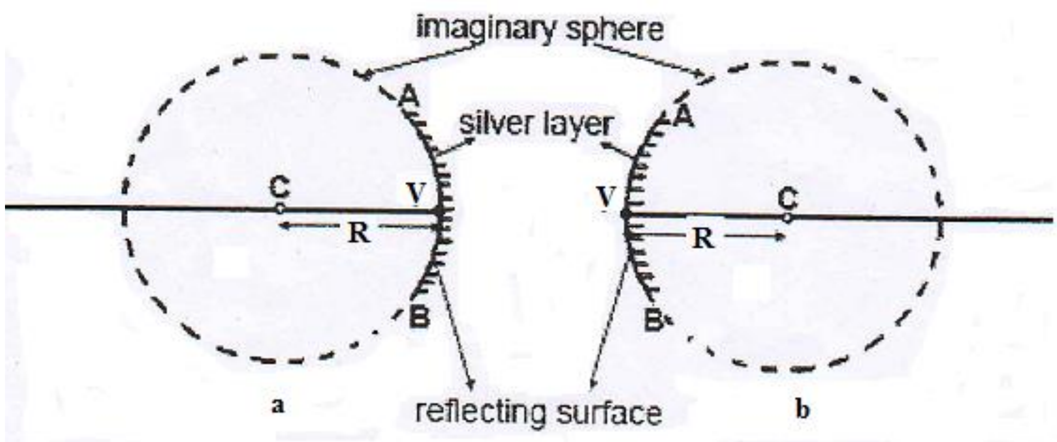
Lecture 5: Reflection of Spherical Mirrors

- Definition: is defined as a reflecting surface that is a part of a sphere.
- There two types of spherical mirror. It is **convex** (curving outwards) and **concave** (curving inwards) mirrors.
- Figures below show the shape of concave and convex mirrors
  (a) Concave (**converging**) mirror. (b) Convex (**Diverging**) mirror.



- Some terms of spherical surface:
- **Center of curvature** (**point C**): is defined as the center of the sphere of which a curved mirror forms a part.
- **Radius of curvature, R:** is defined as the radius of the sphere of which a curved mirror forms a part.
- **Vertex** (**Point V**): is defined as the point at the center of the mirror.
- **Principle axis:** is defined as the straight line through the center of curvature C and vertex V of the mirror.
- **AB** is called the **aperture** of the mirror.

Focal point (F) and Focal length (*f*):

- Consider the ray diagram for concave and convex mirror as shown in figure:
- Point **F** represents The focal point or foucs of the mirrors while distance ***f*** represnts the focal length of the mirror.
  The parallel incident rays represent the object infinitly far away from the spherical mirror e.g. the sun.

- **Focal point or foucs**, **F**

  For concave mirror – is defined as a point where the incident parallel rays converge after reflection on the mirror.

  - Its focal point is real (principle).

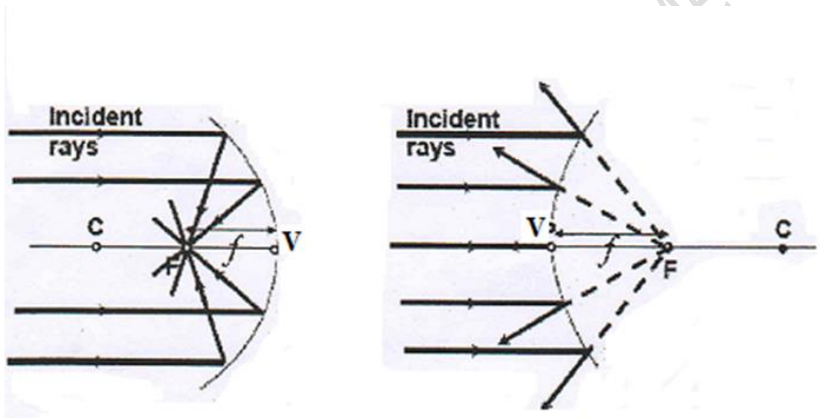
For convex mirror – is defined as a point where the incident parallel rays seem to diverge from a point behind the mirror after reflection.

  - Its focal point is virtual.

