Lecture 1

Air Conditioning

Fundamental properties of air and vapor mixtures:

1- Definitions:

a- Air conditioning: the process of treating air to control simultaneously its temperature, humidity, cleanliness and distribution to meet the requirements of the conditioned space.

b- Ventilation: the process of supplying or removing air by natural or mechanical means to or from any space. Such air may or may not have been conditioned.

c- Air: is a mechanical mixture of gases and water vapour in the form of super-heated steam.

d- The law of partial pressures: (Gibbs-Dalton law).

In a given mixture of gases or vapours each gas or vapour exerts the same pressure it would exert if it occurred alone in the same space and at the same temperature as exists in the mixture. (the pressure exerted by each gas in a mixture of gases is independent of the presence of the other gases).

Corollary: in any gas mixture, the total pressure exerted is the summation of the partial pressures exerted independently by each of the constituent gases.

The total pressure for naturally occurring air.

$P_t = P_{N2} + P_{O2} + P_{o.g} + P_s = P_a + P_s$

(t-total, N_2-nitrogen, O_2- oxygen, o.g- other gases, a-air, s-steam).

P_a = partial pressure of the gases. (dry air).

P_s = partial pressure of the water vapour.

e.g. saturated air at 26°C and atmospheric pressure. Find the partial pressure of dry air and water vapour.

Solution: from steam tables at 26°C, the saturated pressure is

$P_s = 3.36$kpa.

$P_{atm.} = 101325 N/m^2$.

$P_a = P_t - P_s = 101325 - 3360 = 97.965 N/m^2$

e- Saturation: the maximum amount of steam that can exist in a certain space depends on the temperature and is independent of the weight and pressure of the air which may simultaneously exist in the same space. When the space holds the maximum amount it is said to be saturated.

2. The general gas law:

P . V = m . R . T

P= N/m^2, V=m^3, m=kg, R=constant., T=abs. temperature °K.

R=$\frac{R_o}{M}$, R_o= universal gas constant.

R_o = 8314.66 J/kmol.°K.

kmol= is a mass in kilograms numerically equal to the molecular mass of the gas.

$R_a = \frac{8314.66}{28.97} = 287$ J/kg.°K

$R_s = \frac{8314.66}{18.02} = 461$ J/kg.°K

Also remember:

$\rho_a = 1.293$ kg/m^3 at 101325N/m^2 & 0°C.

$\rho_w = 1000$ kg/m^3 at 4°C.

=998.23 kg/m^3 at 20°C.

Barometric pressure. $P_B = 101325$ N/m^2.

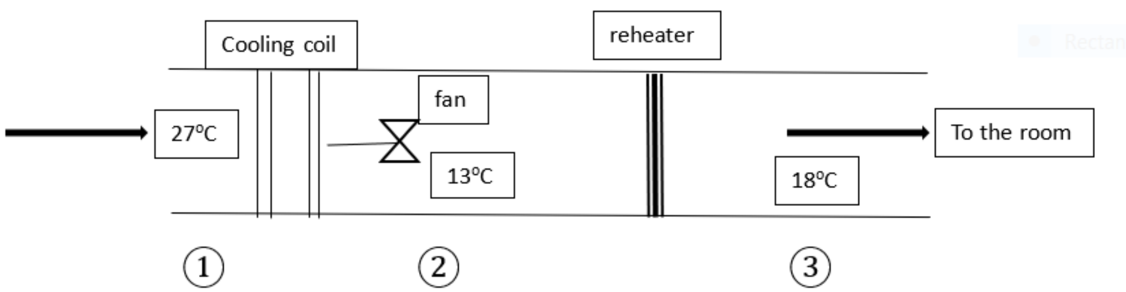
e.g: 15m^3/s of air at a temperature of 27°C passes over a cooling coil which reduces its temp. to 13°C. The air is then handled by a fan & blown over a reheater which increases its temp. to 18°C, then the air is supplied to a room. Calculate the quantity of air: a) handled by the fan. b)supplied to the room.

Solution:

a) PV=mRT

m,R,T = constant.

So, $\frac{V}{T} = C = \frac{mR}{P}$



$\frac{V_2}{T_2} = \frac{V_1}{T_1}$

$V_2 = \frac{T_2.V_1}{T_1} = \frac{15(13+273)}{(27+273)} = 14.3$ m³/sec.

b) $\frac{V_1}{T_1} = \frac{V_3}{T_3}$

or $V_3 = \frac{T_3.V_1}{T_1} = \frac{(18+273)*15}{(27+273)} = 14.55$m³/sec.

or

$$\frac{V_3}{T_3} = \frac{V_2}{T_2}$$

$V_3 = \frac{T_3.V_2}{T_2} = \frac{(18+273)*14.3}{(13+273)} = 14.55$m³/sec.

