Chapter 3

Bending moment and shearing forces

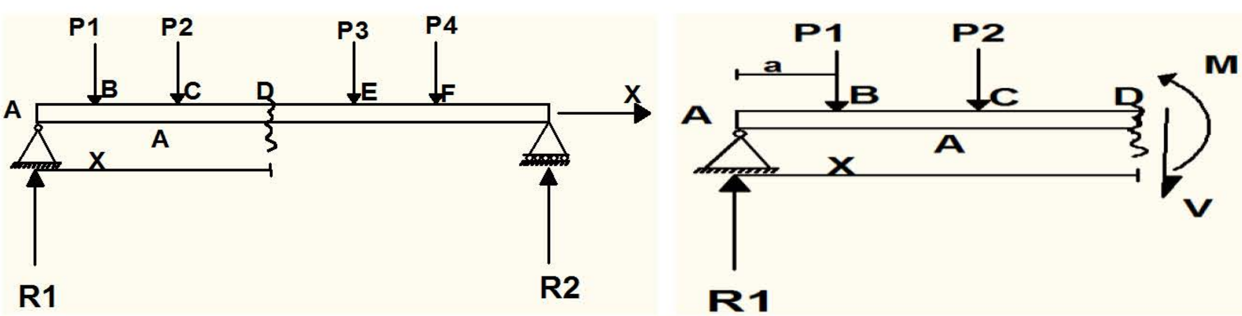
3-1 Introduction

In this chapter we use the term beam or bar. The beam is defined as a bar subjected to forces or couples that lie in a plane containing the longitudinal axis of the bar and acting perpendicular to this axis. During the context of this chapter different types of beams will be used depending on the types of support or loading(i.e. simply supported, cantilever, overhanging beam----etc). The beams also divided as:-

- Statically determinate beams: all the beams are ones in which the reactions of the supports may be determined by use of the equations of static equilibrium. The values of reactions are independent of the deformation of the beam.
- Statically indeterminate beams: if the number of the reactions upon the beam exceeds the number of equations of static equilibrium, then the statics equations must be supplemented by equations based upon the deformations of the beam.

3-2 Internal forces and moments in beams

When the beam loaded by forces and couples, internal stresses arise in the bar. In general, both normal and shearing stresses will occur. The resultant of forces and moment acting are necessary to be found and equations of static equilibrium are applied to find these stresses. Fig. 3-1 shows a simple supported beam with several concentrated loads.



(a) **(b)**

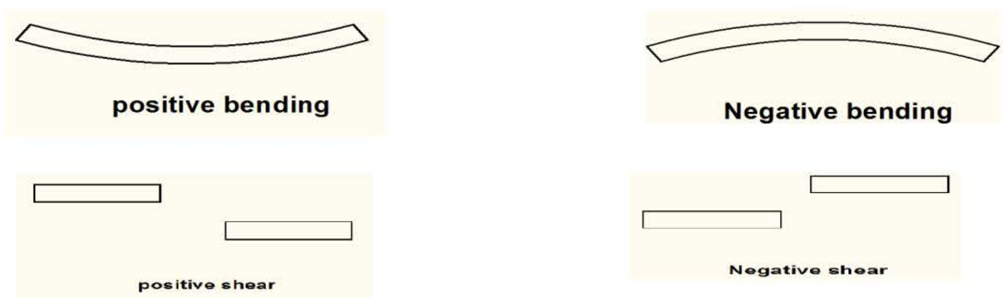
Fig. 3-1

To study the internal stresses at section D let us consider the beam to be cut at D and the portion of the beam to the right of D removed. The portion removed must then be replaced by the effect it exerted upon the portion to the left of D and this effect will consist of a vertical shearing force together with a couple, as represented by vectors V and M respectively in the free body diagram Fig. 3-1b.

The force V and the couple M hold the left portion of the bar in equilibrium under the action of the forces R_1 ,$P_{1,}$ P_2.

3-3. Sign Conventions

The customary sign convention for shearing force and bending moment are represented by the following diagrams.



(a) (b)

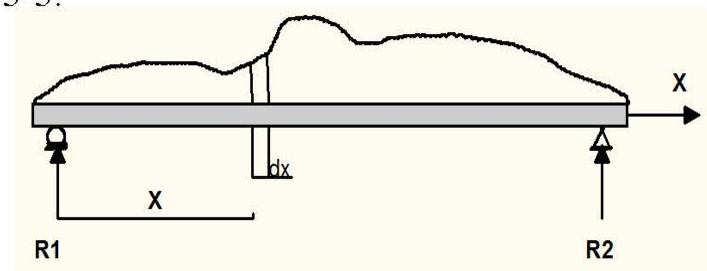
Fig. 3-2

3-4. Shear and Moment Equations

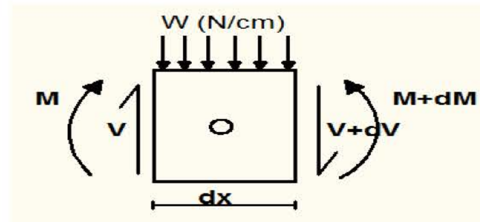
To know the shearing forces and bending moment at all sections along the beam and for this purpose two equations are written, one specifying the shearing force V as a function of the distance x from one end of the beam, the other giving the bending moment M as function of x. The plots of these equations for V and M are known as shearing force and bending moments diagrams respectively. These diagrams represent graphically the variation of shearing force and bending moment at any section along the length of the bar. From these plots it is quite easy to determine the maximum value of each of these quantities.

