# 3.6 Integral of vector function and distance along the curve

## Integral of vector function

**let If F(t)=f(t)i+g(t)j+h(t)k then**

$$\int_a^b F(t)\,dt = \int_a^b f(t)i + \int_a^b g(t)j + \int_a^b h(t)k$$

**Example : Find** $\int_0^{\pi} (\cos t\, i + \sin t\, j + tk)dt$

**Solution//**

$$\int_0^{\pi} \cos t\, i\, dt + \int_0^{\pi} \sin t\, j\, dt + \int_0^{\pi} tk\, dt = 0i + 2j + \frac{\pi^2}{2}k$$

**Exercise:** $find \ \int_0^1 (t^3 i + 7j + (t+1)k)dt$

## Distance along the curve

$$\text{Length of curve} = \int_a^b |V|\, dt$$

$$= \int_a^b \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2 + (\frac{dx}{dt})^2}\ \ \text{dt}$$

**Example: Find the length of curve of one turn of the r= $(\cos t\, i + \sin t\, j + tk)$.**

**solution//**

$$\text{Length of curve} = \int_a^b |V|\, dt$$

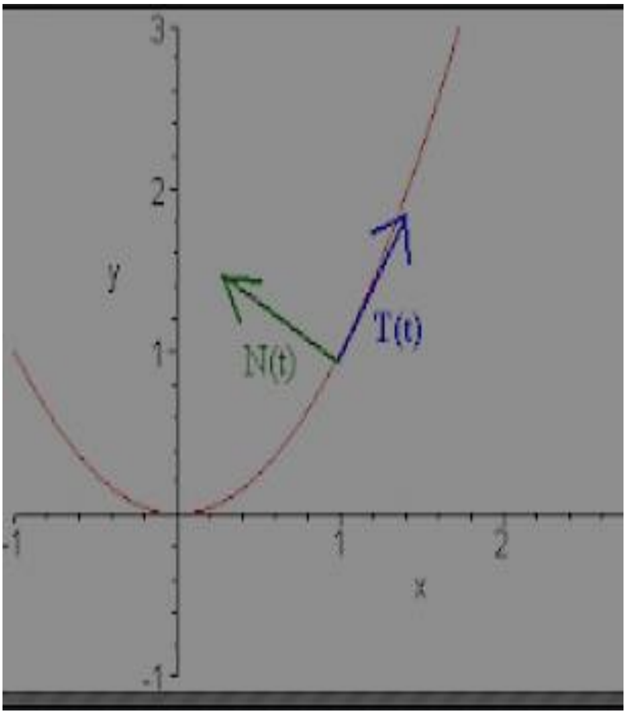
**V=-sin t i+cos t j+ 1k**

$$|V| = \sqrt{(-\sin t)^2 + (\cos t)^2 + (1)^2} = \sqrt{2}$$

$$\text{Length of curve} = \int_0^{2\pi} \sqrt{2}\, dt = 2\pi\sqrt{2}$$

## 2.7 Unit tangent and normal vector for curve

### The Unit Tangent vector



$$S(t)=\int |V| dt$$

$$ds(t)=|V| dt \div dt$$

$$\frac{ds(t)}{dt}=|V|$$

$$\frac{dt}{ds}=\frac{1}{|V|}$$

$$\frac{dr}{ds}=\frac{dr}{dt}.\frac{dt}{ds}$$

$$T=\frac{V}{|V|}=\frac{\frac{dr}{dt}}{\left|\frac{dr}{dt}\right|}$$

**Example: Find unit tangent vector of the curve**

$$r=(\cos t\, i+\sin t\, j)$$

**Solution//**

$$V=\frac{dr}{dt}=-\sin t\, i+\cos t\, j$$

