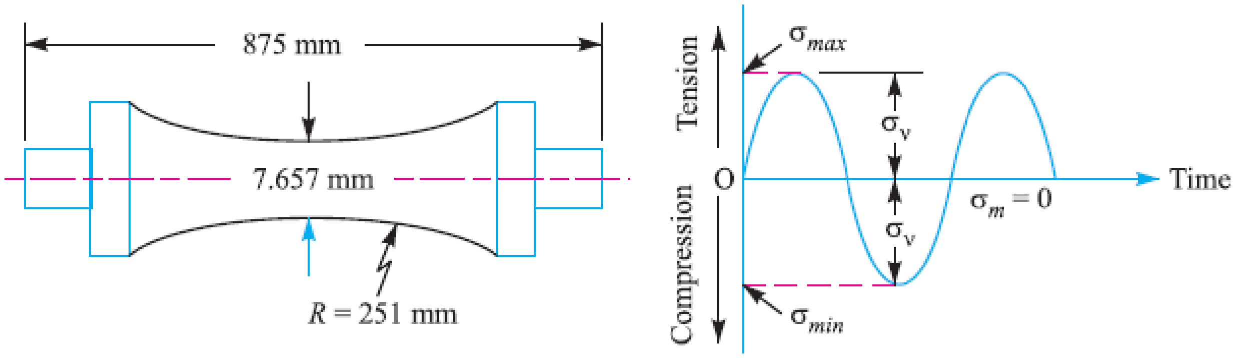
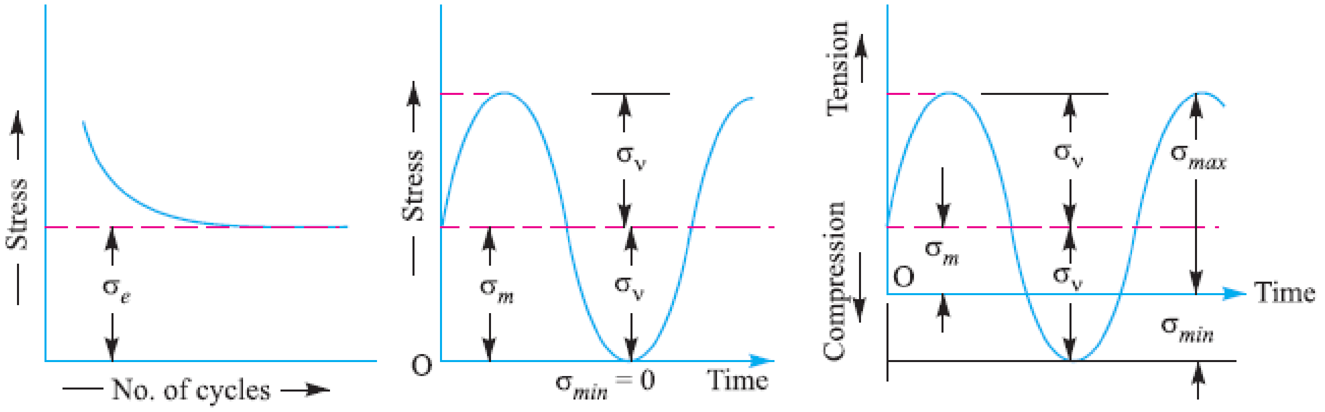
Fatigue and Endurance Limit

It has been found experimentally that when a material is subjected to repeated stresses, it fails at stresses below the yield point stresses. Such type of failure of a material is known as **fatigue.** The failure is caused by means of a progressive crack formation which are usually fine and of microscopic size. The failure may occur even without any prior indication. The fatigue of material is effected by the size of the component, relative magnitude of static and fluctuating loads and the number of load reversals.



(a) Standard specimen.

(b) Completely reversed stress.



(c) Endurance or fatigue limit.

(d) Repeated stress.

(e) Fluctuating stress.

Fig Time-stress diagrams.

The stress ***verses*** time diagram for fluctuating stress having values σ_{min} and σ_{max} is shown in Figure (*e*). The variable stress, in general, may be considered as a combination of steady (or mean or average) stress and a completely reversed stress component σ_v. The following relations are derived from Figure (*e*):

1. Mean or average stress,

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

2. Reversed stress component or alternating or variable stress,

$$\sigma_v = \frac{\sigma_{max} - \sigma_{min}}{2}$$

Note: For repeated loading, the stress varies from maximum to zero (*i.e.* $\sigma_{min} = 0$) in each cycle as shown in Figure (*d*).

$$\sigma_m = \sigma_v = \frac{\sigma_{max}}{2}$$

Effect of Loading on Endurance Limit—Load Factor

The endurance limit (σ_e) of a material as determined by the rotating beam method is for reversed bending load. There are many machine members which are subjected to loads other than reversed bending loads. Thus the endurance limit will also be different for different types of loading. The endurance limit depending upon the type of loading may be modified as discussed below:

Let K_b = Load correction factor for the reversed or rotating bending load. Its value is usually taken as unity.