3-5. Relation Between Shearing Force and Bending Moment

Let us consider a beam subjected to any type of transverse load as shown in Fig. 3-3:



(a)



(b)

Fig. 3-3

Take an element of length dx from the beam and draw a free body diagram of it. From equilibrium of moments about o, we have :

$$\sum M_0 = M - (M + dM) + V\,dx - w(dx)\,(dx/_2) = 0$$

$$dM = Vdx + \frac{w}{2}(dx)^2$$

Since the last term is negligible

Then , $dM = Vdx$

Or $V = \frac{dM}{dX}$

This shearing force is equal to the rate of change of the bending moment with respect to x.

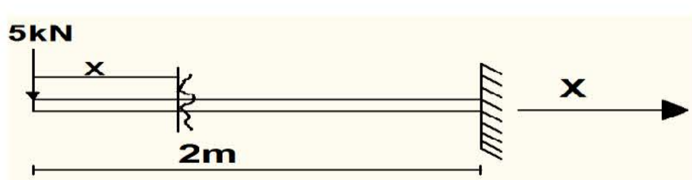
3.6.Point of contra flexure

It is the point, where the bending moment changes sign from positive to negative or inverse. Such point can occur in overhanging beam as in exercise 3-10.

Exercises

3-1 For the cantilever beam shown in Fig.3-4 below, draw the shear force and bending moment diagrams.

Fig. 3-4



Ans: let us consider any vertical section through the beam at a general distance x from the left end. The 5 kN force tends to shear the portion of the beam to the left of the section at x downward with respect to the portion to the write of the section. According to the sign convention in Fig. 3-3 b is a negative shear. Hence the shearing force V at any section x is the algebraic sum of all forces to the left of that section. Then

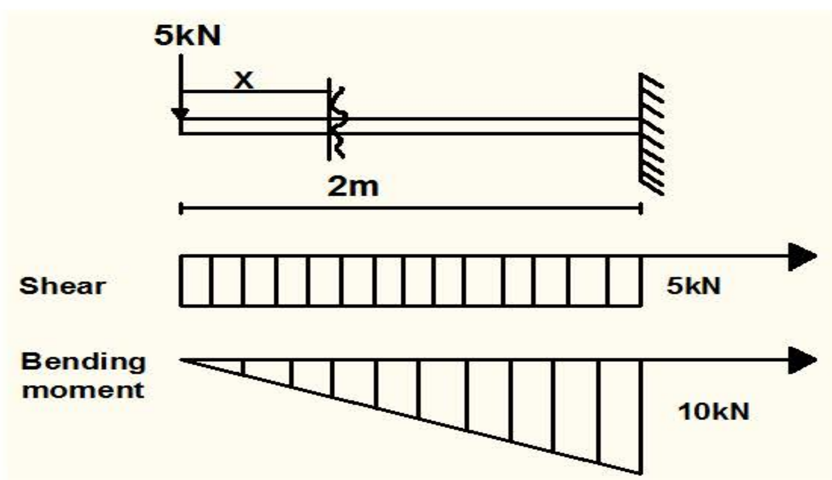
$$V = -\ 5\ \text{kN}$$

Also the bending moment produced from these forces has a negative sign. Thus the equation for the bending moment M is

$$M = -\ 5x\ \text{kN-m}$$

From the above equation of shear it is evident the quantity is constant along the length of the beam and thus should be plotted as a horizontal straight line.

The equation for the bending moment indicates that this quantity is zero at the left end of the beam and at right end of the beam where $x = 2$m takes the value of -5x 2= 10 kN-m. The plots of the shearing force and bending moment have the appearance shown below.



3-2 For the cantilever beam subjected to uniformly distributed load w kN/ m of length as shown in Fig.3-5 below , draw the shear force and bending moment diagrams.

Ans: To determine the shearing force and bending moment at any section of the beam a distance x from the free end , we may replace the portion of the distributed load to the left of this section by its resultant. The resultant is a downward force of

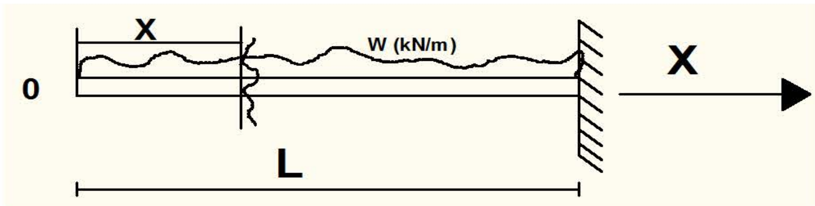


Fig. 3-5 Cantilever beam under uniformly distributed load.

