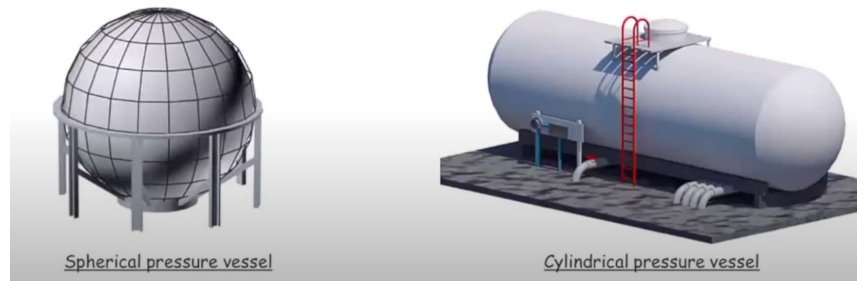


Members subjected to axisymmetric loads

There are two types of pressure vessels

- 1- Spherical pressure vessel
- 2- Cylindrical pressure vessel

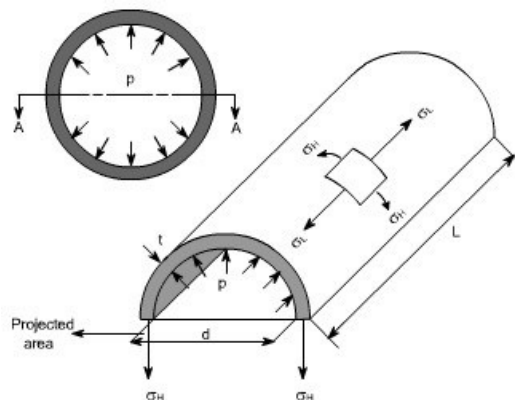


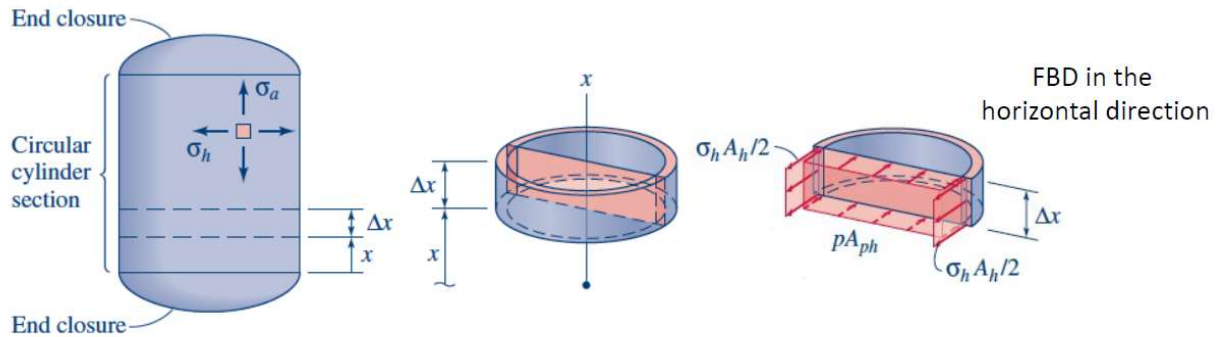
1- pressurized thin-walled cylinder

The thin cylinder has a ratio of inside diameter to thickness of wall (d/t) and in the cylinder this ratio equal to $(d/t) \geq 20$ while the thick cylinder has ratio of $(d/t) \leq 20$. A cylindrical vessel may carry fluid (liquid or gas) under pressure of p and it's subjected to tensile forces which resist the bursting forces developed across longitudinal and transverse section. In the thin wall cylinder two types of stresses are created due to the fluid pressure:

A- Hoop stresses (circumferential or tangential stresses): σ_H

Consider a thin cylindrical shell subjected to an internal pressure as shown in Figure bellow. A tensile stress acting in a direction tangential to the circumference is called circumferential or hoop stress. In other words, it is a tensile stress on longitudinal section (or on the cylindrical walls).





Force due to internal pressure (F) = $p \cdot \text{Area}$
 $= p \cdot (d \cdot L)$

Resisting force due to hoop stress (P) = $2 \cdot \sigma_H \cdot L \cdot t$

$$\therefore F = P$$

$$p \cdot d \cdot L = 2 \cdot \sigma_H \cdot L \cdot t$$

$$\therefore \sigma_H = \frac{p \cdot d}{2t}$$

where σ_H : is the Hoop's stress or circumferential tensile stress

Note: in the cylindrical tank with the circular cross-section the maximum stress is equal to the stress exerted in a circumferential direction which is equal to the hoop's stress.

