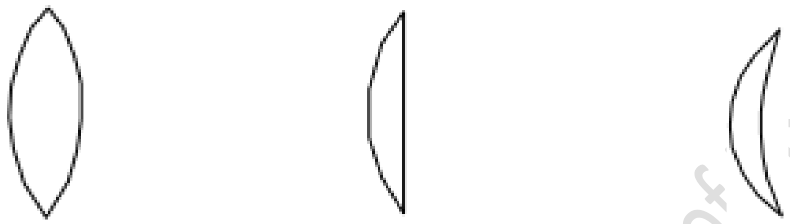
Lecture 6: Thin Lens

- Definition – is defined as a transparent material with two spherical refracting surfaces whose thickness is thin compared to the radii of curvature of the two refracting surfaces.

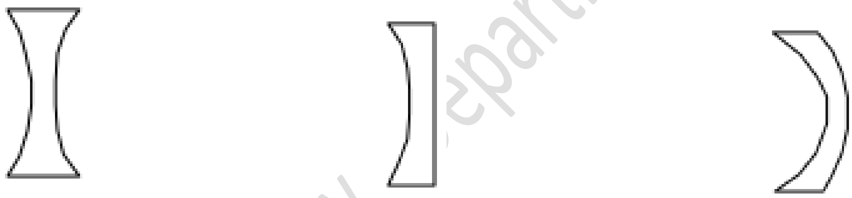
- There are two types of thin lens it is converging and diverging lens.

(a) Converging (Convex) lenses:



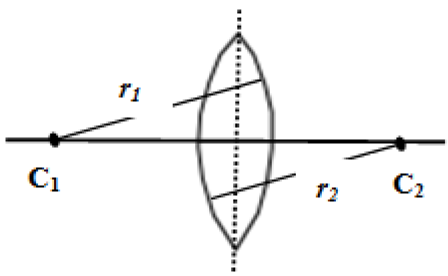
Biconvex **Plano-convex** **Convex meniscus**

(b) Diverging (Concave) lenses:

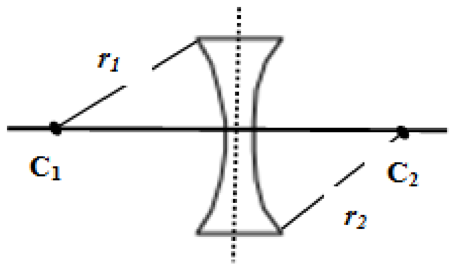


Biconcave **Plano-concave** **Concave meniscus**

(a) Converging lenses

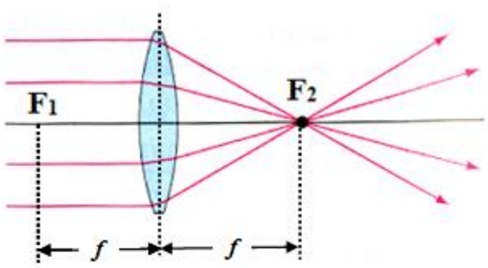


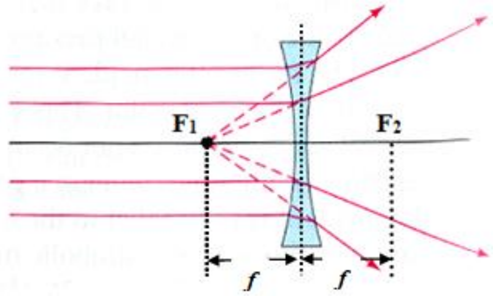
(b) Diverging lenses



- **Center of curvature (point C_1 and C_2):** Defined as the center of the sphere of which the surface of the lens is a part.
- **Radius of curvature (r_1 and r_2):** Defined as the radius of the sphere of which the surface of the lens is a part.
- **Principle (Optical) axis:** Defined as the line joining the two centers of curvature of a lens.
- **Optical center (point O):** Defined as the point at which any rays entering the lens pass without deviation.

- Consider the ray diagrams for converging and diverging lens as shown in this figures below.





- From this figures, points F_1 and F_2 represent the focus of the lens. While distance f represent the focal length of the lens.
- **Focus (point F_1 and F_2)**

For converging (convex) lens – is defined as the point on the principle axis where rays which parallel and close to the principle axis converges after passing through the lens. Its focus is real (principle).

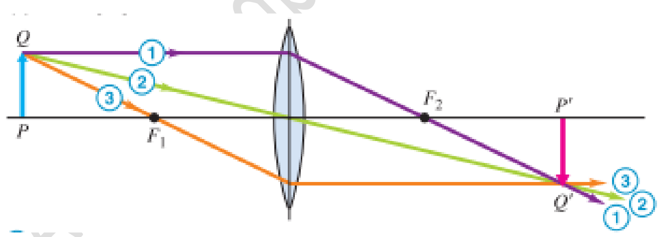
For diverging (concave) lens – is defined as the points on the principle axis where rays are parallel to the principle axis seem to diverge from after passing through the lens. Its focus is virtual.