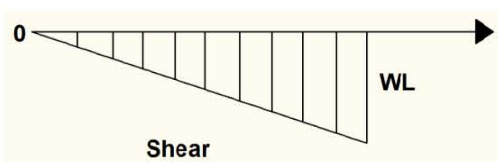
wx kN acting mid-way between 0 and the section x .The shearing force at the section x is defined to be the sum of the forces to the left of the section.

$$V = wx \text{ kN}$$

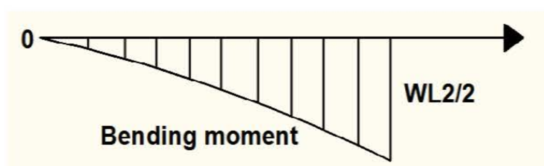
This equation indicates that the shear is zero at $x = 0$ and when $x = L$ it is $-wL$. The bending moment at that section x is defined to be the sum of the moments of the forces to the left of this section about an axis through point A.

$$M = - wx\left(\frac{x}{2}\right) \text{ kN- m}$$

This equation indicates that the bending moment ia zero at x = 0 at the left end and $-wL^2/2$ at the fixed end when $x=L$. The shear force and bending moment diagrams are plotted in Fig. 3-6.



(a)



(b)

Fig. 3-6 shear force and bending moment diagram

3-3 A cantilever beam loaded only by a couple of 3 kN-m applied as shown in Fig.3-7 below. Write the equations of shear force and bending moment at any point along the length of the beam. Draw the shear force and bending moment diagram for this loading.

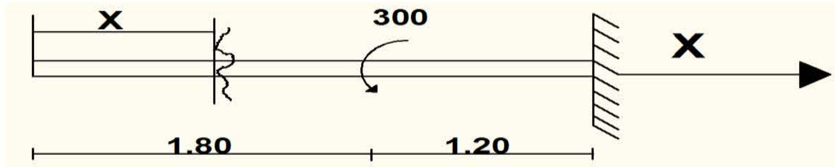


Fig. 3-7 cantilever beam subjected to couple.

Ans. The shearing force at any section along the length of the beam is the algebraic sum of the applied vertical forces to the left of the section chosen. Since there is no vertical forces applied to the bar therefore, the shearing force *V* for all values of *x* is zero. See Fig. 3-

To determine the bending moment to regions must be considered along the length of the beam. One is the 1.80 m length to the left of the applied couple and, the other the 1.20 m length to the right of the couple. Therefore we may write the equations for the bending moment at any section *x* in the form

$M = 0$ *for* $0 < x < 1.8\ m$,

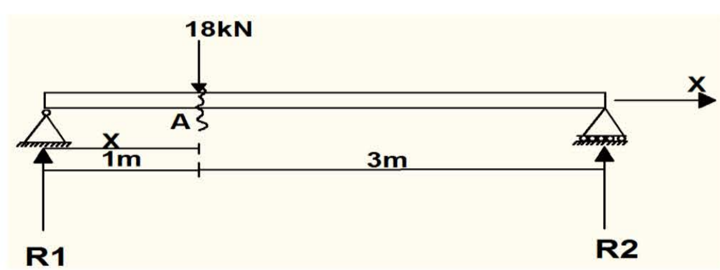
$M = -3kN\text{-}m$ *for* $1.8 < x < 3\ m$

The plots of shear and moment equations are shown in Fig.3.8.

(a) (b)

Fig. 3-8

3-4 Write the equations for the shearing force and bending moment at any position in the simply supported beam subjected to concentrated load of 18kN as shown in Fig.3-9 below.



3-9 Simply supported beam

Ans. First determine the external reactions R_1 and R_2. Taking the moment about point 0.

$\sum M = 4R_2 - 18 \times 1 = 0$

$R_2 = 4.5$ kN

The equations in the vertical direction

$\sum F_v = R_1 + 4.5 - 18 = 0$

$R_1 = 13.5$ kN

If we take first the region to the left of 18kN load at a distance x from the left support , the shearing force consists entirely of the reaction $R_1 = 13.5$ kN because that is the only force at that section. Hence in this region ,

$V = 13.5$ kN (upward) for $0 < x < 1\ m$

After exceeding 1m (to the right of concentrated load)the shearing force is

$V = 13.5 - 18 = 4.5$ for $1 < x < 4\ m$

The moment to the left of 18 kN load at a distance x is

$M = 13.5\,x$ kN-m for $0 < x < 1\ m$

If we consider a section to the right 18 kN load we get

$M = 13.5x - 18(x-1)$ kN-m for $1 < x < 4$ m

The plots of these equations for shear and bending moment diagrams are shown in Fig. 3-10.

