Shafts Subjected to Bending Moment Only

When the shaft is subjected to a bending moment only, then the maximum stress (tensile or compressive) is given by the bending equation. We know that

$$\frac{M}{I} = \frac{\sigma_b}{y} \qquad (1)$$

M = Bending moment,

I = Moment of inertia of cross-sectional area of the shaft about the axis of rotation,

σ_b = Bending stress, and

y = Distance from neutral axis to the outer-most fibre.

We know that for a round solid shaft, moment of inertia,

$$I = \frac{\pi}{64} \times d^4 \quad \text{and} \quad y = \frac{d}{2}$$

Substituting these values in equation **(1)**, we have

$$\frac{M}{\frac{\pi}{64} \times d^4} = \frac{\sigma_b}{\frac{d}{2}} \quad \text{or} \quad M = \frac{\pi}{32} \times \sigma_b \times d^3$$

From this equation, diameter of the solid shaft (d) may be obtained.

We also know that for a hollow shaft, moment of inertia,

$$I = \frac{\pi}{64}\left[(d_o)^4 - (d_i)^4\right] = \frac{\pi}{64}(d_o)^4\,(1 - k^4) \qquad \ldots(\text{where } k = d_i / d_o)$$

$$y = d_o / 2$$

Again substituting these values in equation **(1)**, we have

$$\frac{M}{\frac{\pi}{64}(d_o)^4\,(1 - k^4)} = \frac{\sigma_b}{\frac{d_o}{2}} \quad \text{or} \quad M = \frac{\pi}{32} \times \sigma_b\,(d_o)^3\,(1 - k^4)$$

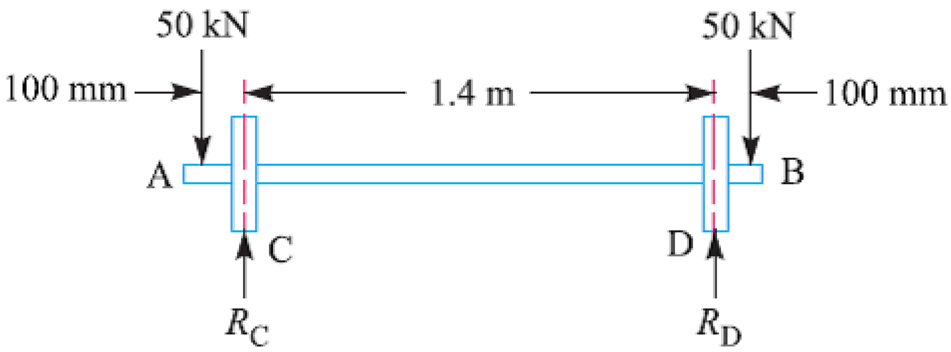
From this equation, the outside diameter of the shaft (d_o) may be obtained.

Problem 3

A pair of wheels of a railway wagon carries a load of 50 kN on each axle box, acting at a distance of 100 mm outside the wheel base. The gauge of the rails is 1.4 m. Find the diameter of the axle between the wheels, if the stress is not to exceed 100 MPa.

Solution

W = 50 kN = 50 × 10^3 N ; L = 100 mm ; x = 1.4 m ; σ_b = 100 MPa = 100 N/mm^2



The maximum B.M. may be obtained as follows :

RC = RD = 50 kN = 50 × 10^3 N

B.M. at A, $M_A = 0$

B.M. at C, $M_C = 50 \times 10^3 \times 100 = 5 \times 10^6$ N-mm

B.M. at D, $M_D = 50 \times 10^3 \times 1500 - 50 \times 103 \times 1400 = 5 \times 10^6$ N-mm

B.M. at B, $M_B = 0$

A little consideration will show that the maximum bending moment acts on the wheels at C and D. Therefore maximum bending moment,

$M = W.L = 50 \times 10^3 \times 100 = 5 \times 10^6$ N-mm

Let d = Diameter of the axle.

We know that the maximum bending moment (M),

$$5 \times 10^6 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 100 \times d^3 = 9.82\, d^3$$

$$d^3 = 5 \times 10^6 / 9.82 = 0.51 \times 10^6 \text{ or } d = 79.8 \text{ say } 80 \text{ mm}$$

Shafts Subjected to Combined Twisting Moment and Bending Moment

Let τ = Shear stress induced due to twisting moment, and

σ_b = Bending stress (tensile or compressive) induced due to bending moment.