K_a = Load correction factor for the reversed axial load. Its value may be taken as 0.8.

K_s = Load correction factor for the reversed torsional or shear load. Its value may be taken as 0.55 for ductile materials and 0.8 for brittle materials.

∴ Endurance limit for reversed bending load, $\sigma_{eb} = \sigma_e.K_b = \sigma_e$ ... ($K_b = 1$)

Endurance limit for reversed axial load, $\sigma_{ea} = \sigma_e.K_a$ and endurance limit for reversed torsional or shear load, $\tau_e = \sigma_e.K_s$

Effect of Surface Finish on Endurance Limit—Surface Finish Factor

Let K_{sur} = Surface finish factor.

∴ Endurance limit,

$\sigma_{e1} = \sigma_{eb}.K_{sur} = \sigma_e.K_b.K_{sur} = \sigma_e.K_{sur}$ ... ($K_b = 1$) ... (For reversed bending load)

$= \sigma_{ea}.K_{sur} = \sigma_e.K_a.K_{sur}$ ... (For reversed axial load)

$= \tau_e.K_{sur} = \sigma_e.K_s.K_{sur}$ ... (For reversed torsional or shear load)

Effect of Size on Endurance Limit—Size Factor

Let K_{sz} = Size factor.

∴ Endurance limit,

$\sigma_{e2} = \sigma_{e1} \times K_{sz}$ ... (Considering surface finish factor also)

$= \sigma_{eb}.K_{sur}.K_{sz} = \sigma_e.K_b.K_{sur}.K_{sz} = \sigma_e.K_{sur}.K_{sz}$ ($Kb = 1$)

$= \sigma_{ea}.K_{sur}.K_{sz} = \sigma_e.K_a.K_{sur}.K_{sz}$ ... (For reversed axial load)

$= \tau_e.K_{sur}.K_{sz} = \sigma_e.K_s.K_{sur}.K_{sz}$ ... (For reversed torsional or shear load)

Notes:

1. The value of size factor is taken as unity for the standard specimen having nominal diameter of 7.657 mm.

2. When the nominal diameter of the specimen is more than 7.657 mm but less than 50 mm, the value of size factor may be taken as 0.85.

3. When the nominal diameter of the specimen is more than 50 mm, then the value of size factor may be taken as 0.75.

For steel, $\sigma_e = 0.5\ \sigma_u$;

For cast steel, $\sigma_e = 0.4\ \sigma_u$;

For cast iron, $\sigma_e = 0.35\ \sigma_u$;

For non-ferrous metals and alloys, $\sigma_e = 0.3\ \sigma_u$

Factor of Safety for Fatigue Loading

When a component is subjected to fatigue loading, the endurance limit is the criterion for failure. Therefore, the factor of safety should be based on endurance limit. Mathematically,

$$\text{Factor of safety } (F.S.) = \frac{\text{Endurance limit stress}}{\text{Design or working stress}} = \frac{\sigma_e}{\sigma_d}$$

Note:

For steel, $\sigma_e = 0.8$ to $0.9\ \sigma_y$

Where σ_e = Endurance limit stress for completely reversed stress cycle, and

σ_y = Yield point stress.

Problem 1

Determine the design stress for a piston rod where the load is completely reversed. The surface of the rod is ground and the surface finish factor is 0.9. There is no stress concentration. The load is predictable and the factor of safety is 2.

Solution

$K_{sur} = 0.9$; $F.S. = 2$

The piston rod is subjected to reversed axial loading. We know that for reversed axial loading, the load correction factor (K_a) is 0.8

If σ_e is the endurance limit for reversed bending load, then endurance limit for reversed axial load,

$$\sigma_{ea} = \sigma_e \times K_a \times K_{sur} = \sigma_e \times 0.8 \times 0.9 = 0.72\ \sigma_e$$

Design stress

$$\sigma_d = \frac{\sigma_{ea}}{F.S.} = \frac{0.72\ \sigma_e}{2} = 0.36\ \sigma_e$$

