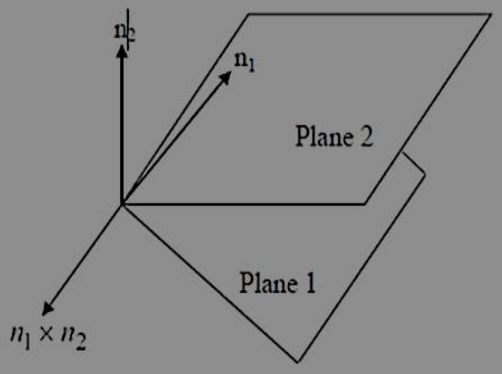
3.3 Vectors analysis/angle between two planes

Angle between planes

The angle between two intersecting planes is defined to be the acute angle between their normal vectors.

$$\theta = \cos^{-1}\left(\frac{n_1.n_2}{|n_1||n_2|}\right)$$



Example: Find the angle between the planes 2*x* - 6*y* - *z* = 5 and *x* + *2y* - 2*z* = 12

Solution//

n_1=2i-6j-k

n2=i+2j-2k

$$|\boldsymbol{n1}| = \sqrt{\mathbf{4+36+1}} = \sqrt{\mathbf{41}}$$

$$|n2| = \sqrt{1+4+4} = \sqrt{9}$$

$$\theta = \cos^{-1}\frac{n1.n2}{|n1|.|n2|} = \cos^{-1}\frac{-8}{\sqrt{41}.\sqrt{9}} = 114.6°$$

3.4 Vectors analysis/intersection line &plane

Example: Find the vector parallel to the line of intersection of the planes 3x-6y-2z=15 , x+2y-z=5.

Solution/

N1=3i-6j-2k

N2=i+2j-k

N=N1 X N2= $\begin{vmatrix} i & j & k \\ 3 & -6 & -2 \\ 1 & 2 & -1 \end{vmatrix}$

=10 i+j+12k

3.5 Vector Functions

A vector –valued function of real variable can be written in component form as:

F(t)=F1(t)i+F2(t)j+F3(t)k

1. Limits

If L=L1i+L2j+L3k is a vector in space

F(t) is a vector function

F(t)=f(t)i+g(t)j+h(t)k

$$\lim_{t\to a} f(t) = \lim_{t\to a} f1(t) + \lim_{t\to a} f2(t) + \lim_{t\to a} f3(t)$$

Example: Find $\lim_{t\to\pi} f(t)$If f(t)=$\cos t\, i + 3\sin t\, j + t^3 k$

Solution//

$$\lim_{t\to\pi} f(t) = \lim_{t\to\pi} (\cos t\, i + 3\sin t\, j + t^3 k$$

$$= \lim_{t\to\pi} \cos t\, i + \lim_{t\to\pi} 3\sin t\, j + \lim_{t\to\pi} t^3 k = -1i + 0j + \pi^3 k$$

2.Derivative

r(t)=f(t)i+g(t)j+h(t)k

$$\Delta r = r(t+\Delta t) - r(t) \dots\dots\dots 1$$

Then if r(t) sub in equation 1

$$\Delta r = \{f(t+\Delta t) - f(t)\}i + \{g(t+\Delta t) - g(t)\}j + \{h(t+\Delta t) - h(t)\}k$$

As $\Delta t = 0$

$$\frac{dr}{dt} = \lim_{\Delta t\to 0} \frac{\Delta r}{\Delta t}$$

$$= \lim_{\Delta t\to 0} \frac{\{g(t+\Delta t) - g(t)\}j}{\Delta t} + \lim_{\Delta t\to 0} \frac{\{h(t+\Delta t) - h(t)\}k}{\Delta t} + \lim_{\Delta t\to 0} \frac{\{f(t+\Delta t) - f(t)\}i}{\Delta t}$$

$$\frac{dr}{dt} = \frac{df}{dt} i + \frac{dg}{dt} j + \frac{dr}{dt} k$$

Notes//

1. Velocity= $\frac{dr}{dt} = \overline{V}$

2. Acceleration a= $\frac{d^2r}{dt^2} = \frac{dv}{dt}$

3. Speed or magnitude of velocity = $|V|$

Or velocity $\overline{V} = speed|V| * direction$

Example: Find speed and direction of r(t) when t=2 If r(t)= t^2**i+**$2t^2 j$ **+5k**

Solution//

$$\frac{dr}{dt} = \overline{V} = 3t^2 i + 4t\, j + 0k$$

speed = $|V| = \sqrt{(3t^2)^2 + (4t)^2}$

At t=2→ $|V| = 14.4$

Direction (at t=2) = $\frac{\overline{v}}{|V|}$

$$= \frac{12\, i + 8\, j + 0k}{14.4}$$

Differential rules

1. $\frac{dc}{dt} = 0 \; if \; c = constant$

Example :c=2i+4j+5k, $\frac{dc}{dt} = 0i + 0j + 0k = 0$

2. if u(t) is a vector function, then $\frac{dcu}{dt} = c.\frac{du}{dt}$

$$\boldsymbol{where\ c\ is\ constant\ a\ vector}$$

3.$\frac{d(u\pm v)}{dt} = \frac{du}{dt} \pm \frac{dv}{dt}$ $(\boldsymbol{u\&v\ are\ vector\ function})$

4.$\frac{d(u.v)}{dt} = u.\frac{dv}{dt} + v.\frac{du}{dt}$ $(\boldsymbol{u\&v\ are\ vector\ function})$

5.$\frac{d(uxv)}{dt} = u\ x\frac{dv}{dt} + v\ x\frac{du}{dt}$ $(\boldsymbol{u\&v\ are\ vector\ function})$

Chain rule

If r(t)=f(t)i+g(t)j+h(t)k is a function of S then

$$\frac{dr}{ds} = \frac{dr}{dt}.\frac{dt}{ds}$$

<u>Note:</u> u(t) is a function vector has constant length then

$$\bar{u}.\overline{\frac{du}{dt}} = 0 \ \ or\ \bar{u}\perp\overline{\frac{du}{dt}}$$

Example: show that u(t)=sin $t\ i + \cos t\ j + 5k$ $\ \ \boldsymbol{has\ constant\ length\ and\ is\ orthogonal\ to\ its\ derivative}$

Solution//

$$\bar{u}.\overline{\frac{du}{dt}} = 0$$

$$u(t)=\sin t\ i + \cos t\ j + 5k$$

$$\frac{du}{dt} = \cos t\ i\text{-}\sin t\ j + 0k$$

$$\bar{u}.\overline{\frac{du}{dt}} = \sin t \cos t - \sin t \cos t + 0 = 0$$