart in 6 seconds. Its speed as it passes the second point is 45 m/s. Find (a) its speed when it was at the first point and (b) its acceleration.

Q17/ A body undergoing uniformly accelerated motion starts moving with a velocity of 5 m/s and after 5 seconds its velocity becomes 20 m/s in the same direction. What is the velocity of the body 10 seconds after the start of the motion?

Q18/ The motion of a particle along a straight line is described by the function, $x = 6 + 4t^2 - t^4$ where x is in meters and t is in seconds. Find the displacement, velocity and acceleration at t = 2 sec.

(8-8)