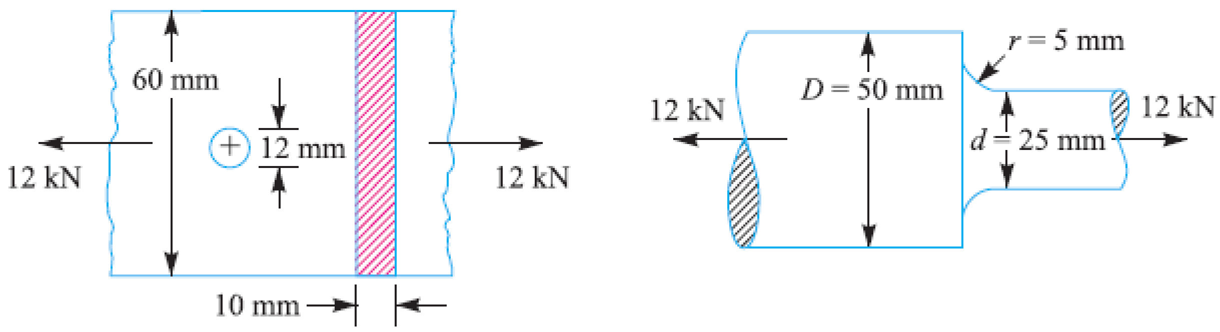
Problem 2

Find the maximum stress induced in the following cases taking stress concentration into account: 1. A rectangular plate 60 mm ×10 mm with a hole 12 diameter as shown in Figure (a) and subjected to a tensile load of 12 kN. 2. A stepped shaft as shown in Figure (b) and carrying a tensile load of 12 kN.



(a) (b)

Solution

Case 1. b = 60 mm ; t = 10 mm ; d = 12 mm ; W = 12 kN = 12 × 103 N We know that cross-sectional area of the plate,

$$A = (b - d)\, t = (60 - 12)\, 10 = 480 \text{ mm}^2$$

$$\text{Nominal stress} = \frac{W}{A} = \frac{12 \times 10^3}{480} = 25 \text{ N/mm}^2 = 25 \text{ MPa}$$

Ratio of diameter of hole to width of plate,

$$\frac{d}{b} = \frac{12}{60} = 0.2$$

From Table 1, we find that for d / b = 0.2, theoretical stress concentration factor, K_t = 2.5

∴ Maximum stress = K_t × Nominal stress = 2.5 × 25 = 62.5 MPa

Case 2.

D = 50 mm ; d = 25 mm ; r = 5 mm ; W = 12 kN = 12 × 103 N We know that cross-sectional area for the stepped shaft,

$$A = \frac{\pi}{4} \times d^2 = \frac{\pi}{4}(25)^2 = 491 \text{ mm}^2$$

$$\text{Nominal stress} = \frac{W}{A} = \frac{12 \times 10^3}{491} = 24.4 \text{ N/mm}^2 = 24.4 \text{ MPa}$$

Ratio of maximum diameter to minimum diameter,

$$D/d = 50/25 = 2$$

Ratio of radius of fillet to minimum diameter,

$$r/d = 5/25 = 0.2$$

From Table 3, we find that for $D/d = 2$ and $r/d = 0.2$, theoretical stress concentration factor,

$$K_t = 1.64.$$

∴ Maximum stress = K_t × Nominal stress = 1.64 × 24.4 = 40 MPa

Fatigue Stress Concentration Factor

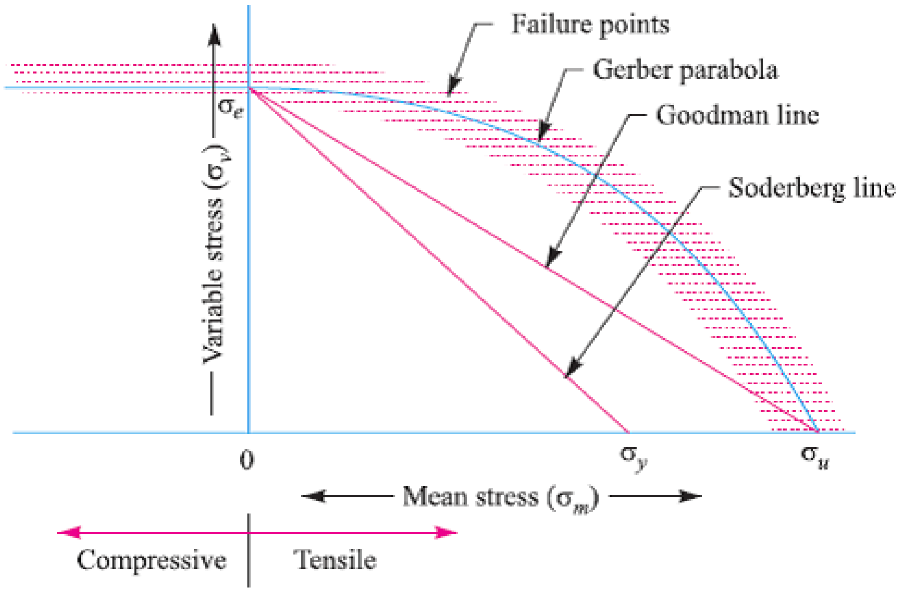
$$K_f = \frac{\text{Endurance limit without stress concentration}}{\text{Endurance limit with stress concentration}}$$