- **Focal length (*f*):** Defined as the distance between the focus F and the optical center of the lens (O).

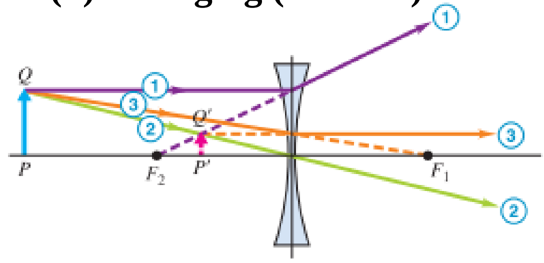
➢ **<u>Ray diagrams for thin lenses</u>**

- Ray diagrams below showing the graphical method of locating an image formed by converging (convex) and diverging (concave) lenses.

(a) Converging (convex) lens



(b) Diverging (concave) lens



At least any two rays for drawing the ray diagram

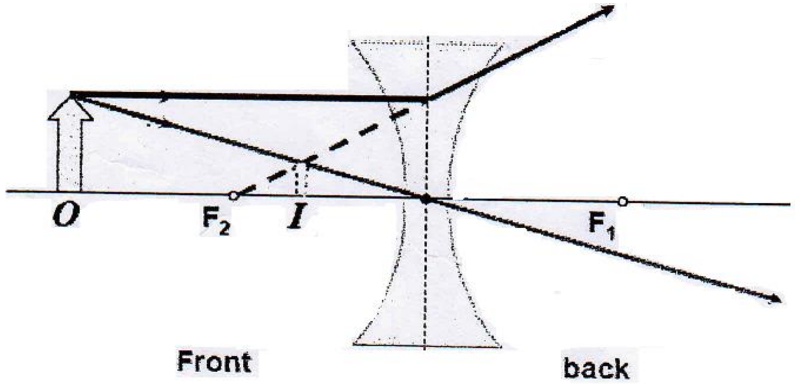
Ray 1- Parallel to the principal axis, after refracting by the lens passes through the focal point F_2 of a converging lens or appears to come from the focal point F_2 of a diverging lens.

Ray 2- Passes through the optical center of the lens does not deviate appreciably.

Ray 3- Passes through the focus F_1 of a converging lens or appears to converge towards the focus F_1 of a diverging lens, after refraction by the lens the ray parallel to the principle axis.

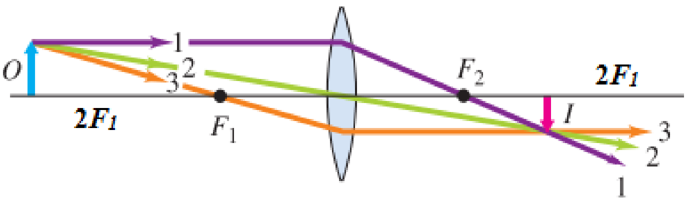
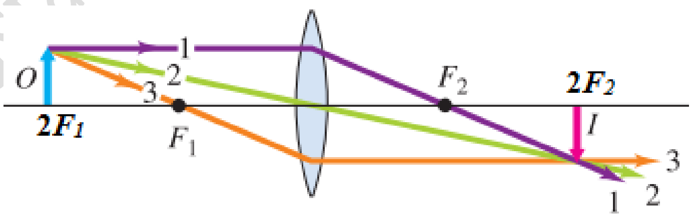
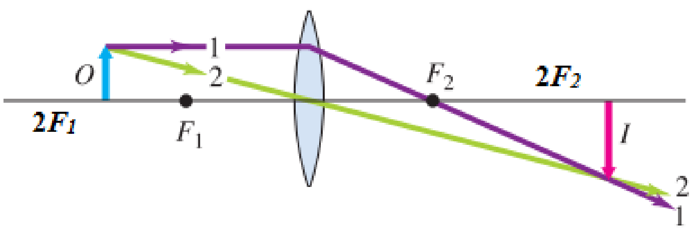
➢ Images formed by a diverging lens