- **Focal length**, ***f***

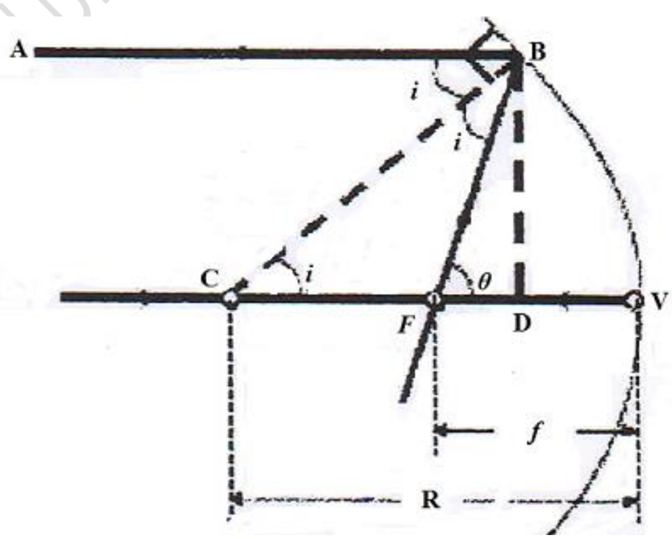
  Definition – is defined as the distance between the focal point (focus) **F** and vertex **V** of the spherical mirror.

- **The paraxial rays** are defined as the rays that parallel to the principle axis.



<u>Relationship between focal length (*f*) and radius of curvature (R):</u>

- Consider a ray AB parallel to the principal axis of concave mirror



- From the figure,

$\Delta BCD \Longrightarrow \tan i = \frac{BD}{CD} \approx i$

$\Delta BFD \Longrightarrow \tan \theta = \frac{BD}{FD} \approx \theta$

Taken the angles are << small by considering the ray AB is paraxial ray.

- By using an isosceles triangle CBF, thus the angle θ is given by:

$$\theta = 2i$$

$$\frac{BD}{FD} = 2\left(\frac{BD}{CD}\right)$$

- Because of AB is paraxial ray, thus point B is too close with vertex v then:

$$CD \approx CV = R$$

$$FD \approx FV = f$$

- Therefore

$$R = 2f$$

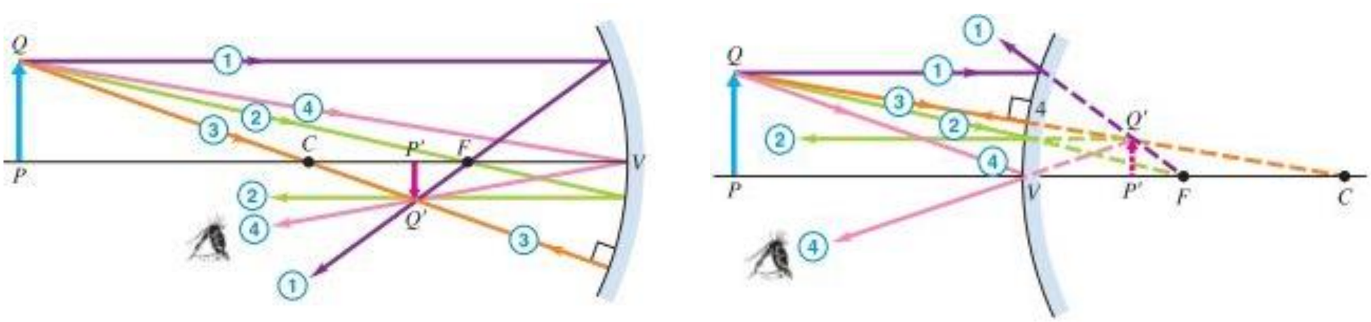
$$f = \frac{R}{2}$$

This relationship also valid for convex mirror

Ray diagrams for spherical mirrors

- Definition - is defined as the simple graphical method to indicate the position of the object and image in a system of mirrors or lenses.
- Ray diagrams below showing the graphical method of locating an image formed by concave and convex mirror.

(a) Concave mirror **(b) Convex mirror**



Ray 1 – Parallel to principle axis, after reflection, pass through the focal point (focus) F of a concave mirror or appears to come from the focal point F of a convex mirror.