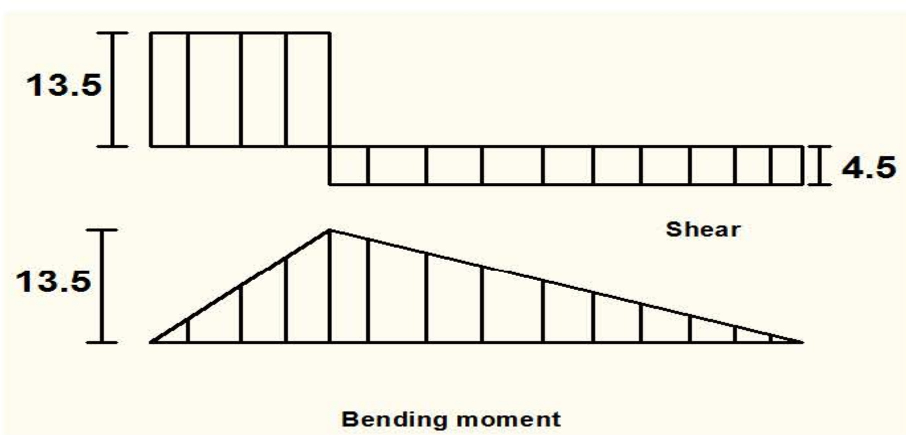


Fig. 3-10 shear force and bending moment

3-5. Write the equation for the shearing force and bending moment diagram at any position in the simply supported beam subjected to uniformly distributed load over the entire length as shown in Fig. 3-11 below.

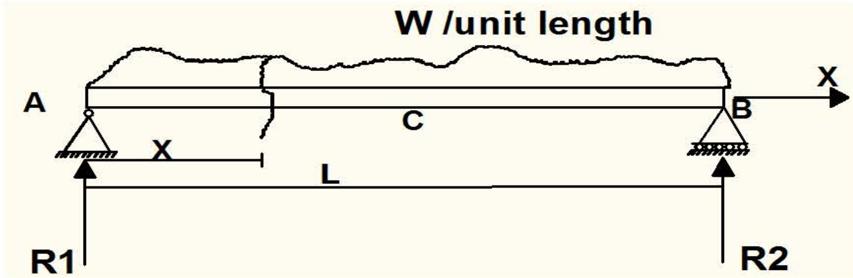


Fig. 3-11. Simply supported beam under uniformly distributed load.

Ans:

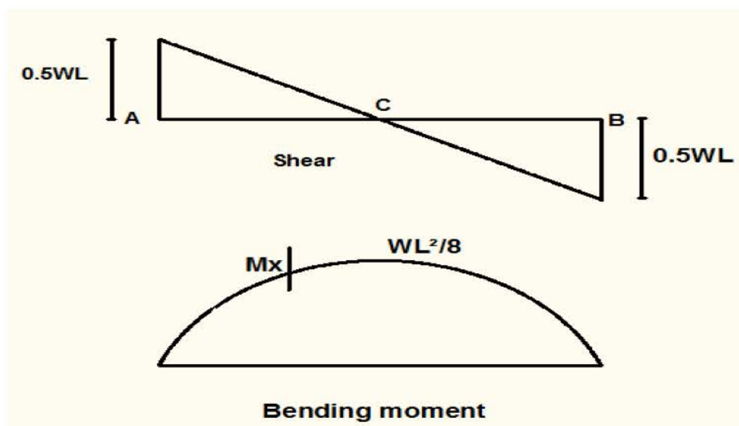
$$R_A = R_B = \frac{wl}{2} = 0.5wl$$

The shear force at any section X at a distance *x* from A,

$$F_X = R_A - w_X = 0.5\,wl - w_X$$

This shows that the shear force decreases uniformly in a straight line law until it reaches zero at mid-span ($x=L/2$) and continue to decrease uniformly to $-0.5wl$ at B $i.e\ R_B$ as shown in Fig.3-12.

Fig. 3-12



The bending moment at any section at a distance *x* from A,

$$M_x = R_A.x - \frac{wx^2}{2} = \frac{wl}{2}x - \frac{wx^2}{2}$$

From the previous equation we see that M=0 at A and B(at $x=0$ and at $x=l$) and has a parabolic curve as shown in Fig.3-12. Thus the bending moment at mid-span (point C),

$M_{c} = \frac{wl}{2}\left(\frac{l}{2}\right) - \frac{w}{2}\left(\frac{l}{2}\right)^2 = \frac{wl^2}{4} - \frac{wl^2}{8} = \frac{wl^2}{8}$

3-6. A simply supported beam 6m long is carrying a uniformly distributed load of 5kN/ m over a length of 3 m from the right end as shown in Fig.3-13 below. Draw the S.F. and B.M. diagrams for the beam and also calculate the maximum B.M. on the section.

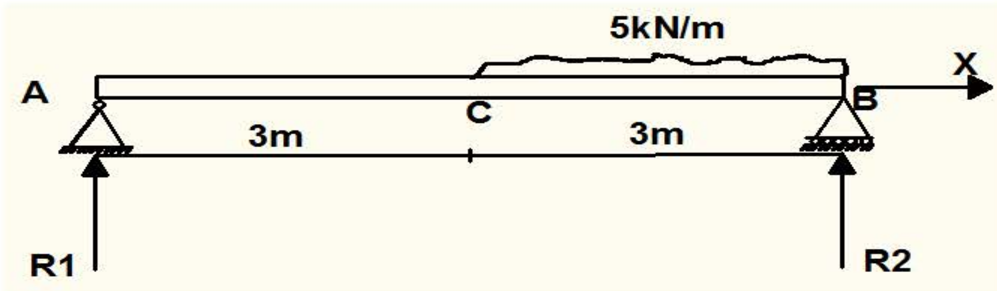


Fig. 3-13. Simply supported beam partially loaded.