B- Longitudinal stresses: σ_L

Consider the cylindrical vessel with closed ends, the internal pressure will tend to push the ends of the cylinder outward causing a longitudinal tensile force in the wall. The resultant force from the fluid pressure will equal the pressure times the projected area.

$$F = P \cdot x \cdot \frac{\pi \cdot D^2}{4}$$

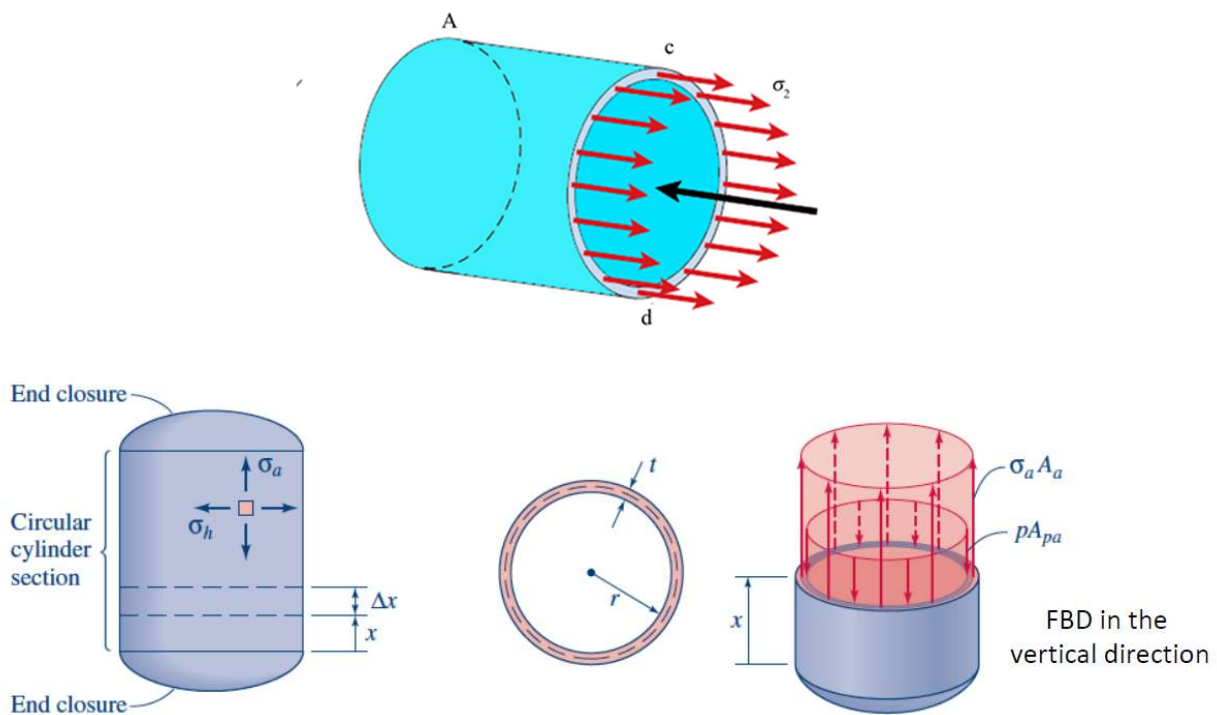
The force in the cylinder must equal the force in above equation and the stress equal to the force divided by the area. So

$$\sigma_L = p \frac{\pi D^2}{4} \left(\frac{1}{\pi D t} \right)$$

$$\sigma_L = \frac{pD}{4t}$$

where σ_L : is the Longitudinal tensile stresses

Notice that in cylindrical sections $\sigma_L = \frac{\sigma_H}{2}$



Example: water is running through a cast iron pipe of a diameter 750 mm at 10 Mpa pressure, calculate the thickness of the pipe if the maximum permissible stress is 120 Mpa

Sol/

Because the pipe is cylindrical with circular cross section this mean that the max stress is equal to hoop' stress so:

$$\sigma_H = \frac{p * d}{2t} = 120 * 10^6 = \frac{10 * 10^6 * 750}{2 * t} = t = 31.25 \text{ mm}$$

Example: A steel storage tank with a wall thickness of 5 mm, a diameter of 8 m, and a height of 25 m is filled with water to a certain height h, assuming a factor of safety of 3.5 and an ultimate strength of the steel in tension of 390 Mpa. Determine the value of h.

Sol/

$$\sigma_H > \sigma_L$$

$$\sigma_{ult} = \sigma_{max} = \sigma_H = \frac{PD}{2t}$$

$$P = \frac{\sigma_H \times 2t}{D} = \frac{390 \times 2 \times 5}{8 \times 10^3} = 0.4875 \text{ N/mm}^2$$

P_{max} or ultimate = 0.4875 Mpa it is the maximum pressure

$$F.S = \frac{P_{ult}}{P_{all}} = P_{all} = \frac{P_{ult}}{F.S} = \frac{0.4875}{3.5} = 0.139 \text{ Mpa}$$

$P_{all} = \gamma \times h_{all}$ where γ is the specific weight of water on Earth at 4° Celsius which is equal to 9810 N/m³

$$\text{So, } h_{all} = 0.139 / 9810 \times 10^{-9} = 14169 \text{ mm} = 14.169 \text{ m}$$

2- Thin wall Spherical vessel

In spherical vessels circumferential stresses are exist due to the fluid pressure.

So:

$$\sigma_L = \frac{p \times d}{4t} = \sigma_H$$