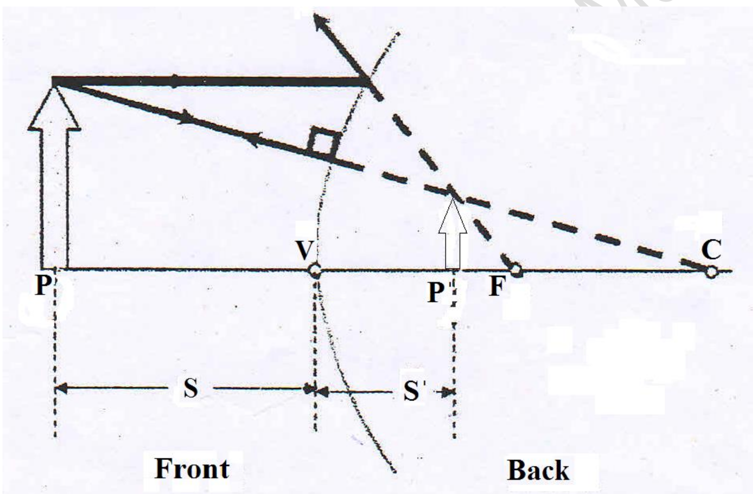
Ray 2 – Passes or directed towards focal point F reflected parallel to principle axis.

Ray 3 – Passes or directed towards center of curvature C, reflected back along the same bath.

Ray 4 – Ray to vertex V, reflects symmetrically around the principle axis.

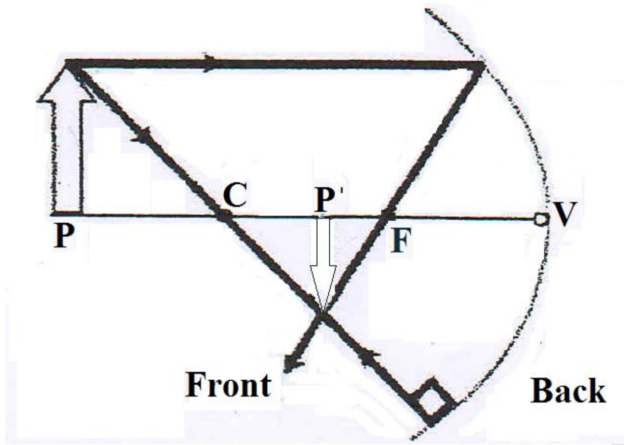
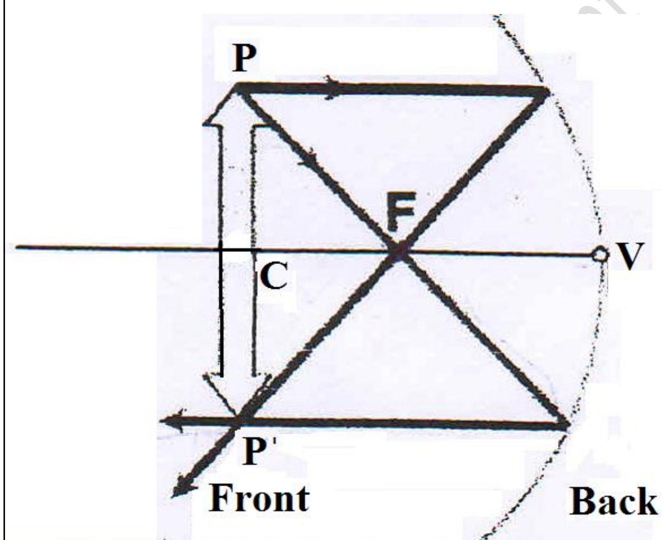
Images formed by a convex mirror