Ans. First find the reactions R_A and R_B. Taking the moments about A ,

R_B x 6 = (5 x 3) x 4.5= 67.5

R_B = 11.25 kN

$\sum F_v = 0$

$R_A + R_B$ = 5 x 3

R_A = 5 x 3 -11.25 = 3.75 kN

The shear force is

$F_A = R_A$ = +3.75 kN

F_C = + 3.75 kN

R_B = 3.75- (5x3) = -11.25 kN

The shear force diagram is shown in Fig. 3-14 a.

The bending moment is,

$M_{A} = 0$

$M_{C} =$ 3.75 x 3 = 11.25 kN

$M_{B} = 0$

The maximum bending moment M will occur where the shear force change its sign (or where the shear force is zero).

Let x be the distance between C and M (Fig.3-14a). Therefore

$$\frac{x}{3.75} = \frac{3-x}{11.25}. \text{ then}$$

$$x = 11.25/15 = 0.75 \text{ m}$$

Therefore,

$$M_M = 3.75\text{x } (3+0.75) - 5\text{x } \frac{0.75}{2}$$

$$= 12.66 \text{ kN -m}$$

The bending moment diagram is shown in Fig. 3-14b

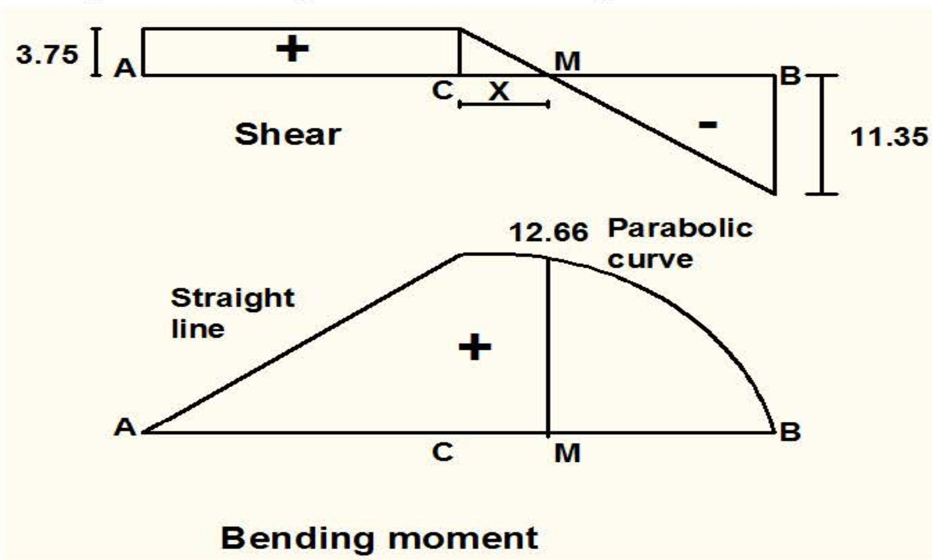
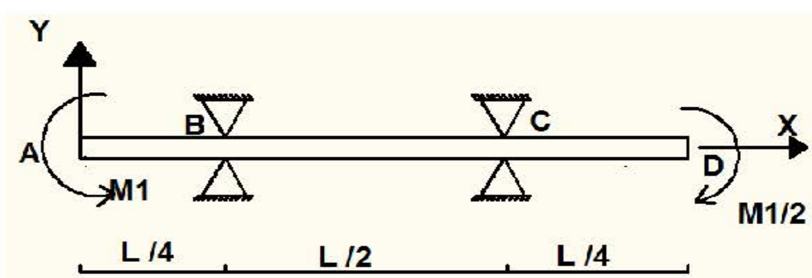


Fig. 3-14 Shear force and bending moment diagram.

3-7. The beam AD in Fig. 3-15 is supported between knife edges at B and C subjected to the end couples indicated. Draw the shearing force and bending moment diagrams.

Fig. 3-15



Ans:

The resultant of the end loadings is a couple

$$M1 - \frac{M1}{2} = \frac{M1}{2}$$

The magnitude of thin couple is in opposite direction.

$$R \cdot \frac{L}{2} = \frac{M1}{2}$$

$$R = \frac{M1}{L} \quad (this\ reaction\ at\ B\ and\ C)$$

The shearing force at any point a distance x to the right of A is given by the sum of all vertical force to the left of x.

