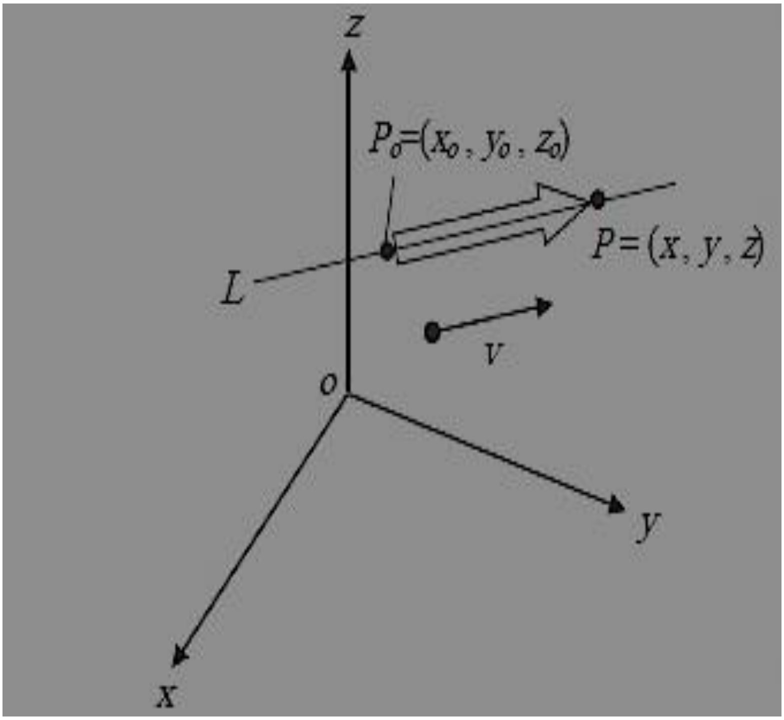
Mathematics II chapter three

3.1 Vectors analysis including parametric equations for lines in space

Equation lines in space.

Suppose that L is a line in space through a point P_o (x_o , y_o , z_o) Parallel to a vector v = ai + bj + ck ,then L is the set of all points p (x , y , z)for which P_oP is parallel to v. thus P_oP = tv for some scalar parameter t. the value of t depend on the location of the point P along the line. the expanded from of the equation:



$$\overrightarrow{P_oP} = tv$$

$$(x - x_o)i + (y - y_o)j + (z - z_o)k = t(ai + bj + ck)$$

$$x - x_o = ta$$

$$y - y_o = tb$$

$$z - z_o = tc$$

From equation above:

The parametric equation for the line through $P_o(x_o, y_o, z_o)$ parallel to $v = ai + bj + ck$ **:**

$$\boxed{\begin{aligned} x &= x_o + ta \\ y &= y_o + tb \\ z &= z_o + tc \end{aligned}}$$

Example: find parametric equation for the line through the points $P(-3,2,-3)$ and $Q(1,-1,4)$

Solution: the vector

$$\overrightarrow{PQ} = (1-(-3))i + (-1-2)j + (4-(-3))k$$

$$\overrightarrow{PQ} = 4i - 3j + 7k$$

$$\therefore \quad a = 4 \quad , \quad b = -3 \quad , c = 7$$

$$\therefore \quad x_o = -3 \quad , \quad y_o = 2 \quad , \quad z_o = -3$$

$\therefore$

$$x = x_o + at \Longrightarrow x = -3 + 4t$$
$$y = y_o + bt \Longrightarrow y = 2 - 3t$$
$$z = z_o + ct \Longrightarrow z = -3 + 7t$$

Example: Find parametric equations for the line through the point $(-2,0,4)$ parallel to the vector $v = 2i + 4j - 2k$

Solution:

With $P_o(x_o, y_o, z_o) = (-2,0,4)$

$$x_o = -2 \quad , \quad y_o = 0 \quad , \quad z_o = 4$$

and $v = ai + bj + ck = 2i + 4j - 2k$

$$a = 2 \quad , \quad b = 4 \quad , \quad c = -2$$

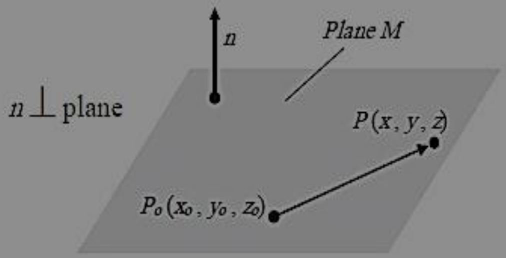
$\therefore$

$$x = x_o + at \Longrightarrow x = -2 + 2t$$
$$y = y_o + bt \Longrightarrow y = 4t$$
$$z = z_o + ct \Longrightarrow z = 4 - 2t$$

3.2 Vectors analysis including parametric equations for planes in space

Equation for plane in space:

Suppose that plane M passes through a point $P_o(x_o, y_o, z_o)$ and is normal to the nonzero vector $n = Ai + Bj + Ck$, then M is the set of all points $P(x,y,z)$ for which $\overrightarrow{P_oP}$ is orthogonal to n. Thus the dot product



$n.\overrightarrow{P_oP} = 0$

This equation is equivalent to:

$$(Ai + Bj + Ck).[(x - x_o)i + (y - y_o)j + (z - z_o)k] = 0$$

$\boxed{A(x - x_o) + B(y - y_o) + C(z - z_o) = 0}$ $\Longrightarrow$ *Component equation*

This becomes:

$$Ax + By + Cz = Ax_o + By_o + Cz_o$$

$\boxed{Ax + By + Cz = D}$ $\Longrightarrow$ *Component equation simplified*

Where $D = Ax_o + By_o + Cz_o$

Example: Find an equation for the plane through *Po* (4,2,1) normal to $\overline{\boldsymbol{N = 5\imath + 2\jmath - 3k}}$.

Solution//

$D=AX_0+BY_0+CZ_O=5*4+2*2-3*1=21$

Ax+By+Cz=D

5x+2y-3z=21

Example: Find an equation for the plane through *A*(1,1,1) ,*B*(3,2,4) and *C*(3,0,3).

Solution//

=(3-1)i+(2-1)j+(4-1)*k*=*2i*+*j*+*3k*$\overline{\boldsymbol{AB}}$

=(3-1)i+(0-1)j+(3-1)*k*=*2i*-*j*+*2k*$\overline{\boldsymbol{AC}}$

$$\overline{\boldsymbol{AB}}\ \boldsymbol{X}\ \overline{\boldsymbol{AC}} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ \mathbf{2} & \mathbf{1} & \mathbf{3} \\ \mathbf{2} & \mathbf{-1} & \mathbf{2} \end{vmatrix}$$

$$= [\mathbf{2-(-3)}\boldsymbol{i} - (\mathbf{4-6})\boldsymbol{j} + (\mathbf{-2-2})\boldsymbol{k}$$

$$= \mathbf{5}\boldsymbol{i} + \mathbf{2}\boldsymbol{j} - \mathbf{4}\boldsymbol{k}]$$

$\mathbf{D=Ax_o+By_o+Cz_o=5*1+2*1-4*1=3}$

5x+2y-4z=3