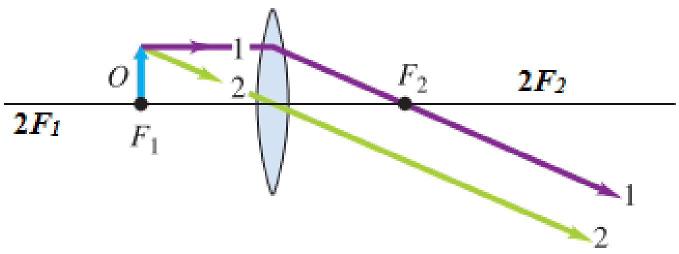
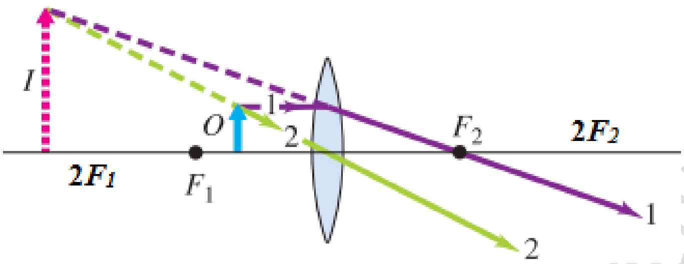
- Ray diagrams below showing the graphical method of locating an image formed by a diverging lens. Properties of image formed are:
  Virtual; Upright; Diminished (smaller than the object); Formed in front of the lens; Object position: At any position in front of the diverging lens.



➢ Images formed by a converging lens

- Table below shows the ray diagrams of locating an image formed by a converging lens for various object distance *S*.

Object distance, S	Ray diagram	Image property
S* > 2*f		○ Real. ○ Inverted. ○ Diminished. ○ Formed between point F_2 and $2F_2$. (at the back of lens)
S* = 2*f		○ Real. ○ Inverted. ○ Same size. ○ Formed at point $2F_2$. (at the back of lens)
f* < *S* < 2*f		○ Real. ○ Inverted. ○ Magnified. ○ Formed at distance greater than *2f* (at the back of lens)

Object distance, S	Ray diagram	Image property
$S = f$		○ Real. ○ Formed at infinity.
$S < f$		○ Virtual. ○ Upright. ○ Magnified. ○ Formed in front of the lens.

- Linear magnification of the thin lens, M is defined as the ratio between the image height, h_i and object height, h_o

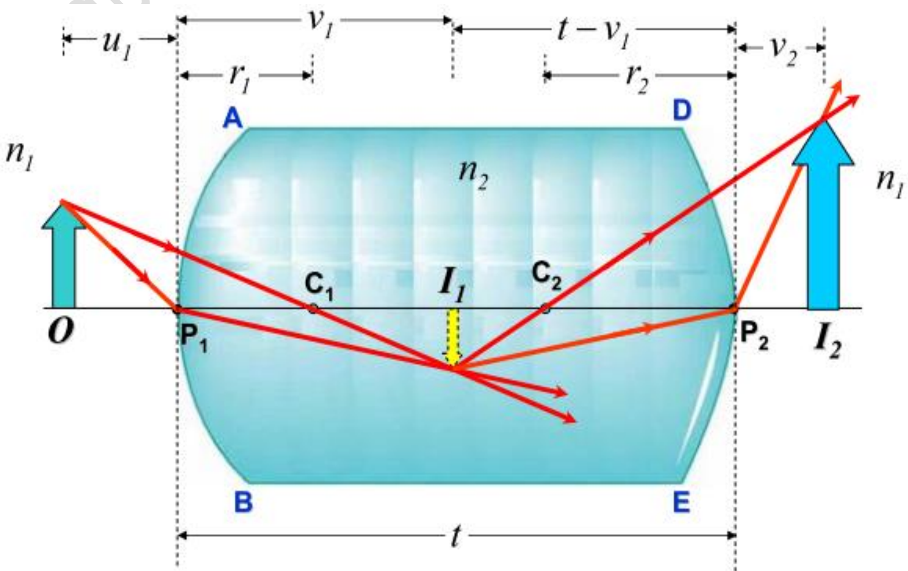
$$M = \frac{h_i}{h_o} = -\frac{v}{u}$$

Where:

v: image distance from optical center; u: object distance from optical center

➢ <u>Thin lens formula and lens marker's equation</u>

- Considering the ray diagram of refraction 2 spherical surfaces as shown in figure below.



- By using the equation of spherical refracting surface, the refracting by first surface **AB** and second surface **DE** are given by the equations:

  - **Convex surface AB ($r = +r_1$)**

$$\frac{n_1}{u_1} + \frac{n_2}{v_1} = \frac{(n_2 - n_1)}{r_1} \quad \text{.........(1)}$$

  - **Concave surface DE ($r = -r_2$)**

$$\frac{n_2}{(t - v_1)} + \frac{n_1}{v_2} = \frac{(n_1 - n_2)}{-r_2}$$

Assuming the lens is very thin thus $t = 0$,

$$\frac{n_2}{-v_1} + \frac{n_1}{v_2} = \frac{(n_1 - n_2)}{-r_2}$$

$$\frac{n_2}{v_1} = -\left[\left(\frac{n_1 - n_2}{-r_2}\right) - \frac{n_1}{v_2}\right]$$

$$\frac{n_2}{v_1} = \frac{n_1}{v_2} - \left(\frac{n_2 - n_1}{r_2}\right) \quad \text{.........(2)}$$