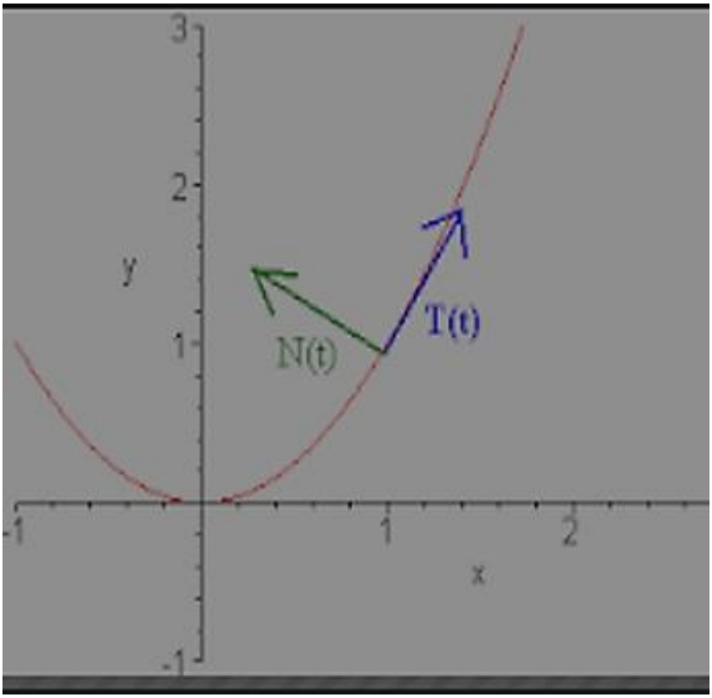
$$|V|=\sqrt{(\sin t)^2+(\cos t)^2}=1$$

$$T=\frac{V}{|V|}=-\sin t\, i+\cos t\, j$$

**Exercise: Find unit tangent vector of the curve**

$$r=(2+t)i-(t+1)j+k$$

# The Unit Normal vector in plane




$$\mathbf{N}= \frac{\frac{dT}{dt}}{\left|\frac{dT}{dt}\right|}$$

Example: Find the tangent and normal vector for curve

$$\mathbf{r}= (\cos 2t\, i + \sin 2t\, j)$$

**Solution//**

$$\mathbf{V}=\frac{dr}{dt} = -2\sin 2t\, i + 2\cos 2t\, j$$

$$|V| = \sqrt{(2\sin 2t)^2 + (2\cos 2t)^2} = 2$$

$$\mathbf{T}=\frac{V}{|V|} = -\sin 2t\, i + \cos 2t\, j$$

$$\frac{dT}{dt} = -2cos\,2t\,i - 2sin\,2t\,j$$

$$\left|\frac{dT}{dt}\right| = 2$$

N= $\frac{\frac{dT}{dt}}{\left|\frac{dT}{dt}\right|}$ =-cos2t i-sin2t j

**Example: Find the tangent and normal vector for curve**

r= $(a\cos t^2\,i + a\,\sin t^2\,j + bt^2 k)$

**Solution//**

V=$\frac{dr}{dt} = -2at\,sin\,t^2\,i + 2at\,cos\,t^2\,j + 2btk$

$$|V| = 2t\sqrt{a^2 + b^2}$$

T=$\frac{V}{|V|}$ ==$\frac{-a\,sint^2\,i}{\sqrt{a^2+b^2}} + \frac{a\,cos\,t^2\,j}{\sqrt{a^2+b^2}} + \frac{b\,k}{\sqrt{a^2+b^2}}$ $\frac{-2at\,sint^2\,i+2at\,cos\,t^2\,j+2btk}{2t\sqrt{a^2+b^2}}$

$$\frac{dT}{dt} = \frac{-2at\,cos\;\;t^2\,i}{\sqrt{a^2+b^2}} - \frac{2at\,sin\;\;t^2\,j}{\sqrt{a^2+b^2}} + 0k$$

$$\left|\frac{dT}{dt}\right| = \frac{2at}{\sqrt{a^2+b^2}}$$

N= $\frac{\frac{dT}{dt}}{\left|\frac{dT}{dt}\right|}$ =-cos$t^2$ i-sin$t^2$ j