Electromagnetic waves

Lecture 1

Vector Analysis and Vector Algebra

By

Ali Jaafar

Tow stage

Department of medical physics

Al-Mustaqbal University-College

1-Vector Analysis and Vector Algebra

1-1Vector is a quantity having both magnitude and direction such as displacement, velocity, force and acceleration.

Example1
Write the vector for each of the following:a. of the vector $(1, -3, -5)$ to($2, -7,0$).
b . of the vector $(2, -7,0)$ to) $1, -3, -5$).c .The location vector to (4,90.(− .) solution
a$\langle -1,4,-5 \rangle$ b$\langle 1,-4,5 \rangle$
The two vectors in a and b are different in sign only, and this shows that they have the same magnitude, but they are opposite

1-2Vector Algebra

Laws of vector algebra. If A, B and C are vectors and m and n are scalars, then
1. A+ B = B + A Commutative Law for Addition
2. A+ (B+C) = (A+B) + C Associative Law for Addition
3. mA = Am Commutative Law for Multiplication
4. m (nA) = (mn) A Associative Law for Multiplication
5. (m+ n) A = mA + nA Distributive Law
6. m (A+ B) = mA + mB Distributive Law

Example:

Let us take A = 10 and B = 5

10 + 5 = 5 + 10

15 = 15

Example:

Prove:- (3+7) = (-3)+(-7)

Proof:

-(10) = -3-7

-10 = -10

L.H.S = R.H.S

Example:

Let us take A = 2, B = 4 and C = 6

L.H.S =A+(B+C) = 2 + (4 + 6)

= 12

R.H.S = (A+B)+C = (2 + 4) + 6

=12

L.H.S = R.H.S

12 = 12

Example:

Let us take A = 2, B = 3 and C = 5

L.H.S $=A \times (B + C)= 2 \times (3+5)$

$= 2 \times 8$

$= 16$

R.H.S $= A \times B + A \times C = 2 \times 3 + 2 \times 5$

$=6+ 10$

$=16$

L.H.S = R.H.S

16 = 16

Example

$A=4,m=5$

$mA = Am$

$5x4=4x5$

$20=20$

Example

$A=5$

$m=3$

$n=2$

$m (nA) = (mn) A$

$3(2x5)=(3x2)5$

3x10=6x5

30=30

1-3 Scalar product

The process of multiplying a vector quantity by another vector quantity, the product of which is a non-vector scalar quantity, which has only an amount..

- The dot product of two vectors is given by the formula $\vec{a} \cdot \vec{b} = |a||b| \cos(\theta)$.

1-3-1 The following laws are valid:

1. A . B = B . A Commutative Law for Dot Products
2. A . (B + C) = A . B + A . C Distributive Law
3. m(A . B) = (mA) . B = A . (mB) = (A . B)m, where m is a scalar.
4. i . i = j . j = k . k = 1, i . j = j . k = k . i = 0
5. If A = Al i + A2 j + A3 k and B = Bl i + B2 j + B3 k, then

A . B = A1B1+A2 B2+A3 B3

A . A = A2 = A1 2 + A2 2 + A3 2

B . B = B 2 = B1 2 + B2 2 + B3 2

6. If A . B = 0 and A and B are not null vectors, then A and B are perpendicular.

Example: Find the scalar product of the vectors a = 2i + 3j - 6k and b = i + 9k.

Solution: To find the scalar product of the given vectors a and b, we will multiply their corresponding components.

a.b = (2i + 3j - 6k).(i + 0j + 9k)

= 2.1 + 3.0 + (-6).9

= 2 + 0 - 54

= -52

<u>Example</u>: Calculate the scalar product of vectors a and b when the modulus of a is 9, modulus of b is 7 and the angle between the two vectors is 60°.

Solution: To determine the scalar product of vectors a and b, we will use the scalar product formula.

a.b = |a| |b| cosθ

= 9 × 7 cos 60°

= 63 × 1/2

= 31.5

<u>Example 1</u>: Find the angle between the two vectors 2i + 3j + k, and 5i -2j + 3k.