- By substituting eq.(2) into eq.(1), thus

$$\frac{n_1}{u_1} + \left[\frac{n_1}{v_2} - \left(\frac{n_2 - n_1}{r_2}\right)\right] = \frac{(n_2 - n_1)}{r_1}$$

$$\frac{n_1}{u_1} + \frac{n_1}{v_2} = \frac{(n_2 - n_1)}{r_1} + \frac{(n_2 - n_1)}{r_2}$$

then

$$\frac{1}{u_1} + \frac{1}{v_2} = \left(\frac{n_2}{n_1} - 1\right)\left(\frac{1}{r_1} + \frac{1}{r_2}\right) \quad \text{.........(3)}$$

- If $u_1 = \infty$ and $v_2 = f$ hence eq. (3) becomes

$$\frac{1}{f} = \left[\frac{n_2}{n_1} - 1\right]\left[\frac{1}{r_1} + \frac{1}{r_2}\right] \Longrightarrow \textbf{lens marker's equation}$$

Where: f: focal length; r_1: radius of curvature of first refracting surface; r_2: radius of curvature of second refracting surface; n_1: refractive index of the medium; n_2: refractive index of the lens material.

- By eq. (3) with lens marker's equation, hence

$$\frac{1}{u_1}+\frac{1}{v_2}=\frac{1}{f}$$

Therefore in general,

$$\boxed{\frac{1}{f}=\frac{1}{u}+\frac{1}{v}} \Rightarrow \textbf{thin lens formula}$$

Note:

- If the medium is air ($n_1 = n_{air} = 1$) thus the lens maker's equation will be

$$\boxed{\frac{1}{f}=(n-1)\left[\frac{1}{r_1}+\frac{1}{r_2}\right]}$$

Where n: refractive index of the lens material

Notes:

- For thin lens formula and lens marker's equation, use the sign convention for refraction.
- The radius of curvature of flat refracting surface is infinity, $r = \infty$.

Example 1: A biconvex lens is made of glass with refractive index 1.52 having the radii of curvature of 20 cm respectively. Determine the focal length of the lens in the following mediums: (a) water with refractive index is 1.33.
(b) carbon disulfide with refractive index is 1.63.

Solution:

$n_w = 1.33$, $n_c = 1.63$, $r_1 = +20$ cm, $r_2 = +20$ cm, $n_g = n_2 = 1.52$

a. Given the refractive index of water, $n_w = n_1$

$$\frac{1}{f}=\left[\frac{n_g}{n_w}-1\right]\left[\frac{1}{r_1}+\frac{1}{r_2}\right]$$

$$f = +07\ cm$$

b. Given the refractive index of carbon disulfide, $n_c = n_1$

$$\frac{1}{f}=\left[\frac{n_g}{n_c}-1\right]\left[\frac{1}{r_1}+\frac{1}{r_2}\right]$$

$$f = -148.18\ cm$$

Example 2:

An object is placed 90 cm from glass lens (n = 1.56) with one concave surface of radius 22 cm and one convex surface of radius 18.5 cm. Determine

(a) the image position.

(b) the linear magnification.

Solution:

u = + 90 cm, n = 1.56, r_1 = - 22 cm, r_2 = +18.5 cm

a. By applying the lens maker equation in air

$$\frac{1}{f} = (n-1)\left[\frac{1}{r_1} + \frac{1}{r_2}\right]$$

f = + 208 cm

By applying the thin lens formula, thus

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

v = - 159

The image forms 159 cm in front of the lens (at the same side of the object placed)

b. By applying the equation of linear magnification for thin lens, thus

$$M = -\frac{v}{u} \Longrightarrow M = 1.77$$

Home works about lecture 5:

Q1: A converging lens with of focal length of 90 cm forms an image of a 3.2 cm tall real object that is to the left of the lens. The image is 4.5 cm tall and inverted. The object position from the lens is:

(A) 150 cm (B) 152 cm (C) 154 cm (D) 156 cm

Q2: By using the information in question 1 calculate the image distance from the lens?

(A) 211 cm (B) 213 cm (C) 215 cm (D) 217 cm

Q3: According to the information from the above question the value of linear magnification is

(A) 1.4 (B) – 1.4 (C) 1.6 (D) – 1.6

Q4: Defined as the point at which any rays entering the lens pass without deviation.

(A) Optical center (B) Principle axis (C) Radius of curvature (D) Center of curvature

Q5: Image formed by a converging lens was diminished when the object distance is:

(A) $S < f$ (B) $S = f$ (C) $S = 2f$ (D) $S > 2f$