Laser Principles

Lecture (4,5 and 6)

Dr.Mohammed H. Al-Mamoori
Assist Lecture.Assel A.Hussein

Lecture 4: The Boltzmann Distribution

The Boltzmann equation determines the relation between the population number of a specific energy level and the temperature:
N_i = const * exp ($-E_i/kT$)
N_i = Population Number = number of atoms per unit volume at certain energy level E_i.

k = Boltzmann constant: $k = 1.38*10^{23}$ [Joule/^{0}K].

E_i = Energy of level i.

Const = proportionality constant. It is not important when we consider population of one level compared to the population of another level as we shall see shortly. T=Temperature in degrees Kelvin [^{0}K] (Absolute Temperature).

The Boltzmann equation shows the dependence of the population number (N_i) on the energy level (E_i) at a temperature T. From this equation we see that:

1. The higher the temperature, the higher the population number.

2. The higher the energy level, the lower the population number.

Relative Population (N_2 / N_1).

The relative population (N_2/N_1) of two energy levels E_2 compared to E_1 is:
N_2/N_1 = const* exp ($-E_2/kT$)/ const* exp ($-E_1/kT$) = exp ($-(E_2-E_1)/kT$).
The proportionality constant (const) is canceled by division of the two population numbers.

Conclusions:
1. The relation between two population numbers (N_2/N_1) does not depend on the values of the energy levels E_1 and E_2, but only on the difference between them: $E_2 - E_1$.

2. For a certain energy difference, the higher the temperature, the bigger the relative population.

3. The relative population can be between 0 and 1.

Population at Thermodynamic Equilibrium

Figure 2.2 shows the population of each energy level at thermodynamic equilibrium.

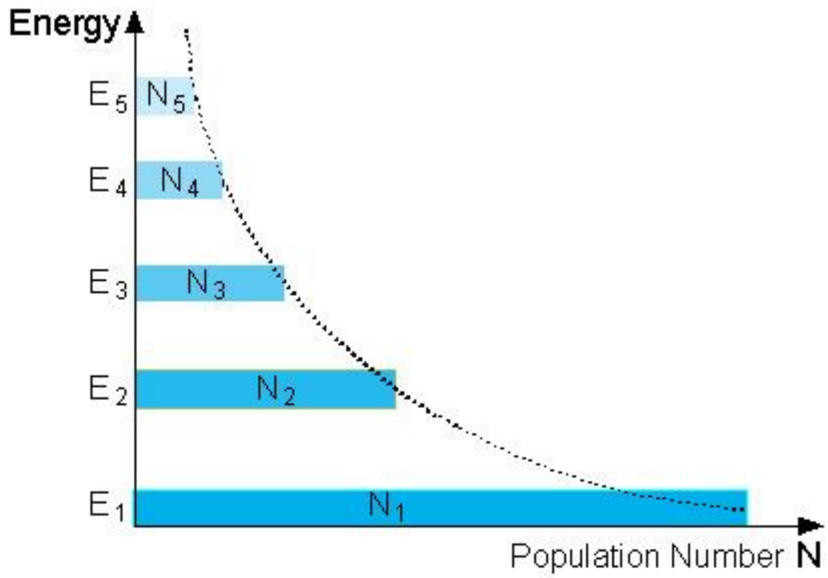


Figure 2.2: Population Numbers at "Normal Population"

Notes :

1- Figure 2.2 (and most of the figures later in this course) plots the energy values of the energy levels on y axis, and the corresponding population numbers on the x axis. If we interchange the axes, we get a histogram in which the height of each column shows the population number of each level. The width of the energy levels in this figure is arbitrary, and does not correspond to the real width of each level.

2. In a thermodynamic equilibrium, the population number of higher energy level is always less than the population number of a lower energy level.

3-Physically, the electrons inside the atom prefer to be at the lowest energy level possible. Even when they are excited to a higher level, they return back to the lowest energy level after a short time.

Example 2.3:

Calculate the ratio of the population inversion (N_2 / N_1) for the two energy levels E_2 and E_1 when the material is at room temperature (300^0K), and the difference between the energy levels is 0.5 [eV]. What is the wavelength (λ) of a photon which will be emitted in the transition from E_2 to E_1?