Solution:

The two given vectors are:

$\vec{a} = 2i + 3i + k$, and $\vec{b} = 5i - 2j + 3k$

$|a| = \sqrt{2^2 + 3^2 + 1^2} = \sqrt{4 + 9 + 1} = \sqrt{14}$

$|b| = \sqrt{5^2 + (-2)^2 + 3^2} = \sqrt{25 + 4 + 9} = \sqrt{38}$

Using the dot product we have $\vec{a} \cdot \vec{b} = 2.(5) + 3.(-2) + 1.(3) = 10 - 6 + 3 = 7$

$$\text{Cos}\theta = \frac{a.b}{|a|.|b|}$$

$$= \frac{7}{\sqrt{14}.\sqrt{38}}$$

$$= \frac{7}{2.\sqrt{7 \times 19}}$$

$$= \frac{7}{2\sqrt{133}}$$

$$\theta = \text{Cos}^{-1}\frac{7}{2\sqrt{133}}$$

$\theta = \text{Cos}^{-1}\, 0.304 = 72.3°$

1-4 Vector products

The process of multiplying a vector quantity by another vector quantity, the product of which is a vector quantity with magnitude and direction.

- The cross product of two vectors is given by the formula $\vec{a} \times \vec{b} = |a||b| \sin(\theta)$.

Cross or vector product

1-4-1 The following laws are valid:

1. A × B = - B × A Commutative Law for Cross Products Fails
2. A × (B + C) = A × B + A × C Distributive Law
3. m(A × B) = (mA) × B = A × (mB) = (A × B)m , where m is a scalar.
4. i × i = j × j = k × k = 0,
i × j = −j × i = k,
j × k = −k × j = i,
k × i = −i × k = j.

5. If A = Al i + A2 j + A3 k and B = Bl i + B2 j + B3 k, then
6. The magnitude of A × B is the same as the area of a parallelogram with sides A and B.
7. If A × B = 0 and A and B are not null vectors, then A and B are parallel.

Example: Find the cross product of two vectors $\vec{a} = (3,4,5)$ and $\vec{b} = (7,8,9)$

Solution:

The cross product is given as,

$$a \times b = \begin{array}{ccc} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 5 \\ 7 & 8 & 9 \end{array}$$

$= [(4\times9)-(5\times8)]\,\hat{i} - [(3\times9)-(5\times7)]\,\hat{j} + [(3\times8)-(4\times7)]\,\hat{k}$

$= (36-40)\hat{i} - (27-35)\hat{j} + (24-28)\,\hat{k} = -4\hat{i} + 8\hat{j} - 4\hat{k}$

Example: Two vectors have their scalar magnitude as $|a|=2\sqrt{3}$ and $|b| = 4$, while the angle between the two vectors is 60°.

Calculate the cross product of two vectors.

Solution:

We know that $\sin 60^{\circ} = \sqrt{3}/2$

The cross product of the two vectors is given by, $\vec{a} \times \vec{b} = |a||b|\sin(\theta)\hat{n} = 2\sqrt{3}\times4\times\sqrt{3}/2 = 12\hat{n}$

Example If $\vec{a} = (2, -4, 4)$ and $\vec{b} = (4, 0,3)$, find the angle between them.

Solution

$\vec{a} = 2i - 4j + 4k$

$\vec{b} = 4i + 0j + 3k$

The magnitude of $\vec{a}$ is

$|a| = \sqrt{(2^2+4^2+4^2)} = \sqrt{36} = 6$

The magnitude of $\vec{b}$ is

$|b| = \sqrt{(4^2+0^2+3^2)} = \sqrt{25} = 5$

As per the cross product formula, we have

$$\vec{a} \times \vec{b} = \begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -4 & 4 \\ 4 & 0 & 3 \end{matrix}$$

$$= [(-4 \times 3) - (4 \times 0)]\hat{i}$$
$$-[(3 \times 2) - (4 \times 4)]\hat{j}$$

$$+[(2 \times 0) - (-4 \times 4)]\hat{k}$$

$$= -12\hat{i} + 10\hat{j} + 16\hat{k}$$

$\vec{a} \times \vec{b} = (-12, 10, 16)$

The length of the $\vec{c}$ is

$|c| = \sqrt{(-(12)^2 + 10^2 + 16^2)}$

$= \sqrt{(144+100+256)}$

$= \sqrt{500}$

$= 10\sqrt{5}$

$\vec{a} \times \vec{b} = |a||b| \sin\theta$

$$\sin\theta = \frac{\vec{a} \times \vec{b}}{|a||b|}$$

$\sin\theta = 10\sqrt{5}/(5\times 6)$

$\sin\theta = \sqrt{5}/3$

$\theta = \sin^{-1}(\sqrt{5}/3)$

$\theta = \sin^{-1}(0.74)$

$\theta = 48°$