For the three regions of the beam we have:

$$V = 0 \qquad 0 < x < \frac{L}{4}$$

$$V = \frac{M1}{L} \qquad \frac{L}{4} < x < \frac{3L}{4}$$

$$V = 0 \qquad \frac{3L}{4} < x < L$$

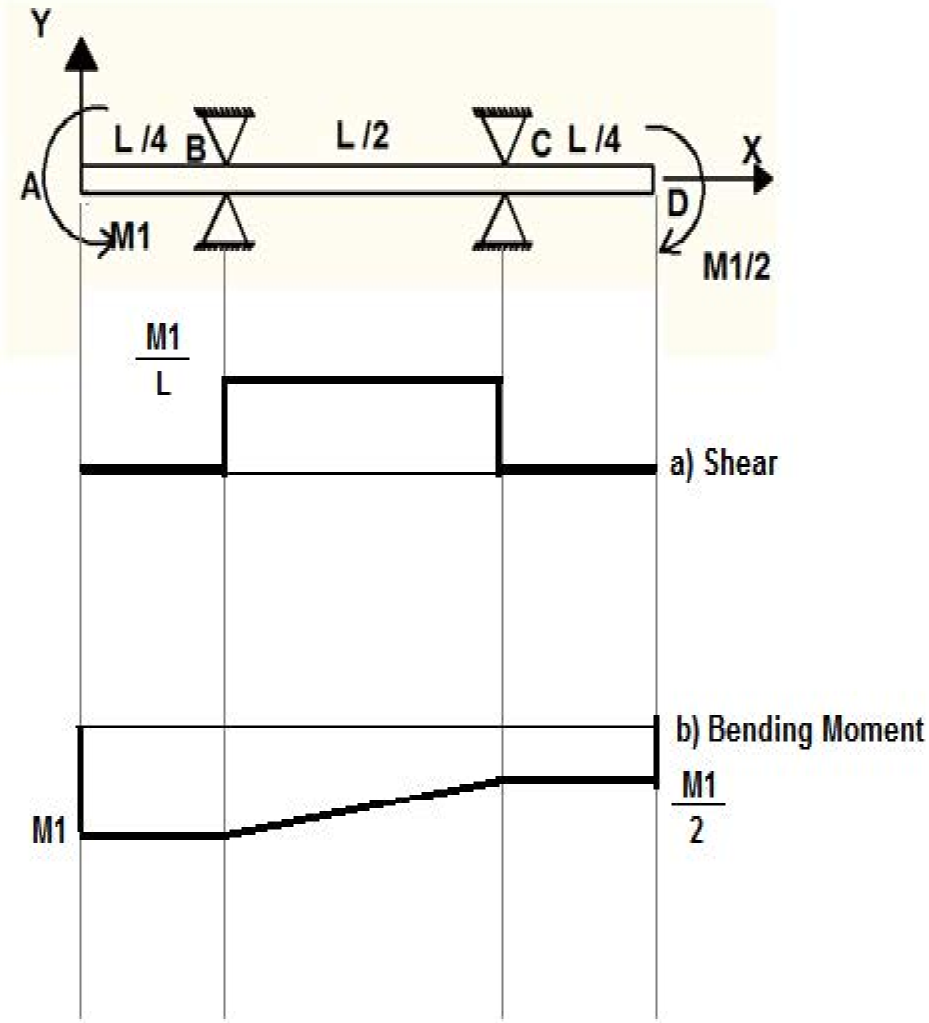
and

$$M = -M1 \qquad 0 < x < \frac{L}{4}$$

$$M = -M1 + \frac{M1}{L}\left(x - \frac{L}{4}\right) \qquad \frac{L}{4} < x < \frac{3L}{4}$$

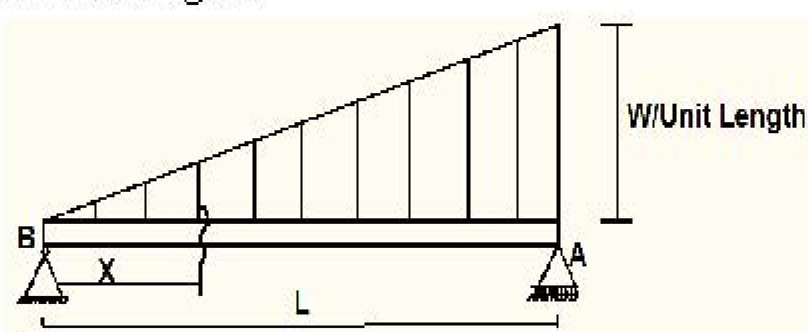
$$M = -M1 + \frac{M1}{L} \cdot \frac{L}{2} \qquad \frac{3L}{4} < x < L$$

The shear force and bending moment diagram is shown in figure below.



3-8. A simply supported beam AB of a length L and carrying a gradually varying load zero at one end and w per unit length at the other as shown in Fig. 3-16. Draw the Shearing force and bending moment diagram.

