



## Lecture One

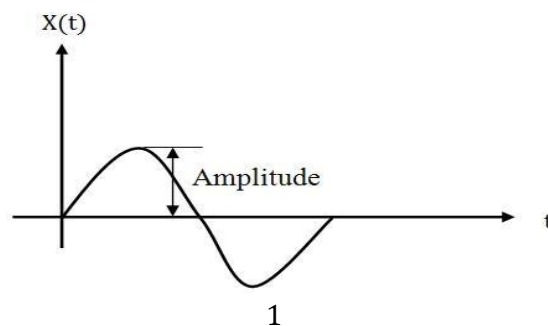
### Background

#### 1- Some Mathematical Relations

- $\sin at = \frac{e^{jat} - e^{-jat}}{2j}$
- $\cos at = \frac{e^{jat} + e^{-jat}}{2}$
- $\text{sa}(t) = \frac{\sin t}{t}$  (sinc function)
- $\sin(A + B) = \sin(A)\cos(B) + \cos(A)\sin(B)$
- $\sin(A - B) = \sin(A)\cos(B) - \cos(A)\sin(B)$
- $\cos(A + B) = \cos(A)\cos(B) - \sin(A)\sin(B)$
- $\cos(A - B) = \cos(A)\cos(B) + \sin(A)\sin(B)$
- $\cos A \cos B = \frac{1}{2} [\cos(A-B) + \cos(A+B)]$
- $\sin A \sin B = \frac{1}{2} [\cos(A-B) - \cos(A+B)]$

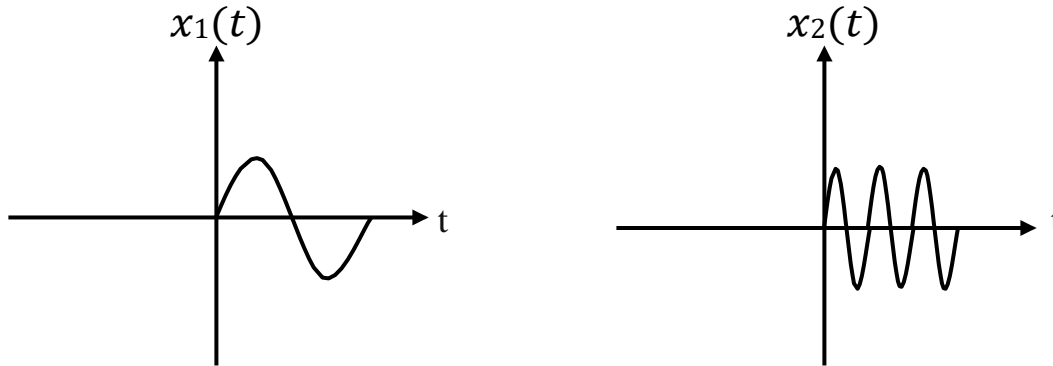
#### 2- Signal Characteristics:

- **Amplitude (A)** is the maximum displacement of a particle in a wave from its equilibrium position. It is measured in meters (m).



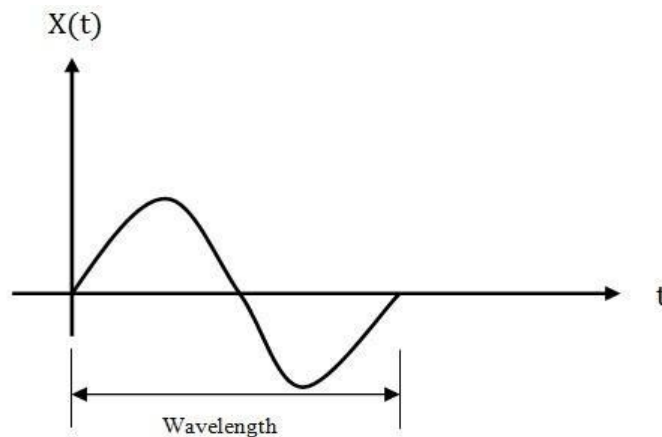


- **Frequency (f)** is the number of complete waves passing a point in one second. It is measured in hertz (Hz).



Note//  $x_2(t)$  has higher frequency than  $x_1(t)$ .

- **Wavelength ( $\lambda$ )** is the distance between two identical points on a wave (i.e. one full wave). It is measured in meters (m).



- **Wave speed (c)** is measured in meters per second (m/s).

Wave speed (c), frequency (f) and wavelength ( $\lambda$ ) are linked together in the following equation.

$$c = f \lambda$$

- $c$  = wave speed (m/s)
- $\lambda$  = wavelength (m)



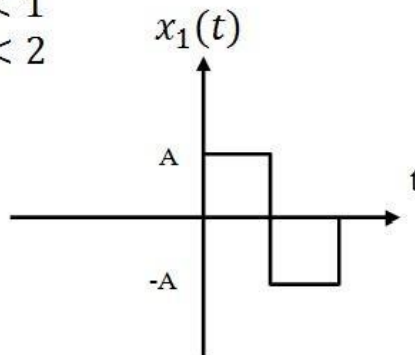
- **Phase**

Points on a wave which are always travelling in the same direction, rising and falling together, are **in phase** with each other.

