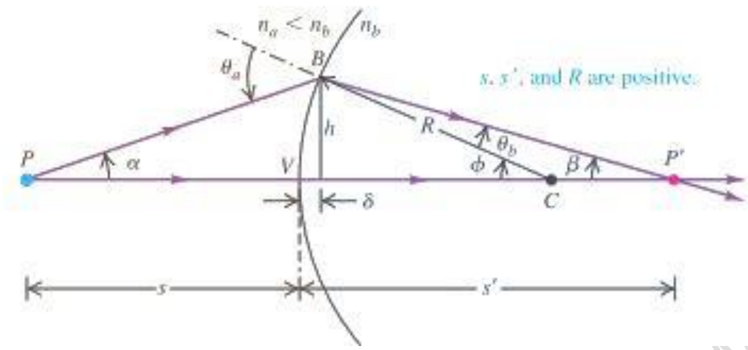
Lecture 4: Refraction at a Spherical Surface

- Figure below shows a spherical surface with radius ***R*** forms an interface between two media with refractive indices n_a and n_b.



- The surface form an image ***P***ˈ of an object point ***p*** as shown in figure above.
- The incident ray ***PB*** making an angle θ_a with the normal and is refracted to ray ***BP***ˈ making an angle θ_b where $n_a < n_b$.
- Point ***C*** is the center of curvature of the spherical surface.
- From the figure

Δ **PBC** $\Longrightarrow$ $\theta_a = \alpha + \phi$ ------ (1)

Δ **PˈBC** $\Longrightarrow$ $\phi = \beta + \theta_b$

$\theta_b = \phi - \beta$ ------ (2)

<u>**Hint:**</u> an exterior angle of triangle equals to the sum of the two opposite interior angles.

- From the Snell's law

$$n_a \sin \theta_a = n_b \sin \theta_b$$

Also, the tangents of α, β and ϕ are:

$$\tan \alpha = \frac{h}{s+\delta}; \quad \tan \beta = \frac{h}{s'-\delta}; \quad \tan \phi = \frac{h}{R-\delta}$$

By considering point B is very close to the vertex V, hence

$\sin \theta_a \approx \theta_a$: $\sin \theta_b \approx \theta_b$: $\tan \alpha \approx \alpha$: $\tan \beta \approx \beta$: $\tan \phi \approx \phi$

$s + \delta = s$: $s' - \delta = s'$: $R - \delta = R$

Then Snell's law can be written as

$$n_a \theta_a = n_b \theta_b \text{ ------- (3)}$$

- By substituting eq.(1) and (2) into eq.(3) , thus

$$n_a (\alpha + \phi) = n_b (\phi - \beta)$$

$$\boldsymbol{n_a}\,\alpha + \boldsymbol{n_b}\,\beta = (\boldsymbol{n_b} - \boldsymbol{n_a})\,\phi$$

$$\boldsymbol{n_a}\left(\frac{h}{s}\right) + \boldsymbol{n_b}\left(\frac{h}{s'}\right) = (\boldsymbol{n_b} - \boldsymbol{n_a})\left(\frac{h}{R}\right)$$

$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{(n_b - n_a)}{R}$$

⇒ **Equation of spherical refracting surface**

Where:

s: object distance from vertex.

s': image distance from vertex.

n_a: refractive index of medium 1 (medium containing the incident ray).

n_b: refractive index of medium 2 (medium containing the refracted ray).

R: Radius of curvature.

- Note: If the refraction surface is plane (flat) ⟶ R = ∞, then $\frac{n_a}{s} + \frac{n_b}{s'} = 0$

- The equation of linear magnification of refraction by the spherical surface is given by

$$M = \frac{h'}{h} = -\frac{n_a\, s'}{n_b\, s}$$

Where:

h: height of object.

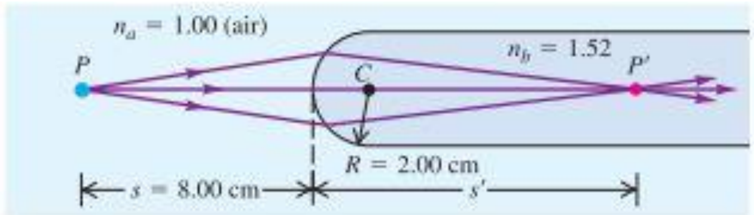
h': height of image.