Solution to example 2.3:

When substituting the numbers in the equation, we get:

$$\frac{N_2}{N_1} = \exp\left(-\frac{E_2 - E_1}{k_B \cdot T}\right) = \exp\left[-\frac{(0.5 \cdot eV)\cdot\left(1.6\cdot 10^{-19}\cdot\frac{J}{eV}\right)}{\left(1.38\cdot 10^{-23}\cdot\frac{J}{K}\right)\cdot(300K)}\right]$$

$= 4 * 10^{-9}$

This means that at room temperature, for every 1,000,000,000 atoms at the ground level (E_1), there are 4 atoms in the excited state (E_2)!!!

To calculate the wavelength:

$$\lambda = \frac{h\cdot c}{\Delta E} = \frac{(6.626\cdot 10^{-34}\cdot J\cdot sec)\cdot\left(3.10^{8}\cdot\frac{m}{sec}\right)}{(0.5\cdot eV)\cdot\left(1.6\cdot 10^{-19}\cdot\frac{J}{eV}\right)} = 2.48\cdot\mu m$$

This wavelength is in the Near Infra-Red (NIR) spectrum.

Question 2.4:

A material is in thermodynamic equilibrium at room temperature (300^0K).The wavelength of the photon emitted in the transition between two levels is 500 nm (visible radiation). Calculate the ratio of the population numbers for these energy levels.

2.6 Population Inversion

We saw that in a thermodynamic equilibrium Boltzmann equation shows us that:

$N_1 > N_2 > N_3$

Thus, the population numbers of higher energy levels are smaller than the population numbers of lower ones. This situation is called "Normal Population" (as described in Figure 2.3a). In a situation of normal population a photon impinging on the material will be absorbed, and raise an atom to a higher level. By putting energy into a system of atoms, we can achieve a situation of "Population Inversion". In population inversion, at least one of the higher energy levels has more atoms than a lower energy level.

An example is described in Figure 2.3b. In this situation there are more atoms (N_3) in a higher energy level (E_3), than the number of atoms (N_2) in a lower energy level (E_2).

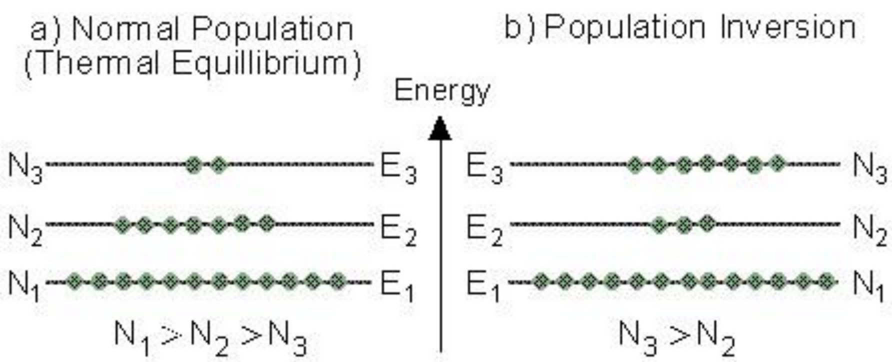


Figure 2.3: "Normal Population" compared to "Population Inversion".

As we shall see later, this is one of the necessary conditions for lasing. The process of raising the number of excited atoms is called "Pumping". If this process is done by optical excitation (electromagnetic beam), it is called "Optical Pumping".