Combined Steady and Variable Stress

The failure points from fatigue tests made with different steels and combinations of mean and variable stresses are plotted in Figure as functions of variable stress (σ_v) and mean stress (σ_m).

There are several ways in which problems involving this combination of stresses may be solved, but the following are important from the subject point of view :

1. Gerber method, **2.** Goodman method, and **3.** Soderberg method.



Gerber Method for Combination of Stresses

The relationship between variable stress (σ_v) and mean stress (σ_m) for axial and bending loading for ductile materials are shown in Figure. The point σ_e represents the fatigue strength corresponding to the case of complete reversal ($\sigma_m = 0$) and the point σ_u represents the static ultimate strength corresponding to $\sigma_v = 0$.

$$\sigma_v = \sigma_e \left[\frac{1}{F.S.} - \left(\frac{\sigma_m}{\sigma_u} \right)^2 F.S. \right]$$

or

$$\frac{1}{F.S.} = \left(\frac{\sigma_m}{\sigma_u} \right)^2 F.S. + \frac{\sigma_v}{\sigma_e}$$

Where $F.S.$ = Factor of safety,

σ_m = Mean stress (tensile or compressive),

σ_u = Ultimate stress (tensile or compressive), and

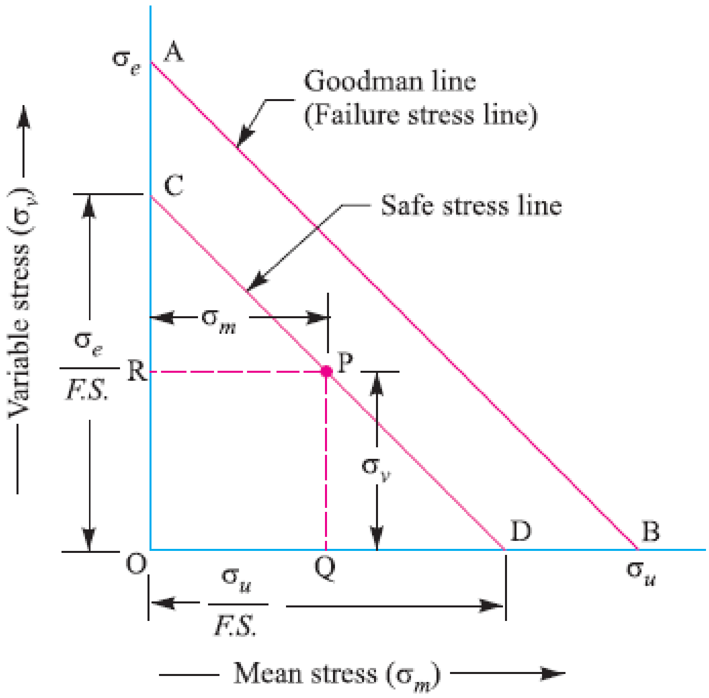
σ_e = Endurance limit for reversal loading.

Considering the fatigue stress concentration factor (Kf), the equation may be written as

$$\frac{1}{F.S.} = \left(\frac{\sigma_m}{\sigma_u} \right)^2 F.S. + \frac{\sigma_v \times K_f}{\sigma_e}$$

Goodman Method for Combination of Stresses

A straight line connecting the endurance limit (σe) and the ultimate strength (σu), as shown by line *AB* in Figure, follows the suggestion of Goodman. A Goodman line is used when the design is based on ultimate strength and may be used for ductile or brittle materials.