Stress Concentration

The theoretical or form stress concentration factor is defined as the ratio of the maximum stress in a member (at a notch or a fillet) to the nominal stress at the same section based upon net area. Mathematically, theoretical or form stress concentration factor,

$$K_t = \frac{\text{Maximum stress}}{\text{Nominal stress}}$$

The value of K_t depends upon the material and geometry of the part

Stress Concentration due to Holes and Notches

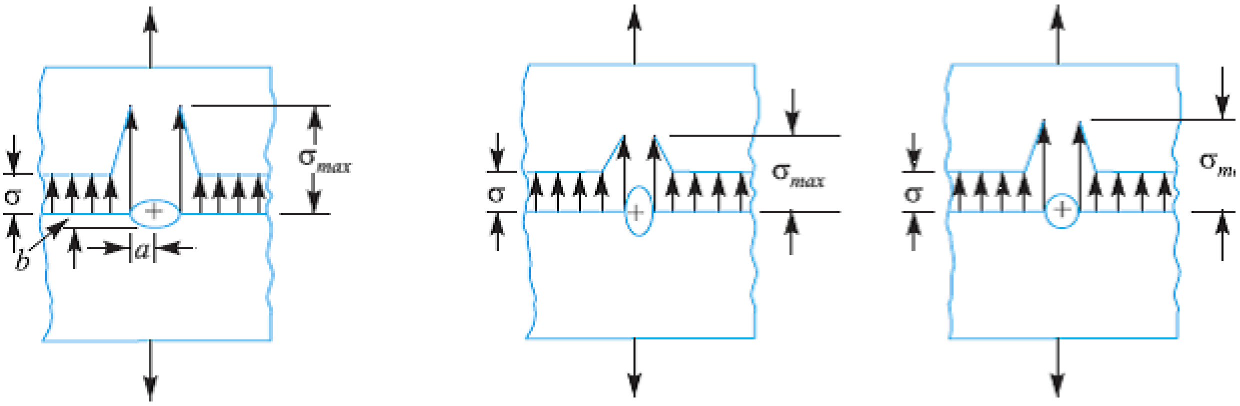
Consider a plate with transverse elliptical hole and subjected to a tensile load as shown in Figure (*a*). We see from the stress-distribution that the stress at the point away from the hole is practically uniform and the maximum stress will be induced at the edge of the hole. The maximum stress is given by

$$\sigma_{max} = \sigma\left(1 + \frac{2a}{b}\right)$$

and the theoretical stress concentration factor,

$$K_t = \frac{\sigma_{max}}{\sigma} = \left(1 + \frac{2a}{b}\right)$$

When a/b is large, the ellipse approaches a crack transverse to the load and the value of K_t becomes very large. When a/b is small, the ellipse approaches a longitudinal slit [as shown in Figure (*b*)] and the increase in stress is small. When the hole is circular as shown in Figure (*c*), then $a/b = 1$ and the maximum stress is three times the nominal value.



$a/b = 2$
$\sigma_{max} = 5\sigma$
(*a*)

$a/b = 1/2$
$\sigma_{max} = 2\sigma$
(*b*)

$a/b = 1$
$\sigma_{max} = 3\sigma$
(*c*)

. Stress concentration due to holes.

The following tables show the theoretical stress concentration factor for various types of members.

Table 1. Theoretical stress concentration factor (K_t) for a plate with hole (of diameter d) in tension.

$\frac{d}{b}$	0.05	0.1	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55
K_t	2.83	2.69	2.59	2.50	2.43	2.37	2.32	2.26	2.22	2.17	2.13

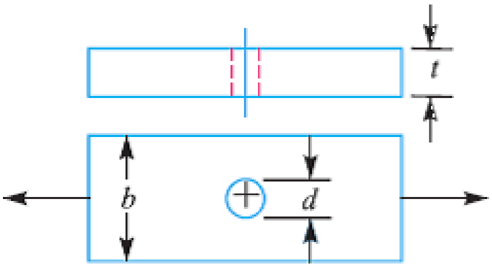


Fig. for Table 1

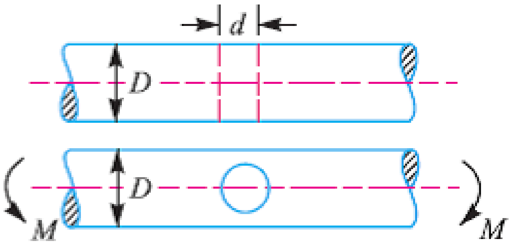


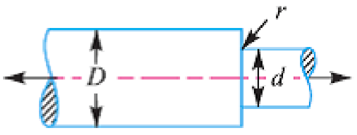
Fig. for Table 2

Table 2. Theoretical stress concentration factor (K_t) for a shaft with transverse hole (of diameter d) in bending.