$V_3 > V_2$ as heating increase the volume.

e.g: 15grams of water vapour exist in 1m³ of air at 24°C and standard atmospheric pressure, calculate the partial pressure of air.

Solution:

PV=mRT

$P_s=\frac{mRT}{V}=\frac{\left(\frac{15}{1000}\right)*461*(24+273)}{1}=2053.76 N/m^2$ less than $P_{saturated\ at\ 24}{}^oC$

$P_{sat.\ at24}{}^oC=3.003\ N/m^2$.

So, air is not saturated as $P_s<P_{sat.}$

$P_a=P_{atm.}-P_s=101325-2053.76=99271.2\ N/m^2$

3- Vapour pressure of steam in moist air:

The vapour pressure of saturated steam is obtained from steam tables at the given air dry bulb temperature. This represents the partial pressure of saturated mixture associated with the air. For unsaturated air (i.e. vapour is not saturated but superheated) the partial pressure of the moisture is obtained from the empirical equation:-

$P = P_{sw} - P_B *A*(t_d - t_w)$

Where P= required partial pressure of vapour.

P_{sw} =saturation pressure at (t_w), the wet bulb temperature (from steam tables).

P_B =Barometric pressure.

t_d = dry bulb temperature.

t_w =wet bulb temperature.

(A)- constant equal to: $66.66*10^{-4}\ {}^oC^{-1}$ for $t_w \geq 0^oC$.

$5.94*10^{-4}\ {}^oC^{-1}$ for $t_w < 0^oC$.

Note: at saturation $t_d = t_w$, $P = P_{sw}$.

e.g: calculate the vapour pressure of moist air at 20°C dry bulb and 15°C wet bulb & the barometric pressure of 95kp_a .

sol: P= Psw – P_B .A.($t_d - t_w$)

p_{sw} = 1.7051 kp_a fro table at t_w =15°C.

$p = 1.7051 - 95*6.66*10^{-4} *(20-15) = 1.3887 kp_a$.

4- Moist air terminology:

a- Humidity (moisture): is the water vapour superheated or saturated mixed with atmospheric air.

b- Vapour density (d): is the weight of water vapour in kg occurring in each cubic meter of space.

$d=\frac{1}{\upsilon}$, υ= specific volume (v/m) (m=1kg).

c- Relative humidity (ϕ):

1- ϕ is the ratio of the partial pressure of water vapour in moist air to the pressure which saturated water vapour exerts at the same temperature.

$\phi=\left(\frac{P}{P_s}\right)_{t_d}$

2- Also, ϕ is the ratio of the density of water vapour in the air to the density of saturated water vapour at the air dry bulb temperature.

$\phi=\left(\frac{d}{d_s}\right)_{t_d}$

e.g: Air at 20°C d.b. & 15°C w.b. & 95kp_a barometric pressure, find a) the vapour density (d). & b) the relative humidity (ϕ).

Sol: from previous example P = 1.3887kp_a.

$P_s)_{20}$°C =2.339kp_a

a) $P\upsilon = RT$ & $d=\frac{1}{\upsilon}$ which give:

$d=\frac{P}{R.T_{d.b}}=\frac{1000*1.3887}{461*(20+273)}=0.0103 kg/m^3$

b) $\phi=\left(\frac{P}{P_s}\right)_{t_d}=\frac{1.3887}{2.339}=0.5937=59.37\%$

Or $\phi=\left(\frac{d}{d_s}\right)_{t_d}$, $d_s=\frac{P_s}{R_s .T_s}=\frac{2339}{461*(20+273)}=0.0173 kg/m^3$

or $d_s=\frac{1}{\upsilon_s}=\frac{1}{57.84\ from\ steam\ table\ at\ t_{d.b.}}=0.0173 kg/m^3$

$\phi=\frac{0.0103}{0.0173}=0.595=59.5\%$

e.g: At 26°C d.b. the vapour density was found to be ($0.018 kg/m^3$) when the barometric pressure was $90kp_a$. find the relative humidity (ϕ).