Fig. 3-16



Ans:

$$RB \times L = \left(\frac{0+W}{2}\right) \times L \times \frac{L}{3} = \frac{WL^2}{6}$$

$$\therefore\ RB = \frac{WL}{6}$$

$$but\ W = \frac{WL}{2} \quad (W = total\ load)$$

$$Or\ WL = 2W$$

Therefore

$$RB = \frac{W}{3}$$

and

$$RA = \frac{WL}{2} - \frac{WL}{6} - \frac{WL}{3} = \frac{2WL}{3}$$

The shear force at any section x at a distance x from B,

$$Fx = -RB + \frac{Wx^2}{2l}$$

$$Fx = \frac{Wx^2}{2l} - \frac{W}{3}$$

When x=0 the shear force in equal to $-\frac{W}{3}$ at B, and the shear force $= \frac{2W}{3}$ at x=L at A. the increase between these two value is a parabolic curve.

The bending moment is

$$Mx = RB \cdot x - \frac{Wx}{L} \cdot \frac{x}{2} \cdot \frac{x}{3}$$

$$Mx = \frac{WLx}{6} - \frac{Wx^3}{6L}$$

The bending moment is zero at A and B at x=0 and x=L.

The bending moment increase in the form of a cubic curve as given by the previous equation.The maximum bending moment position can be obtained by equating the for following equation to zero.

$$0 = \frac{Wx^2}{2L} - \frac{W}{3}$$

$$O = \frac{Wx^2}{2L} - \frac{W}{2(3)}$$

$$O = \frac{Wx^2}{2L} - \frac{W}{6}$$

$$or \quad \frac{Wx^2}{2L} = \frac{WL}{6}$$

$$Wx^2 = \frac{WL^2}{3}$$

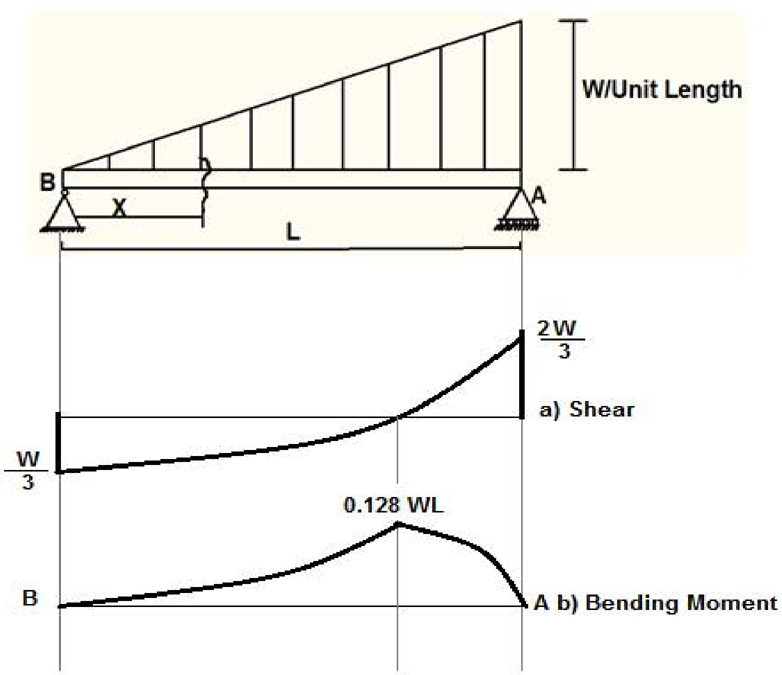
$$x^2 = \frac{L^2}{3} \quad or \quad x = \frac{L}{\sqrt{3}} = 0.577L$$

$$Mx = \frac{WL}{6}\left(\frac{L}{\sqrt{3}}\right) - \frac{W}{6L}\left(\frac{L}{\sqrt{3}}\right)^3$$

$$Mx = \frac{WL^2}{9\sqrt{3}}$$

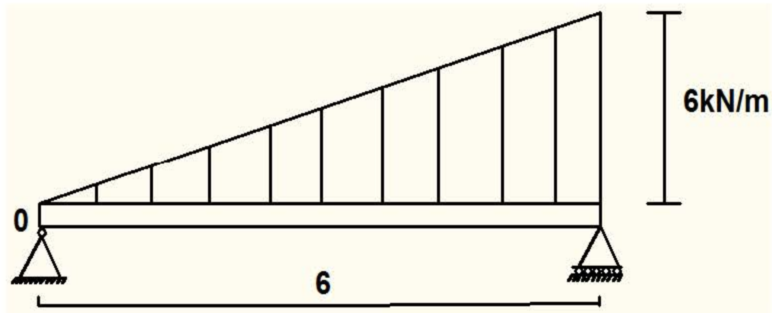
$$= \frac{2WL}{9\sqrt{3}} = 0.128W \quad (W = \frac{WL}{2})$$

The shear force and bending moment diagrams are plotted in figure below.



3-9. The simply supported beam shown in Fig. 3-17 carries a vertical load that increases uniformly from zero at the left to a maximum value of 6kN/m of length at the right end. Draw the shearing force and the bending moment diagrams.

Fig. 3-17



Ans:

Take the moment about B

$$RA \times 6 - 18000 \times 2 = 0$$

$$\therefore \; RA = \frac{18000 \times 2}{6}$$

$$RA = 6000\ N$$

And

$$RB \times 6 - 18000 \times 4 = 0$$

$$\therefore RB = \frac{18000 \times 4}{6}$$

$$RB = 12000\ N$$

To draw the shear force and bending moment diagrams we must consider the a section at a distance x from the left end as in figure.