Now from similar triangles *COD* and *PQD*,

$$\frac{PQ}{CO} = \frac{QD}{OD} = \frac{OD - OQ}{OD} = 1 - \frac{OQ}{OD} \qquad ...(\because\ QD = OD - OQ)$$

$$\therefore \qquad \frac{*\,\sigma_v}{\sigma_e / F.S.} = 1 - \frac{\sigma_m}{\sigma_u / F.S.}$$

$$\sigma_v = \frac{\sigma_e}{F.S.}\left[1 - \frac{\sigma_m}{\sigma_u / F.S.}\right] = \sigma_e\left[\frac{1}{F.S.} - \frac{\sigma_m}{\sigma_u}\right]$$

$$\text{or} \qquad \frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v}{\sigma_e}$$

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e}$$

Where $F.S.$ = Factor of safety,

σ_m = Mean stress,

σ_u = Ultimate stress,

σ_v = Variable stress,

σ_e = Endurance limit for reversed loading, and

K_f = Fatigue stress concentration factor.

Considering the load factor, surface finish factor and size factor, the equation may be written as

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_{eb} \times K_{sur} \times K_{sz}} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e \times K_b \times K_{sur} \times K_{sz}}$$

$$= \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e \times K_{sur} \times K_{sz}} \quad \ldots(\because\ \sigma_{eb} = \sigma_e \times K_b \text{ and } K_b = 1)$$

K_b = Load factor for reversed bending load,

K_{sur} = Surface finish factor, and

K_{sz} = Size factor.

Note: Here we have assumed the same factor of safety ($F.S.$) for the ultimate tensile strength (σ_u) and endurance limit (σ_e). In case the factor of safety relating to both these stresses is different, then the following relation may be used

$$\frac{\sigma_v}{\sigma_e / (F.S.)_e} = 1 - \frac{\sigma_m}{\sigma_u / (F.S.)_u}$$

$(F.S.)_e$ = Factor of safety relating to endurance limit, and

$(F.S.)_u$ = Factor of safety relating to ultimate tensile strength.

Thus for brittle materials, the equation **may** be written as

$$\frac{1}{F.S.} = \frac{\sigma_m \times K_t}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_{eb} \times K_{sur} \times K_{sz}}$$

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_{ea} \times K_{sur} \times K_{sz}} \quad \text{...(For ductile materials)}$$

$$\frac{1}{F.S.} = \frac{\sigma_m \times K_t}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_{ea} \times K_{sur} \times K_{sz}} \quad \text{...(For brittle materials)}$$

Similarly, for reversed torsional or shear loading,

$$\frac{1}{F.S.} = \frac{\tau_m}{\tau_u} + \frac{\tau_v \times K_{fs}}{\tau_e \times K_{sur} \times K_{sz}} \quad \text{...(For ductile materials)}$$

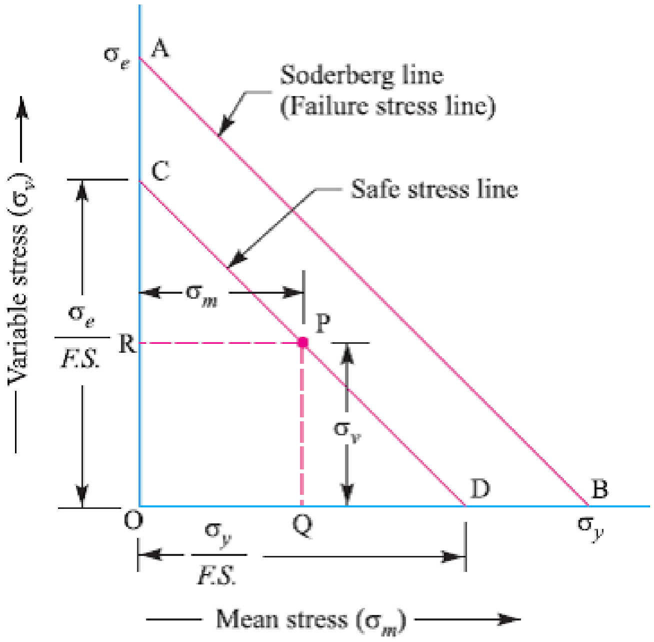
$$\frac{1}{F.S.} = \frac{\tau_m \times K_{ts}}{\tau_u} + \frac{\tau_v \times K_{fs}}{\tau_e \times K_{sur} \times K_{sz}} \quad \text{...(For brittle materials)}$$

Where suffix '*s*' denotes for shear. For reversed torsional or shear loading, the values of ultimate shear strength (τ_u) and endurance shear strength (τ_e) may be taken as follows:

$\tau_u = 0.8\ \sigma_u$; and $\tau_e = 0.8\ \sigma_e$

Soderberg Method for Combination of Stresses

A straight line connecting the endurance limit (σ_e) and the yield strength (σ_y), as shown by the line *AB* in Figure, follows the suggestion of Soderberg line. This line is used when the design is based on yield strength.