According to maximum shear stress theory, the maximum shear stress in the shaft,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

Substituting the values of τ and σ_b from **<u>Twisting Moment Only</u>** and **<u>Bending Moment Only</u>**

$$\tau_{max} = \frac{1}{2}\sqrt{\left(\frac{32M}{\pi d^3}\right)^2 + 4\left(\frac{16T}{\pi d^3}\right)^2} = \frac{16}{\pi d^3}\left[\sqrt{M^2 + T^2}\right]$$

$$\frac{\pi}{16} \times \tau_{max} \times d^3 = \sqrt{M^2 + T^2} \qquad (1)$$

Equivalent twisting moment and is denoted by T_e

$$T_e = \sqrt{M^2 + T^2} = \frac{\pi}{16} \times \tau \times d^3 \qquad (2)$$

From this expression, diameter of the shaft (d) may be evaluated.

Now according to maximum normal stress theory, the maximum normal stress in the shaft,

$$\sigma_{b(max)} = \frac{1}{2}\sigma_b + \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

$$= \frac{1}{2}\times\frac{32M}{\pi d^3} + \frac{1}{2}\sqrt{\left(\frac{32M}{\pi d^3}\right)^2 + 4\left(\frac{16T}{\pi d^3}\right)^2}$$

$$= \frac{32}{\pi d^3}\left[\frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right)\right]$$

$$\frac{\pi}{32}\times\sigma_{b\,(max)}\times d^3 = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right] \qquad (4)$$

Equivalent bending moment and is denoted by M_e.

$$M_e = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right] = \frac{\pi}{32}\times\sigma_b\times d^3 \qquad (5)$$

From this expression, diameter of the shaft (d) may be evaluated.

In case of a hollow shaft, the equations **(2)** and **(5)** may be written as

$$T_e = \sqrt{M^2 + T^2} = \frac{\pi}{16}\times\tau\,(d_o)^3\,(1 - k^4)$$

$$M_e = \tfrac{1}{2}\left(M + \sqrt{M^2 + T^2}\right) = \frac{\pi}{32}\times\sigma_b\,(d_o)^3\,(1 - k^4)$$

It is suggested that diameter of the shaft may be obtained by using both the theories and the larger of the two values is adopted.

نعتمد قيمة القطر الاعلى في اي المعادلتين

Problem 4

A solid circular shaft is subjected to a bending moment of 3000 N-m and a torque of 10000 N-m. The shaft is made of 45 C 8 steel having ultimate tensile stress of 700 MPa and a ultimate shear stress of 500 MPa. Assuming a factor of safety as 6, determine the diameter of the shaft.

Solution

Given : M = 3000 N-m = 3×10^6 N-mm ; T = 10 000 N-m = 10×10^6 N-mm ; σ_{tu} = 700 MPa = 700 N/mm^2 ; τ_u = 500 MPa = 500 N/mm^2

We know that the allowable tensile stress,

$$\sigma_t \text{ or } \sigma_b = \frac{\sigma_{tu}}{F.S.} = \frac{700}{6} = 116.7 \text{ N/mm}^2$$

and allowable shear stress,

$$\tau = \frac{\tau_u}{F.S.} = \frac{500}{6} = 83.3 \text{ N/mm}^2$$

d = Diameter of the shaft in mm.

According to maximum shear stress theory, equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(3 \times 10^6)^2 + (10 \times 10^6)^2} = 10.44 \times 10^6 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$10.44 \times 10^6 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 83.3 \times d^3 = 16.36\, d^3$$

$$d^3 = 10.44 \times 10^6 / 16.36 = 0.636 \times 10^6 \text{ or } d = 86 \text{ mm}$$

According to maximum normal stress theory, equivalent bending moment,

$$M_e = \frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right) = \frac{1}{2}(M + T_e)$$

$$= \frac{1}{2}(3 \times 10^6 + 10.44 \times 10^6) = 6.72 \times 10^6 \text{ N-mm}$$

We also know that the equivalent bending moment (M_e),

$$6.72 \times 10^6 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 116.7 \times d^3 = 11.46\, d^3$$

$$d^3 = 6.72 \times 10^6 / 11.46 = 0.586 \times 10^6 \text{ or } d = 83.7 \text{ mm}$$

Taking the larger of the two values, we have

d = 86 say 90 mm

Homework

A shaft made of mild steel is required to transmit 100 kW at 300 r.p.m. The supported length of the shaft is 3 meters. It carries two pulleys each weighing 1500 N supported at a distance of 1 meter from the ends respectively. Assuming the safe value of stress, determine the diameter of the shaft.

Hint