$\frac{d}{D}$	0.02	0.04	0.08	0.10	0.12	0.16	0.20	0.24	0.28	0.30
K_t	2.70	2.52	2.33	2.26	2.20	2.11	2.03	1.96	1.92	1.90

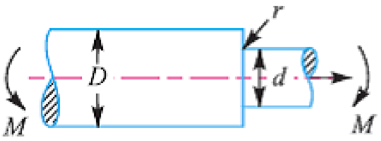
Table 3. Theoretical stress concentration factor (K_t) for stepped shaft with a shoulder fillet (of radius r) in tension.

$\frac{D}{d}$	Theoretical stress concentration factor (K_t)									
	r/d									
	0.08	0.10	0.12	0.16	0.18	0.20	0.22	0.24	0.28	0.30
1.01	1.27	1.24	1.21	1.17	1.16	1.15	1.15	1.14	1.13	1.13
1.02	1.38	1.34	1.30	1.26	1.24	1.23	1.22	1.21	1.19	1.19
1.05	1.53	1.46	1.42	1.36	1.34	1.32	1.30	1.28	1.26	1.25
1.10	1.65	1.56	1.50	1.43	1.39	1.37	1.34	1.33	1.30	1.28
1.15	1.73	1.63	1.56	1.46	1.43	1.40	1.37	1.35	1.32	1.31
1.20	1.82	1.68	1.62	1.51	1.47	1.44	1.41	1.38	1.35	1.34
1.50	2.03	1.84	1.80	1.66	1.60	1.56	1.53	1.50	1.46	1.44
2.00	2.14	1.94	1.89	1.74	1.68	1.64	1.59	1.56	1.50	1.47



$A = \frac{\pi}{4} \times d^2$

Fig. for Table .3



$Z = \frac{\pi}{32} \times d^3$

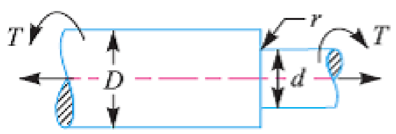
Fig. for Table .4

Table 4. Theoretical stress concentration factor (K_t) for a stepped shaft with a shoulder fillet (of radius r) in bending.

$\frac{D}{d}$	Theoretical stress concentration factor (K_t)									
	r/d									
	0.02	0.04	0.08	0.10	0.12	0.16	0.20	0.24	0.28	0.30
1.01	1.85	1.61	1.42	1.36	1.32	1.24	1.20	1.17	1.15	1.14
1.02	1.97	1.72	1.50	1.44	1.40	1.32	1.27	1.23	1.21	1.20
1.05	2.20	1.88	1.60	1.53	1.48	1.40	1.34	1.30	1.27	1.25
1.10	2.36	1.99	1.66	1.58	1.53	1.44	1.38	1.33	1.28	1.27
1.20	2.52	2.10	1.72	1.62	1.56	1.46	1.39	1.34	1.29	1.28
1.50	2.75	2.20	1.78	1.68	1.60	1.50	1.42	1.36	1.31	1.29
2.00	2.86	2.32	1.87	1.74	1.64	1.53	1.43	1.37	1.32	1.30
3.00	3.00	2.45	1.95	1.80	1.69	1.56	1.46	1.38	1.34	1.32
6.00	3.04	2.58	2.04	1.87	1.76	1.60	1.49	1.41	1.35	1.33

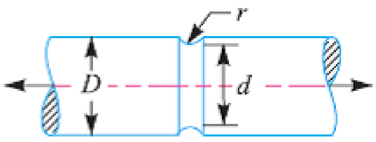
Table 5. Theoretical stress concentration factor (K_t) for a stepped shaft with a shoulder fillet (of radius r) in torsion.

$\frac{D}{d}$	Theoretical stress concentration factor (K_t)									
	r/d									
	0.02	0.04	0.08	0.10	0.12	0.16	0.20	0.24	0.28	0.30
1.09	1.54	1.32	1.19	1.16	1.15	1.12	1.11	1.10	1.09	1.09
1.20	1.98	1.67	1.40	1.33	1.28	1.22	1.18	1.15	1.13	1.13
1.33	2.14	1.79	1.48	1.41	1.35	1.28	1.22	1.19	1.17	1.16
2.00	2.27	1.84	1.53	1.46	1.40	1.32	1.26	1.22	1.19	1.18



$$\frac{J}{d/2} = \frac{\pi d^3}{16}$$

Fig. for Table 5



$$A = \frac{\pi}{4} \times d^2$$

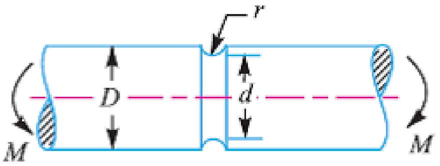
Fig. for Table 6

Table 6. Theoretical stress concentration factor (K_t) for a grooved shaft in tension.