Now from similar triangles *COD* and *PQD*,

$$\frac{PQ}{CO} = \frac{QD}{OD} = \frac{OD - OQ}{OD}$$

$$= 1 - \frac{OQ}{OD} \qquad \ldots(\because QD = OD - OQ)$$

$$\therefore \quad \frac{\sigma_v}{\sigma_e / F.S.} = 1 - \frac{\sigma_m}{\sigma_y / F.S.}$$

or

$$\sigma_v = \frac{\sigma_e}{F.S.}\left[1 - \frac{\sigma_m}{\sigma_y / F.S.}\right] = \sigma_e\left[\frac{1}{F.S.} - \frac{\sigma_m}{\sigma_y}\right]$$

$$\therefore \quad \frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v}{\sigma_e}$$

For machine parts subjected to fatigue loading, the fatigue stress concentration factor (K_f) should be applied to only variable stress (σ_v).

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times K_f}{\sigma_e}$$

Considering the load factor, surface finish factor and size factor

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times K_f}{\sigma_{eb} \times K_{sur} \times K_{sz}}$$

Since $\sigma_{eb} = \sigma_e \times K_b$ and $K_b = 1$ for reversed bending load, therefore $\sigma_{eb} = \sigma_e$ may be substituted in the above equation.

When a machine component is subjected to reversed axial loading

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times K_f}{\sigma_{ea} \times K_{sur} \times K_{sz}}$$

When a machine component is subjected to reversed shear loading

$$\frac{1}{F.S.} = \frac{\tau_m}{\tau_y} + \frac{\tau_v \times K_{fs}}{\tau_e \times K_{sur} \times K_{sz}}$$

Problem 3

A machine component is subjected to a flexural stress which fluctuates between + 300 MN/m2 and – 150 MN/m2. Determine the value of minimum ultimate strength according to 1. Gerber relation; 2. Modified Goodman relation; and 3. Soderberg relation. Take yield strength = 0.55 Ultimate strength; Endurance strength = 0.5 Ultimate strength; and factor of safety = 2.

Solution

$\sigma_1 = 300$ MN/m2 ; $\sigma_2 = -150$ MN/m2 ; $\sigma y = 0.55\ \sigma u$; $\sigma e = 0.5\ \sigma u$; $F.S. = 2$

Let σ_u = Minimum ultimate strength in MN/m2.

We know that the mean or average stress,

$$\sigma_m = \frac{\sigma_1 + \sigma_2}{2} = \frac{300 + (-150)}{2} = 75 \text{ MN/m}^2$$

$$\sigma_v = \frac{\sigma_1 - \sigma_2}{2} = \frac{300 - (-150)}{2} = 225 \text{ MN/m}^2$$

1. **According to Gerber relation**

$$\frac{1}{F.S.} = \left(\frac{\sigma_m}{\sigma_u}\right)^2 F.S. + \frac{\sigma_v}{\sigma_e}$$

$$\frac{1}{2} = \left(\frac{75}{\sigma_u}\right)^2 2 + \frac{225}{0.5\sigma_u} = \frac{11\,250}{(\sigma_u)^2} + \frac{450}{\sigma_u} = \frac{11\,250 + 450\,\sigma_u}{(\sigma_u)^2}$$

$$(\sigma_u)^2 = 22\,500 + 900\,\sigma_u$$

or $$(\sigma_u)^2 - 900\,\sigma_u - 22\,500 = 0$$

$\therefore$ $$\sigma_u = \frac{900 \pm \sqrt{(900)^2 + 4 \times 1 \times 22\,500}}{2 \times 1} = \frac{900 \pm 948.7}{2}$$

$$= 924.35 \text{ MN/m}^2 \textbf{ Ans.}$$ ...(Taking +ve sign)

2. According to modified Goodman relation

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v}{\sigma_e}$$

$$\frac{1}{2} = \frac{75}{\sigma_u} + \frac{225}{0.5\,\sigma_u} = \frac{525}{\sigma_u}$$

$$\sigma_u = 2 \times 525 = 1050 \text{ MN/m}^2$$

3. According to Soderberg relation

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v}{\sigma_e}$$

$$\frac{1}{2} = \frac{75}{0.55\ \sigma_u} + \frac{255}{0.5\ \sigma_u} = \frac{586.36}{\sigma_u}$$

$$\sigma_u = 2 \times 586.36 = 1172.72\ \text{MN/m}^2$$