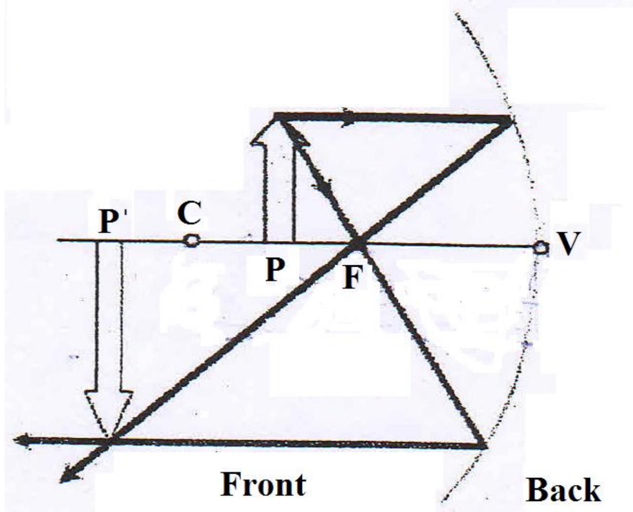
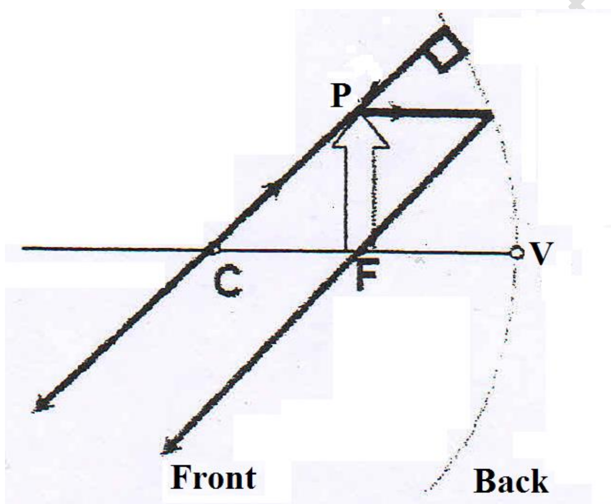
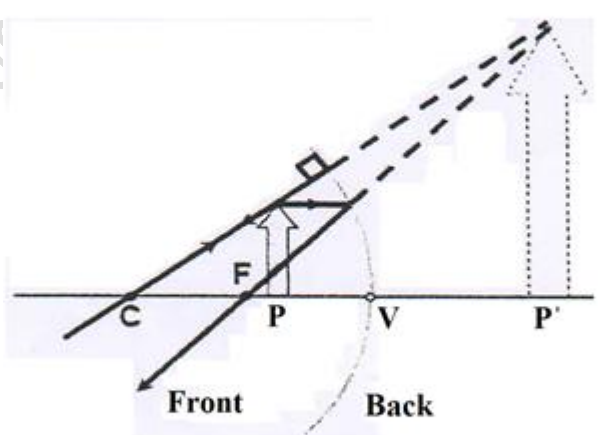
- Ray diagram below showing the graphical method of locating an image formed by a convex mirror.



- Properties of image formed are: P'
  - ✓ virtual
  - ✓ upright
  - ✓ diminished (smaller than the object)
  - ✓ formed at the back of the mirror
- Object position ⟶ any position in front of the convex mirror.

Images formed by a concave mirror

Object distance, *S*	Ray diagram	Image property
S* > *R		○ Real ○ Inverted ○ Diminished ○ Formed between point C and F.
S* = *R		○ Real ○ inverted ○ Same size ○ Formed at point C.

Object distance, *S*	Ray diagram	Image property
$f < S < R$		○ Real ○ Inverted ○ Magnified ○ Formed at a distance greater than CV.
$S = f$		○ Real ○ Formed at infinity.
$S < f$		○ Virtual ○ Upright ○ Magnified ○ Formed at the back of mirror.

Linear (lateral) magnification of the spherical mirror, M is defined as the ratio between image height, h' and object height, h

$$M = \frac{h'}{h} = -\frac{S'}{S}$$

Where: S' is image distance from vertex; S: object distance from vertex.