$\frac{D}{d}$	Theoretical stress concentration (K_t)									
	r/d									
	0.02	0.04	0.08	0.10	0.12	0.16	0.20	0.24	0.28	0.30
1.01	1.98	1.71	1.47	1.42	1.38	1.33	1.28	1.25	1.23	1.22
1.02	2.30	1.94	1.66	1.59	1.54	1.45	1.40	1.36	1.33	1.31
1.03	2.60	2.14	1.77	1.69	1.63	1.53	1.46	1.41	1.37	1.36
1.05	2.85	2.36	1.94	1.81	1.73	1.61	1.54	1.47	1.43	1.41
1.10	..	2.70	2.16	2.01	1.90	1.75	1.70	1.57	1.50	1.47
1.20	..	2.90	2.36	2.17	2.04	1.86	1.74	1.64	1.56	1.54
1.30	..	..	2.46	2.26	2.11	1.91	1.77	1.67	1.59	1.56
1.50	..	..	2.54	2.33	2.16	1.94	1.79	1.69	1.61	1.57
2.00	..	..	2.61	2.38	2.22	1.98	1.83	1.72	1.63	1.59
∞	..	..	2.69	2.44	2.26	2.03	1.86	1.74	1.65	1.61

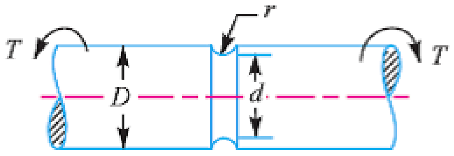
Table 7. Theoretical stress concentration factor (K_t) of a grooved shaft in bending.

$\frac{D}{d}$	Theoretical stress concentration factor (K_t)									
	r/d									
	0.02	0.04	0.08	0.10	0.12	0.16	0.20	0.24	0.28	0.30
1.01	1.74	1.68	1.47	1.41	1.38	1.32	1.27	1.23	1.22	1.20
1.02	2.28	1.89	1.64	1.53	1.48	1.40	1.34	1.30	1.26	1.25
1.03	2.46	2.04	1.68	1.61	1.55	1.47	1.40	1.35	1.31	1.28
1.05	2.75	2.22	1.80	1.70	1.63	1.53	1.46	1.40	1.35	1.33
1.12	3.20	2.50	1.97	1.83	1.75	1.62	1.52	1.45	1.38	1.34
1.30	3.40	2.70	2.04	1.91	1.82	1.67	1.57	1.48	1.42	1.38
1.50	3.48	2.74	2.11	1.95	1.84	1.69	1.58	1.49	1.43	1.40
2.00	3.55	2.78	2.14	1.97	1.86	1.71	1.59	1.55	1.44	1.41
∞	3.60	2.85	2.17	1.98	1.88	1.71	1.60	1.51	1.45	1.42



$$Z = \frac{\pi}{32} \times d^3$$

Fig. for Table 7



$$\frac{J}{d/2} = \frac{\pi d^3}{16}$$

Fig. for Table 8

Table 8. Theoretical stress concentration factor (K_t) for a grooved shaft in torsion.

$\frac{D}{d}$	Theoretical stress concentration factor (K_{ts})									
	r/d									
	0.02	0.04	0.08	0.10	0.12	0.16	0.20	0.24	0.28	0.30
1.01	1.50	1.03	1.22	1.20	1.18	1.16	1.13	1.12	1.12	1.12
1.02	1.62	1.45	1.31	1.27	1.23	1.20	1.18	1.16	1.15	1.16
1.05	1.88	1.61	1.40	1.35	1.32	1.26	1.22	1.20	1.18	1.17
1.10	2.05	1.73	1.47	1.41	1.37	1.31	1.26	1.24	1.21	1.20
1.20	2.26	1.83	1.53	1.46	1.41	1.34	1.27	1.25	1.22	1.21
1.30	2.32	1.89	1.55	1.48	1.43	1.35	1.30	1.26	—	—
2.00	2.40	1.93	1.58	1.50	1.45	1.36	1.31	1.26	—	—
∞	2.50	1.96	1.60	1.51	1.46	1.38	1.32	1.27	1.24	1.23