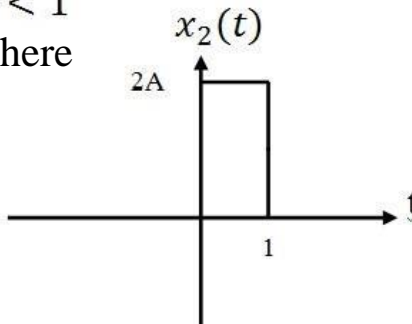
Points on a wave which are always traveling in opposite directions to each other, one is rising while the other is falling, are in **anti-phase** with each other.

### 3- Mathematical Representation of Some Function:

$$1- x_1(t) = \begin{cases} A & 0 < t < 1 \\ -A & 1 < t < 2 \end{cases}$$

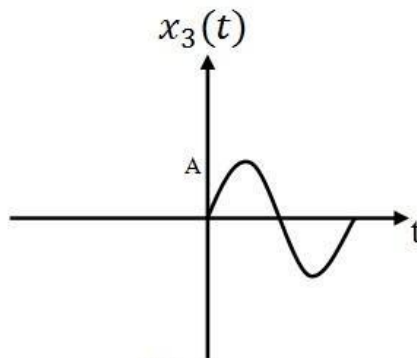


$$2- x_2(t) = \begin{cases} 2A & 0 < t < 1 \\ 0 & \text{elsewhere} \end{cases}$$

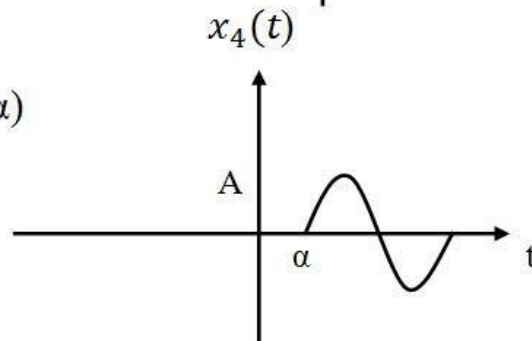




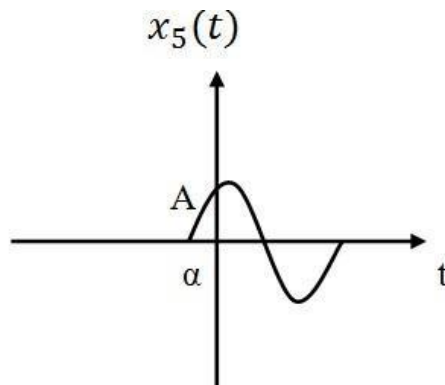
3-  $x_3(t) = A \sin t$



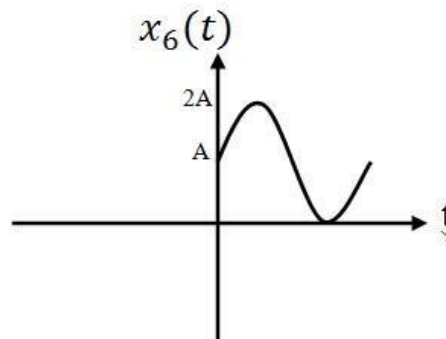
4-  $x_4(t) = A \sin(t - \alpha)$

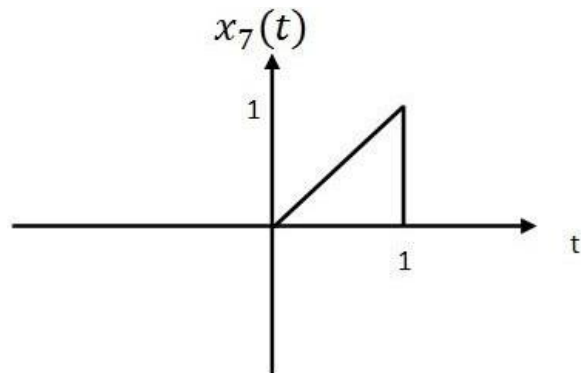


5-  $x_5(t) = A \sin(t + \alpha)$



6-  $x_6(t) = A \sin(t) + A$





By using slope law:

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{y - y_1}{x - x_1}$$

Or,

$$\frac{1-0}{1-0} = \frac{y-0}{x-0}$$

$$y = x$$

$$\text{Or, } x_7(t) = t \quad \text{for } 0 < t < 1$$