Derivation of spherical mirror equation

- Figure below shows an object P at a distance S and on the principal axis of a concave mirror. A ray from the object P is incident at a point B which closes to the vertex V of the mirror.
- From the figure,

  ΔBPC ⟹ $\phi = \alpha + \theta$ ------ (1)

  ΔBC P' ⟹ $\beta = \phi + \theta$ ------ (2)

  Then, eq. (1) - (2):

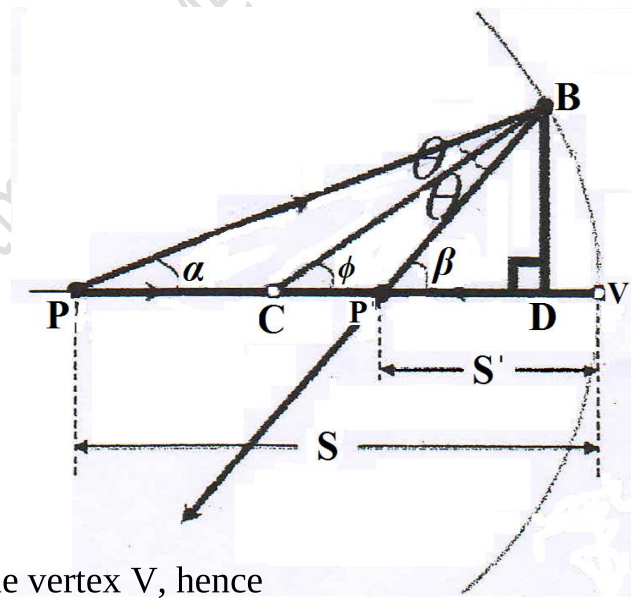
  $$\phi - \beta = \alpha - \phi$$

  $$\alpha + \beta = 2\,\phi \text{ ------- (3)}$$

  By using ΔBPD, ΔBCD and ΔBP'D

  Thus,

  $$\tan\alpha = \frac{BD}{PD}\ ;\ \tan\phi = \frac{BD}{CD}\ ;\ \tan\beta = \frac{BD}{P'D}$$



- By considering point B is very close to the vertex V, hence

  $\tan\alpha \approx \alpha$; $\tan\phi \approx \phi$; $\tan\beta \approx \beta$

  $PD \approx PV = S$; $CD \approx CV = R$; $P'D \approx P'V = S'$

  Then

  $$\alpha = \frac{BD}{S}\ ;\ \phi = \frac{BD}{R}\ ;\ \beta = \frac{BD}{S'}$$ **substituting this value in eq. (3)**

  $$\frac{BD}{S} + \frac{BD}{S'} = 2\left(\frac{BD}{R}\right)\ ;\ \frac{1}{S} + \frac{1}{S'} = \frac{2}{R}\ ;$$ Where $R = 2f$

$$\frac{1}{S} + \frac{1}{S'} = \frac{1}{f}$$ ⟹ **Equation (formula) of spherical mirror**

- Table below shows the sign convention for equation of spherical mirror

Physical quaintly	Positive sign (+)	Negative sign (-)
Object distance, *S*	**Real object** (in front of the mirror)	**Virtual object** (at the back of mirror)
Image distance, *S*'	**Real image** (same side of the object)	**Virtual image** (opposite side of the object)
Focal length, f	**Concave mirror**	**Convex mirror**
Linear magnification, M	**Upright image**	**Inverted image**

Example 1: An object is placed 10 cm in front of a concave mirror whose focal length is 15 cm. Determine: A- position of image, B. linear magnification and state the properties of image.

Solution: $S = +10$ cm, $f = +15$ cm

a. By applying the equation of spherical mirror, thus

$$\frac{1}{S} + \frac{1}{S'} = \frac{1}{f}$$

$$\frac{1}{10} + \frac{1}{S'} = \frac{1}{15}$$

$$S' = -30\ cm$$

The image is 30 cm from the mirror on the opposite side of the object (or 30 cm at the back of the concave mirror).

b. The linear magnification is given by:

$$M = -\frac{S'}{S} = -\frac{(-30)}{10}$$

$$M = 3$$

The properties of image are:

- ✓ Virtual
- ✓ Upright
- ✓ Magnified

Example 2: A 1.5 cm high diamond ring is placed 20 cm in front of a concave mirror whose radius of curvature is 30 cm. Determine: A- the position of the image, B- its size.