- Sign of convention for refraction

Table below shows a sign of convention for spherical refraction

Physical Quantity	**Positive Sign (+)**	**Negative Sign (-)**
Object distance, *s*	**Real object** (in front of the refracting surface)	**Virtual object** (at the back of the refracting surface)
Image distance, *s*'	**Real image** (opposite side of the object)	**Virtual image** (same side of the object)
Focal length, *f*	**Convex surface**	**Concave surface**
Radius of curvature, *R*	**Convex surface**	**Concave surface**
Linear magnification, *M*	**Upright image**	**Inverted image**

Example 1: A cylindrical glass rod in air as shown in figure below has refractive index 1.52. One end is ground to a hemispherical surface with radius, R = 2 cm. A small object is placed on the axis of the rod, 8.00 cm to the left of the vertex. Find? (a) The image distance (b) The linear magnification. (Given the refractive index of air, $n_a = 1$)



Solution: $n_a = 1$, $n_b = 1.52$, $s = 8$ cm, $R = +2$ cm

a. By applying the equation of spherical refracting surface

$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{(n_b - n_a)}{R}$$

$$\frac{1}{8\,cm} + \frac{1.52}{s'} = \frac{1.52 - 1}{+2\,cm} \qquad s' = +11.26 \text{ cm}$$

The image distance s' is positive; the image is formed 11.3 cm at the back of the convex surface and it is real image.

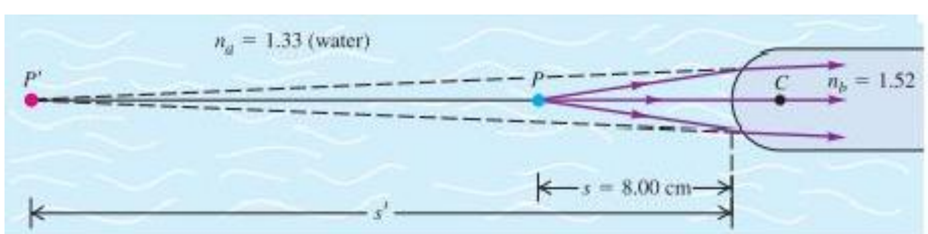
b. By using the equation of linear magnification for refracting surface

$$M = \frac{h}{h'} = -\frac{n_a\, s'}{n_b\, s}$$

$$M = -\frac{1 \times 11.26}{1.52 \times 8} = -0.93$$

Negative value of M indicates that the image is somewhat smaller than the object and it is inverted.

Example 2: The glass rod of Example 1 is immersed in water, which has index of refraction n = 1.33 as shown in figure below. The object distance is 8.00 cm. Find: (a) The image distance. (b) The linear magnification.



Solution: $n_a = 1.33$, $n_b = 1.52$, $s = 8$ cm, $R = +2$ cm

a. By applying the equation of spherical refracting surface

$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{(n_b - n_a)}{R}$$

$$\frac{1.33}{8\ cm} + \frac{1.52}{s'} = \frac{1.52 - 1.33}{+2\ cm} \qquad s' = -21.3\ cm$$

The image distance s' is negative; the image is formed 21.3 cm to the left of the vertex and it is virtual image.

b. By using the equation of linear magnification for refracting surface

$$M = \frac{h'}{h} = -\frac{n_a\, s'}{n_b\, s}$$

$$M = -\frac{(1.33)\,(-21.3\ cm)}{(1.52)\,(8\ cm)} = +2.33$$

The image is upright (because M is positive) and 2.23 times as large as the object.

Home works about lecture 4:

Q1: The end of a solid glass rod of index 1.5 is ground and polished to a hemispherical surface of radius 1 cm. A small object is placed in air on the axis 4 cm to the left of the vertex. Find the position of the image.

A- (2 cm), B- (4 cm), C- (6 cm), D- (8 cm)

Q2: The equation of spherical refracting surface can be expressed as:

A- $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{(n_b - n_a)}{R}$, B- $\frac{n_a}{s'} + \frac{n_b}{s} = \frac{(n_b - n_a)}{R}$, C- $\frac{nb}{s'} + \frac{na}{s} = \frac{(n_b - n_a)}{R}$, D- $\frac{n_a}{s'} + \frac{n_b}{s} = \frac{(n_a - n_b)}{R}$

Q3: Focal length (f) of convex spherical refracting surface is:

A- Negative sine B- zero C- positive sine D- none of them

Q4- The formula of linear magnification for refracting surface can be expressed as:

A- $M = -\frac{n_a\, s'}{n_b\, s}$, B- $M = \frac{n_a\, s'}{n_b\, s}$ C- $M = -\frac{n_b\, s'}{n_a\, s}$ D- $M = \frac{n_a\, s}{n_b\, s'}$

Q5- Radius of curvature (R) of concave spherical refracting surface is

A- Positive sine B- negative sine C- zero D- none of them