



Lecture 3

Signal Spectrum using Fourier Transform

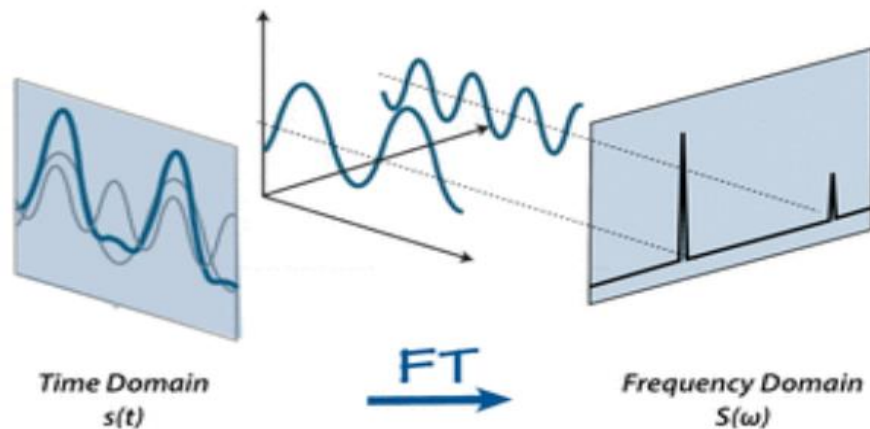
3.1 Fourier Transform

The Fourier transform is used to convert a continuous and non-periodic time-domain signal into the frequency domain. The resulting frequency domain representation from performing the Fourier transform is continuous and non-periodic. The Fourier transform of $x(t)$

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t} dt$$

The inverse Fourier transform of $X(\omega)$:

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)e^{j\omega t} d\omega$$





Example: Find the Fourier Transform of the rectangular pulse signal $x(t)$ defined by

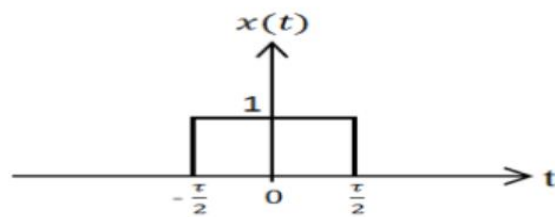
$$x(t) = \begin{cases} 1 & |t| < \frac{\tau}{2} \\ 0 & |t| > \frac{\tau}{2} \end{cases}$$

Solution:

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$X(\omega) = \int_{-\tau/2}^{\tau/2} 1 e^{-j\omega t} dt$$

$$X(\omega) = \frac{1}{-j\omega} \left[e^{-\frac{j\omega\tau}{2}} - e^{\frac{j\omega\tau}{2}} \right]$$

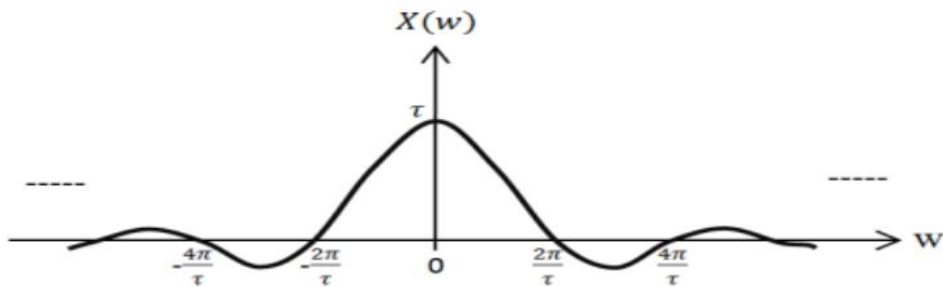


$$X(\omega) = \frac{1}{j\omega} \left[e^{\frac{j\omega\tau}{2}} - e^{-\frac{j\omega\tau}{2}} \right]$$

$$X(\omega) = \frac{2}{\omega} \left[\frac{e^{\frac{j\omega\tau}{2}} - e^{-\frac{j\omega\tau}{2}}}{2j} \right]$$

$$X(\omega) = \frac{2}{\omega} \sin\left(\frac{\omega\tau}{2}\right)$$

$$X(\omega) = \tau \operatorname{sinc}\left(\frac{\omega\tau}{2}\right)$$



Example: Suppose that a signal gets turned on at $t=0$ and then decays exponentially, So that

$$f(t) = \begin{cases} e^{-at} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