Sol: 1) from steam tables at 26°C dry bulb temperature(d.b.).

$$d_s = \frac{1}{\upsilon_s} = \frac{1}{41.266} = 0.02423 kg/m^3.$$

$$\phi = \left(\frac{d}{d_s}\right)_{td} = \frac{0.018}{0.02423} = 74.28\%$$

or 2) $P\upsilon = RT$

$P = d.\ R\ .T = 0.018*461*299 = 2481.1 N/m^2$

$P_s)_{26}{}^{o}C = 3.3844 kP_a$

$$\phi = \left(\frac{P}{P_s}\right)_{td} = \frac{2481.1}{3384.4} = 73.3\%.$$

e.g: Air at 24°C & 40% relative humidity (R.H.) in a space where the barometric read $92kP_a$. find

a) vapour density (d). b) partial pressure of water vapour (P).

Sol: a) $\phi = \left(\frac{d}{d_s}\right)_{td}$

$$d_s = \frac{1}{\upsilon_s} = \frac{1}{46.246} = 0.02162 kg/m^3$$

$d = \phi.d_s = 0.4*0.02162 = 0.00865 kg/m^3$

b) 1) $P = d.R.T = 0.00865*461*(24+273) = 1184.332 N/m^2$

or 2) $P = \phi * P_s$

$P_s = 3.003 kP_a = 3003 N/m^2$

$P = 0.4*3003 = 1201.2 N/m^2$

d- Humidity ratio (W): specific humidity (or) moisture content.

Defined as the mass of water vapour in kilograms which is associated with one kilogram of dry air in an air water mixture.

W= $\frac{m_s}{m_a}$

[PV=mRT] gives [$P_aV_a=m_aR_aT_a$] &[$P_sV_s=m_sR_sT_s$]

W=$\frac{P_sV_s}{R_sT_s}$ / $\frac{P_aV_a}{R_aT_a}$

$T_s=T_a$ & $V_s=V_a$

W=$\frac{R_aP_s}{R_sP_a}=\frac{287}{461}*\frac{P}{P_a}=0.622\frac{P}{P_B-P}$ kg vapour/kg of dry air

As $P_a=P_B$-P

e.g: the temperature in a certain room is 24°C and the relative humidity (r.h.) is 30%, P_B=95kP_a. find the specific humidity.

Sol: 1) P_s=3.003N/m^2 from steam table or chart at 24°C

P=ϕ.P_s =0.3*3.003=0.9009kPa

W=0.622*$\frac{0.9009}{95-0.9009}$ = 0.005955kg vapour/kg of dry air.

Or 2) W=$\frac{m_s}{m_a}=\frac{d_s\upsilon_s}{d_a\upsilon_a}=\frac{d_s}{d_a}=d_s.\upsilon_a=d.\upsilon_a$.

Where (d) is the vapour density and(υ_a) specific volume occupied by 1kg of dry air.

d=ϕ.d_s=0.3*($\frac{1}{\upsilon_s}$) = 0.3 * $\frac{1}{46.246}$ = 0.00648kg/m^3 (as υ_s=46.246)

Now $P_a.\upsilon_a$=R.T_a

$\upsilon_a=\frac{R.Ta}{P_a}$, $P_a=P_B$-P= 95-0.9009=94.0991kP_a

$\upsilon_a=\frac{287*(24+273)}{9409.91}$ = 0.9058m^3/kg.

W=d. υ_a=0.00648*0.9058=0.00586kg vapour/kg dry air.

e.g: Air at 40°C 80% r.h. enters an air conditioner which supplies conditioned air at16°C &96% r.h. find the amount of moisture removed by the cooling coil. P_B=100kPa.

Sol: let air at40°C 80% r.h. be condition (1)

And air at16°C &96% r.h. be condition (2)

$P_1=\phi_1.P_{s1}=0.8*7.384|_{\text{saturated pressure at }40^{o}C}= 5.9kP_a$

$P_2=\phi_2.P_{s2}=0.96*1.8318|_{\text{saturated pressure at }16^{o}C}= 1.758kP_a$

$W_1= 0.622\frac{P_1}{P_B-P_1}=0.622*\frac{5.9}{100-5.9} = 0.03899$ kg /kg dry air

$W_2= 0.622\frac{P_2}{P_B-P_2} = 0.622 * \frac{1.758}{100-1.758} = 0.01113$ kg/kg dry air.

