Digital Signal Processing

System Properties

What is System?

- Systems process input signals to produce output signals
- A system is combination of elements that manipulates one or more signals to accomplish a function and produces some output.

Systems Characteristic

Classification of systems

In the analysis or design of a system, it is desirable to classify the system according to some generic properties that the system satisfies.
For a system to possess a given property, the property must hold true for all possible input signals that can be applied to the system.
If a property holds for some input signals but not for others, the system does not satisfy that property.
we classify systems into five basic categories:

linear and non-linear systems;
time-invariant and time-varying systems;
systems with and without memory;
causal and non-causal systems;

System Properties

Notation: Let $\mathcal{H}$ represent the system, $x(t) \xrightarrow{\mathcal{H}} y(t)$ represent a system with input $x(t)$ and output $y(t)$.

1. Stability: BIBO (bounded input $\Rightarrow$ bounded output) stability

$$|x(t)| \leq M_x < \infty \implies |y(t)| \leq M_y < \infty$$
$$|x[n]| \leq M'_x < \infty \implies |y[n]| \leq M'_y < \infty$$

System Properties

2.Memory /Memory less

- Memory system: present output value depend on future/past input.
- Memory less system: present output value depend only on present input.
- Example

Memory systems:

$$y(t) = 5x(t) + \int_{-\infty}^{t} x(\tau)d\tau$$

$$y[n] = \sum_{m=n-5}^{n+5} x[m]$$

Memoryless systems:

$$y[n] = x[n] + x^2[n]$$

System Properties

3. Causal/noncausal

   - Causal: present output depends on present/past values of input.
   - Noncausal: present output depends on future values of input.

   Note: Memoryless $\Rightarrow$ causal, but causal not necessarily be memoryless.

4. Time invariance (TI): time delay or advance of input $\Rightarrow$ an identical time shift in the output.
   Let us define a system mapping $y(t) = \mathcal{H}(x(t))$. The system is time-invariant if

$$x(t - t_0) \xrightarrow{\mathcal{H}} y(t - t_0)$$

$$x[n - n_0] \xrightarrow{\mathcal{H}} y[n - n_0]$$

Solution : (i) $y(t) = \sin x(t)$

Let us determine the output of the system for delayed input $x(t-t_1)$. i.e.,

$$y(t, t_1) = f[x(t-t_1)]$$

$$= \sin x(t-t_1) \qquad \text{... (1.7.6)}$$

Here $y(t, t_1)$ represents output due to delayed input.

Now delay the output $y(t)$ by t_1. Hence we have to replace t by $t-t_1$ in $y(t) = \sin x(t)$. i.e.,

$$y(t-t_1) = \sin x(t-t_1)$$

On comparing the above equation with equation 1.7.6 we find that,

$$y(t, t_1) = y(t-t_1)$$

This satisfies equation 1.7.5. Hence the system is time invariant.

System Properties

Linear and Nonlinear Systems

A system is said to be linear if Superposition theorem applies to that system. Consider the two systems defined as follows :

$y_1(t) = f[x_1(t)]$ i.e. $x_1(t)$ is excitation and $y_1(t)$ is response.

$y_2(t) = f[x_2(t)]$ i.e. $x_2(t)$ is excitation and $y_2(t)$ is response.

Then for the linear system,

$$f[a_1 x_1(t) + a_2 x_2(t)] = a_1 y_1(t) + a_2 y_2(t)$$

Here a_1 and a_2 are constants.

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(ii) $y(t) = x^2(t)$

The output of the system to two inputs $x_1(t)$ and $x_2(t)$ becomes,

$$y_1(t) = f[x_1(t)] = x_1^2(t)$$

$$y_2(t) = f[x_2(t)] = x_2^2(t)$$

Hence linear combination of these outputs become,

$$\begin{aligned} y_3(t) &= a_1 y_1(t) + a_2 y_2(t) \\ &= a_1 x_1^2(t) + a_2 x_2^2(t) \end{aligned}$$

Now let us find the response of the system to linear combination of inputs. i.e.,

$$\begin{aligned} y'_3(t) &= f[a_1 x_1(t) + a_2 x_2(t)] \\ &= [a_1 x_1(t) + a_2 x_2(t)]^2 \\ &= a_1^2 x_1^2(t) + a_2^2 x_2^2(t) + 2 a_1 a_2 x_1(t) x_2(t) \end{aligned}$$

Here note that $y'_3(t) \neq y_3(t)$. Hence this is not linear system.