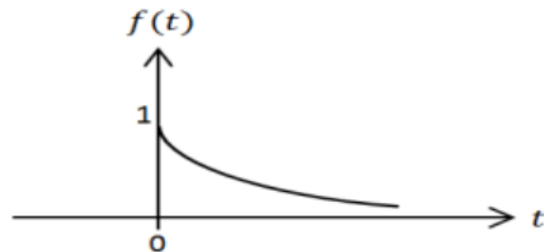
For $a > 0$. Find the fourier transform of this signal.

Solution:

$$F(w) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$$

$$F(w) = \int_0^{\infty} e^{-at} e^{-j\omega t} dt$$

$$F(w) = \int_0^{\infty} e^{-t(a+j\omega)} dt$$



$$F(w) = \frac{1}{-(a + j\omega)} [e^{-\infty} - e^0]$$

$$F(w) = \frac{1}{(a + j\omega)}$$



3.2 Properties of the Continuous-Time Fourier Transform

There are a number of properties of the Fourier transform that may be used to simplify the evaluation of the Fourier transform and its inverse. Some of these properties are described below:

Property	Signal	Fourier Transform
Linearity	$ax(t) + by(t)$	$aX(w) + bY(w)$
Time-Shift	$x(t - t_0)$	$e^{-jw t_0} X(w)$
Time-Reversal	$x(-t)$	$X(-w)$
Frequency Shifting	$e^{jw_0 t} x(t)$	$X(w - w_0)$
Time Scaling	$x(at)$	$\frac{1}{ a } X\left(\frac{w}{a}\right)$
Convolution	$x(t) * y(t)$	$X(w) Y(w)$
Time differentiation	$\frac{dx(t)}{dt}$	$jwX(w)$
Parseval's relation	$\int_{-\infty}^{\infty} x(t) ^2 dt$	$\frac{1}{2\pi} \int_{-\infty}^{\infty} X(w) ^2 dw$

3.3 Frequency response

In Lecture1 we showed that the output $y(t)$ of a continuous-time LTI system equals the convolution of the input $x(t)$ with the impulse response $h(t)$; that is



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$$y(t) = x(t) \otimes h(t)$$

Applying the convolution property, we obtain

$$Y(\omega) = X(\omega) \cdot H(\omega)$$

Where $Y(\omega)$, $X(\omega)$ and $H(\omega)$ are the Fourier transforms of $y(t)$, $x(t)$ and $h(t)$, respectively. we have

$$H(\omega) = \frac{Y(\omega)}{X(\omega)}$$

Example: Consider the linear time-invariant system characterized by the second-order linear constant coefficient difference equation

$$y'(t) + 2y(t) = x(t) + x'(t)$$

By using Fourier Transform, find the Impulse response, $h(t)$.

Solution:

$$j\omega Y(\omega) + 2Y(\omega) = X(\omega) + j\omega X(\omega)$$

$$Y(\omega)[j\omega + 2] = X(\omega)[1 + j\omega]$$

$$\frac{Y(\omega)}{X(\omega)} = \frac{j\omega + 1}{j\omega + 2}$$

$$H(\omega) = \frac{j\omega + 2 - 1}{j\omega + 2}$$

$$H(\omega) = 1 - \frac{1}{j\omega + 2}$$

$$h(t) = \delta(t) - e^{-2t}u(t)$$



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Assignment:

1. Find the Fourier transform of the unit impulse signal $x(t)$ defined by:

$$x(t) = \delta(t)$$

2. Consider a continuous- time LTI system described by

$$\frac{dy(t)}{dt} + 2y(t) = x(t)$$

Using the Fourier transform to find the output $y(t)$ if the input signal is:

$$x(t) = e^{-t}u(t)$$