Moisture removed=W_1-W_2=0.03899-0.01113=0.02786kg/kg dry air.

e) Saturation ratio: (μ) percentage saturation or degree of saturation, defined as the ratio of the moisture content of moist air at a given temperature to the moisture content of saturated air at the same temperature.

$$\mu=\frac{W}{W_s}$$

$$\mu=\frac{0.622[\frac{P}{P_B-P}]}{0.622[\frac{P_s}{P_B-P_s}]}=\frac{P}{P_s} * \frac{P_B-P_s}{P_B-P}= \phi*\frac{1-\frac{P_s}{P_B}}{1-\frac{P}{P_B}} = \phi*\frac{1-\frac{P_s}{P_B}}{1-\frac{P}{P_B}*\frac{P_s}{P_s}}$$

$$\mu= \phi*[\frac{1-\frac{P_s}{P_B}}{1-\phi*\left(\frac{P_s}{P_B}\right)}]$$

e.g: calculate the degree of saturation for air at 24°C & 40% r.h. P_B=100kPa.

Sol: $\mu= \phi*[\frac{1-\frac{P_s}{P_B}}{1-\phi*\left(\frac{P_s}{P_B}\right)}]$ $P_s=3.003kP_a$ from table at 24°C

$\mu = 0.4*[\frac{1-\frac{3.003}{100}}{1-0.4*\left(\frac{3.003}{100}\right)}]=0.3927$

f) Specific volume of humid air: (humid volume). This is the volume of one kilogram of dry air and the moisture associated with it. It can be found by the general law from:

1- mass and partial pressure of dry air.

2-mass and partial pressure of moisture.

[same volume occupied by both dry air and moisture but each at its own partial pressure].

e.g: find the specific volume of moist air at 40°C & 80% r.h. P_B=100kPa.

Sol: $P=\phi.P_s=0.8*7.384|_{\text{sat. press. at } 40^oC}=5.9kP_a$

1) $V_a=m_aR_aT_a/P_a$ (m_a=1kg), so $V_a=\upsilon_a$).

$P_a=P_B-P=100-5.9=94.1kP_a$

$V_a=\frac{1*287*(40+273)}{94100}=0.955m^3/kg$ of dry air. [(m_a=1kg), so $V_a=\upsilon_a$)].

Or 2) $V_s=m_sR_sT_s/P$ (P= steam pressure=5.9kPa as calculated above).

$W=0.622*\frac{P}{P_B-P}=0.622*\frac{5.9}{100-5.9}=0.03899$kg/kg dry air.

$W=\frac{m_s}{m_a}=\frac{m_s}{1}$ ------ $W=m_s=0.03899$kg/kg dry air.

$V_s=\frac{0.03899*461*(40+273)}{5900}=0.9535m^3/kg$ dry air.

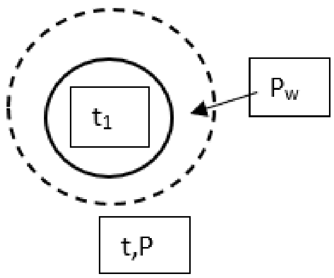
5) Dry bulb, Wet bulb and dew point temperatures: -

1- Dry bulb temperature: is the temperature of air measured by an ordinary thermometer.

2-Wet bulb temperature: is the temperature of air obtained by a thermometer whose bulb is wetted with water and exposed to an unsaturated air stream.

Phenomena off wet bulb depression:

Consider a water droplet suspended in an unsaturated air stream. Droplet temperature (t_1) with partial pressure of vapour film surrounding the droplet (P_w). Air temperature (t) and vapour pressure (P) and surface area (A).



When equilibrium is reached the water droplet will be at a temperature (t_w) known as the thermodynamic wet bulb temperature.

At equilibrium: sensible heat gain into water droplet = latent heat loss from droplet, as sensible heat used as a latent heat of evaporation in the droplet.

The rate of evaporation is proportional to the pressure difference (P_B-P). [+ve evaporation & -ve condensation].

Heat balance at equilibrium.

$(h_c+h_r).A.(t-t_w)=\alpha.A.h_{fg}(P_{sw}-P)$.

hc &h_r are the convective and radiative heat transfer coefficients.

α- diffusion coefficient through vapour film surrounding the droplet.

P_{sw}- saturated vapour pressure at (t_w)

h_{fg}- latent heat of evaporation at equilibrium.

$P_{sw}-P=\frac{h_c+h_r}{\alpha.h_{fg}}\,(t-t_w)$

$P=P_{sw}-\frac{h_c+h_r}{\alpha.h_{fg}}\,(t-t_w)$

The term [$\frac{h_c+h_r}{\alpha.h_{fg}}$] is a function of barometric pressure P_B and temperature. This equation is the basis for calculating the vapour partial pressure from dry and wet bulb temperatures as given previously:

$P=P_{sw}-P_B.A.(t_d-t_w)$. [A constant does not represent the surface area].