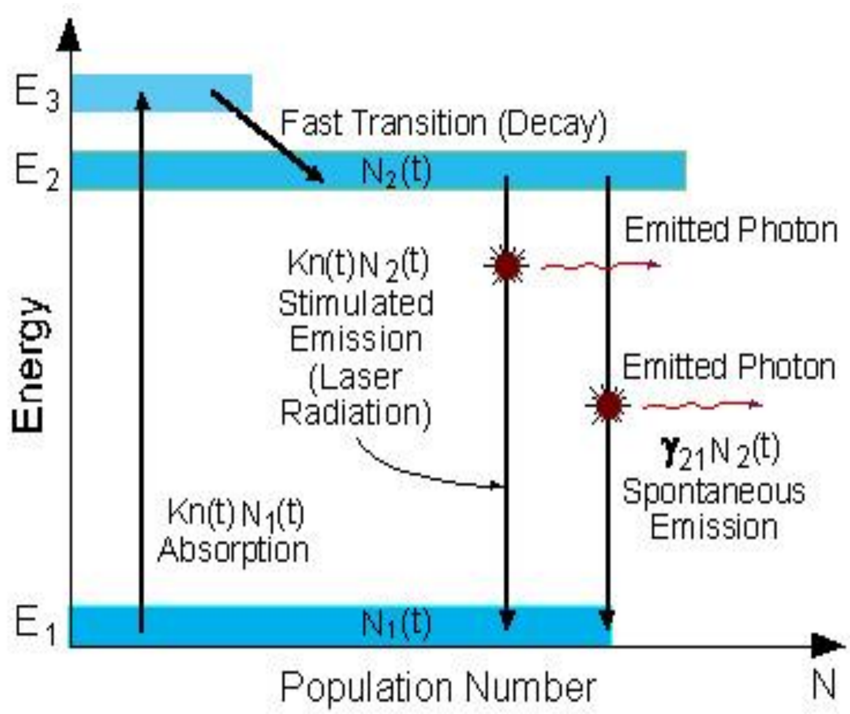
Three Level Laser

A schematic energy level diagram of a laser with three energy levels is shown in figure below . The two energy levels between which lasing occur are: the lower laser energy level (E1), and the upper laser energy level (E2).

To achieve lasing, energy must be pumped into the system to create population inversion. So that more atoms will be in energy level E_2 than in the ground level (E_1). Atoms are pumped from the ground state (E_1) to energy level E_3. They stay there for an average time of 10^{-8} [sec], and decay (usually with a non-radiative transition) to the meta-stable energy level E_2.

Since the lifetime of the meta-stable energy level (E_2) is relatively long (of the order of 10^{-3} [sec], many atoms remain in this level. If the pumping is strong enough, then after pumping more than 50% of the atoms will be in energy level E_2, a population inversion exists, and lasing can occur.

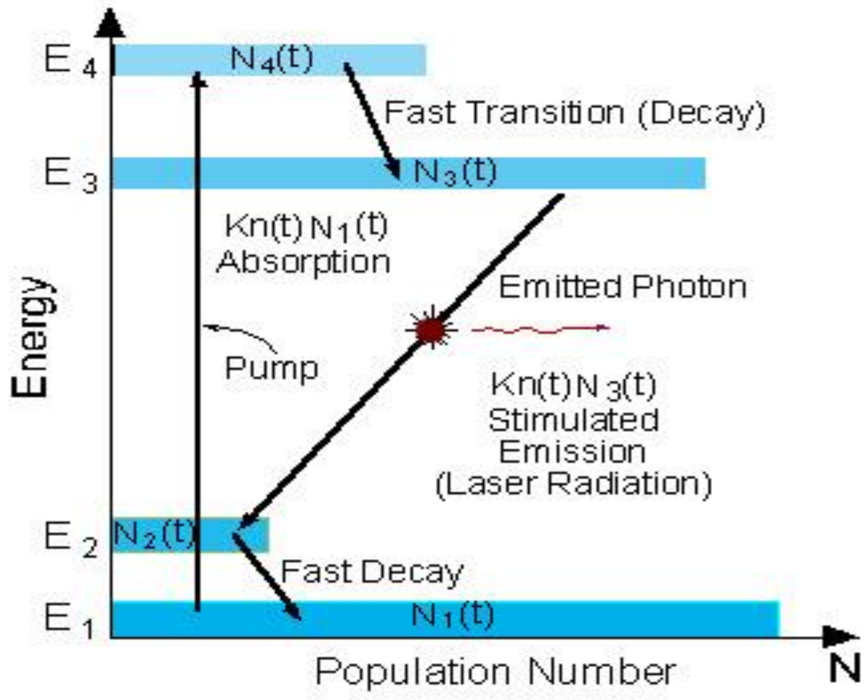
A popular example of a three-level laser medium is ruby, as used by Maiman for the first laser



Three laser levels

Four-Level Systems:

The schematic energy level diagram of a four level laser is shown in figure below . Compared to the equivalent diagram of a three level laser, there is an extra energy level above the ground state. This extra energy level has a very short lifetime.



The pumping operation of a four level laser is similar to the pumping of a three level laser. This is done by a rapid population of the upper laser level (E_3), through the higher energy level (E_4). The advantage of the four level laser is the low population of the lower laser energy level (E_2). To create population inversion, there is no need to pump more than 50% of the atoms to the upper laser level.

The population of the lower laser level (N_2 (t)) is decaying rapidly to the ground state, so practically it is empty. Thus, a continuous operation of the four level laser is possible even if 99% of the atoms remain in the ground state.

Advantages of four level lasers Compared to three level lasers:

- The lasing threshold of a four level laser is lower.
- The efficiency is higher.
- Required pumping rate is lower.
- Continuous operation is possible.

The most popular four-level solid-state gain medium is Nd:YAG. Neodymium ions can also be directly pumped into the upper laser level, e.g. with pump light around 880 nm for Nd:YAG. The gain usually rises linearly with the absorbed pump power.