Solution: $h = 1.5$ cm, $S = +20$ cm, R = + 30 cm, $f = +15$ cm

a. By applying the equation of spherical mirror, thus

$$\frac{1}{S} + \frac{1}{S'} = \frac{1}{f}$$

$$\frac{1}{20} + \frac{1}{S'} = \frac{1}{15}$$

$$S' = +60\ cm$$

The image is 60 cm from the mirror on the same side of the object. (Real image)

b. The linear magnification is given by:

$$M = -\frac{S'}{S} = -\frac{60}{20} = -3$$

$$h' = M\,h = -3 \times 1.5 = -4.5\text{cm}$$

- ✓ Negative sign indicates that the image is inverted

Example 3: The image behind a convex mirror (radius of curvature = 68 cm) is located 22 cm from the mirror. (a) Where is the object located? (b) What is the magnification of the mirror? Determine whether the image is (c) upright or inverted and (d) larger or smaller than the object.

<u>Solution</u>:

R = - 68 cm, f = ½ R = -34 cm, S' = - 22 cm

(a) $\frac{1}{S} + \frac{1}{S'} = \frac{1}{f} \longrightarrow \frac{1}{S} = \frac{1}{f} - \frac{1}{S'}$

$$\frac{1}{S} = \frac{1}{-34\ cm} - \frac{1}{-22\ cm} = 0.016\ cm^{-1}$$

$S = 1/0.016\ cm^{-1} \longrightarrow S = 62$ cm

(b) $M = -\frac{S'}{S} = -\frac{(-22)}{62} = 0.35$

(c) Since $M > 0 \longrightarrow$ image is upright

(d) Since $|M| < 1 \longrightarrow$ image is smaller than the object

Example 4: A concave mirror has a radius of curvature of 24 cm. How far an object from the mirror if an image is formed that is (a) virtual and 3 times the size of the object, (b) real and 3 times the size of the object?

Solution: R = 24 cm; so that for a concave mirror: $f = ½ R$ = 12 cm

(a) Virtual images are upright so $M > 0$

$$M = -\frac{S'}{S} = 3 \longrightarrow S' = -3\,S$$

$$\frac{1}{S} + \frac{1}{S'} = \frac{1}{f} \longrightarrow \frac{1}{S} - \frac{1}{3\,S} = \frac{1}{f} \longrightarrow \frac{2}{3\,S} = \frac{1}{12\,cm} \longrightarrow S = 8\,cm$$

(b) Real images are inverted so $M < 0$

$$M = -\frac{S'}{S} = -3 \longrightarrow S' = 3\,S$$

$$\frac{1}{S} + \frac{1}{S'} = \frac{1}{f} \longrightarrow \frac{1}{S} + \frac{1}{3\,S} = \frac{1}{f} \longrightarrow \frac{4}{3\,S} = \frac{1}{12\,cm} \longrightarrow S = 16\,cm$$

Home works about lecture 5:

Q1: The value of focal length (f) of concave mirror is

A- Negative sine, B- zero, C- positive sine D- none of them

Q2: The value of object distance (S) of real object is

A- Negative sine, B- zero, C- positive sine D- none of them

Q3: The equation of spherical mirror can be given as:

A- $\frac{1}{S} + \frac{1}{S'} = \frac{1}{f}$ B- $\frac{1}{S} + \frac{1}{S'} = \frac{1}{R}$ C- $\frac{1}{S} - \frac{1}{S'} = \frac{1}{f}$ D- $\frac{1}{S} + \frac{1}{S'} = \frac{1}{2f}$

Q4- The formula of linear (lateral) magnification of the spherical mirror is

A- $M = \frac{h'}{h} = -\frac{S}{S'}$ B- $M = \frac{h'}{h} = \frac{S'}{S}$ C- $M = \frac{h'}{h} = -\frac{S'}{S}$ D- $M = \frac{h}{h'} = -\frac{S'}{S}$

Q5- An object its height (4 cm) was placed at distance (20 cm) from convex mirror has focal length (12 cm). Find position of the image

A- (7.5 cm) B- (- 5.5 cm) C- (- 7.5 cm) D- (5.5 cm)

Q6- Image which formed by a concave mirror was diminished when the location of object is

A- $S = f$ B- $S < f$ C- $S = R$ D- $S > R$