Causal & Noncausal Systems

- Causal system : A system is said to be *causal* if the present value of the output signal depends only on the present and/or past values of the input signal.
- Example: $y[n]=x[n]+1/2x[n-1]$

Causal & Noncausal Systems

- Noncausal system : A system is said to be *noncausal* if the present value of the output signal depends only on the future values of the input signal.

- Examples of causal systems:

$$y(t) = \frac{1}{C}\int_{-\infty}^{t} x(\tau)d\tau \qquad \text{or} \qquad y[n] = x[n-1].$$

- Examples of non-causal systems:

$$y(t) = x(-t) \qquad \text{or} \qquad y[n] = \frac{1}{3}(x[n-1] + x[n] + x[n+1]).$$

Static & Dynamic Systems

- A static system is memoryless system
- It has no storage devices
- its output signal depends on present values of the input signal
- For example

$$i(t) = \frac{1}{R} v(t)$$

Static & Dynamic Systems

- A dynamic system possesses memory
- It has the storage devices
- A system is said to possess *memory* if its output signal depends on past values and future values of the input signal

- Examples of memoryless systems:

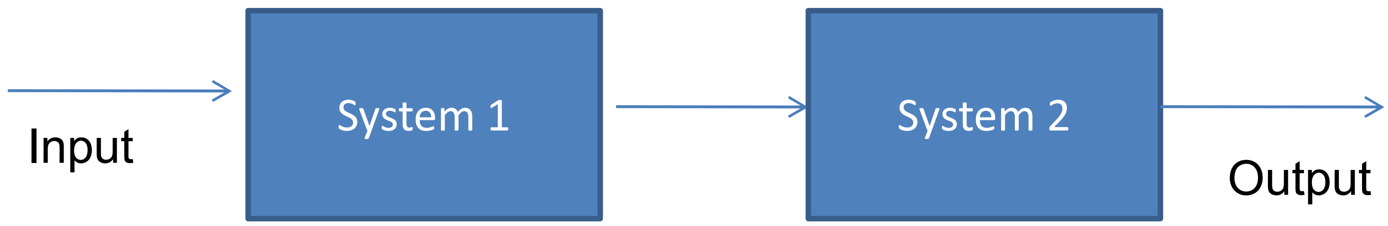
$$y(t) = Rx(t) \qquad \text{or} \qquad y[n] = \left(2x[n] - x^2[n]\right)^2.$$

- Examples of systems with memory:

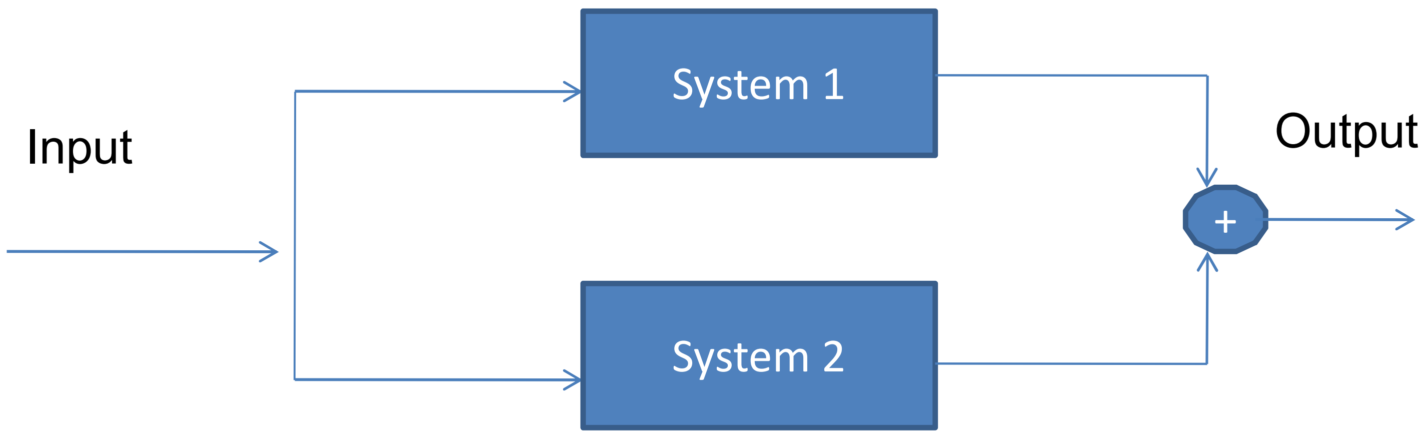
$$y(t) = \frac{1}{C}\int_{-\infty}^{t} x(\tau)d\tau \qquad \text{or} \qquad y[n] = x[n-1].$$

Interconnection of systems

- Series(cascade) Interconnection



- Parallel, Interconnection



Interconnection of systems

- Feedback Interconnection