The average intensity of the load over the length x is:

$$\frac{x}{6} = \frac{W}{6000} \quad or \quad W = \frac{x}{6} \cdot 6000\ N/m$$

The average intensity of the load over the length x is $\frac{1}{2}\left(\frac{x}{6}\right)6000$.

The shearing force and bending moment at A are now readily found to be:

$$V = 6000 - \frac{1}{2}\left(\frac{x}{6}6000\right)x = 6000 - 500x^2$$

$$M = 6000x - \frac{1}{2}\left(\frac{x}{6}6000\right) \cdot x \cdot \frac{x}{3} = 6000x - \frac{500x^3}{3}$$

There equation can be applied along the entire length of the beam the shear force has a parabola shape.

When x=0 its value is 6000 N

When x=6 its value is 12000 N

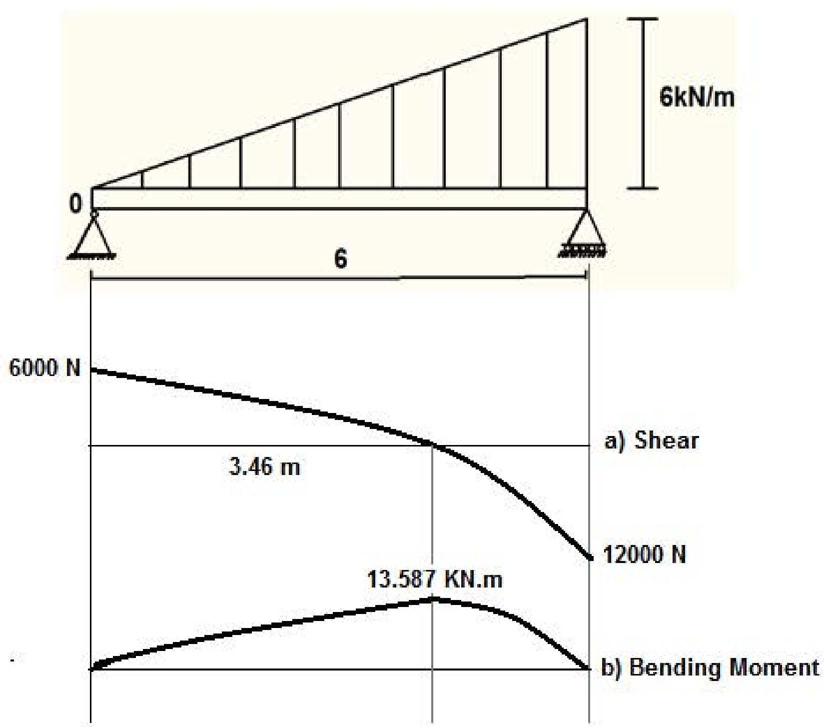
The bending moment is third degree polynomial. Its maximum in at zero shear, the therefore

$$0 = 6000 - 500x^2$$

$$\therefore x = 3.46\, m$$

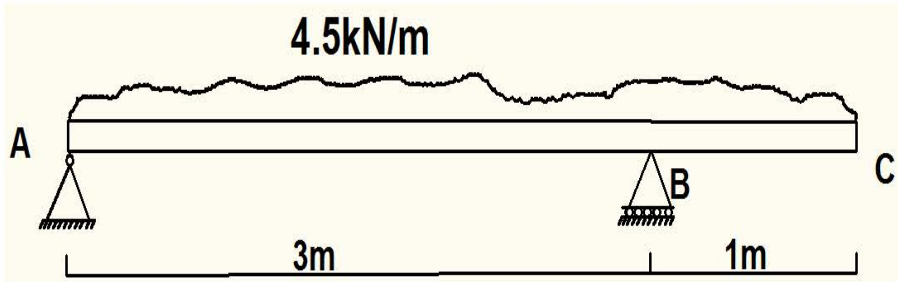
$$then, MC = 6000(3.46) - \frac{500}{3}(3.46)^3$$

$$MC = 20760 - 6903.6 = 13857\ N.m = 13.857\ KN.m$$



3-10. An overhanging beam shown in Fig. 3-18 is under uniformly distributed load of intensity 4.5 kN/m. Draw the shear force and bending moment diagrams and find if there is any point of contra flexure and its position.

Fig. 3-18



Ans:

R_B x = (4.5)(4)x2 = 36

R_B = 36/3 = 12 kN ,

and R_A = (4.5x4) = 6 kN

Shear force :

$F_A = R_A$ = 6 kN

F_B = 6 – (4.5 x3) + 12 = 4.5 kN

F_C = 4.5 – (4.5x1) = 0

The bending moment :

M_A= 0

M_B = - (4.5x1) x $\frac{1}{2}$ = 2.25 kN.m

M_c = 0

The maximum bending moment will occur at point D .

From shear diagram

$$\frac{x}{6} = \frac{3-x}{7.5}$$

7.5x = 6(3-x)

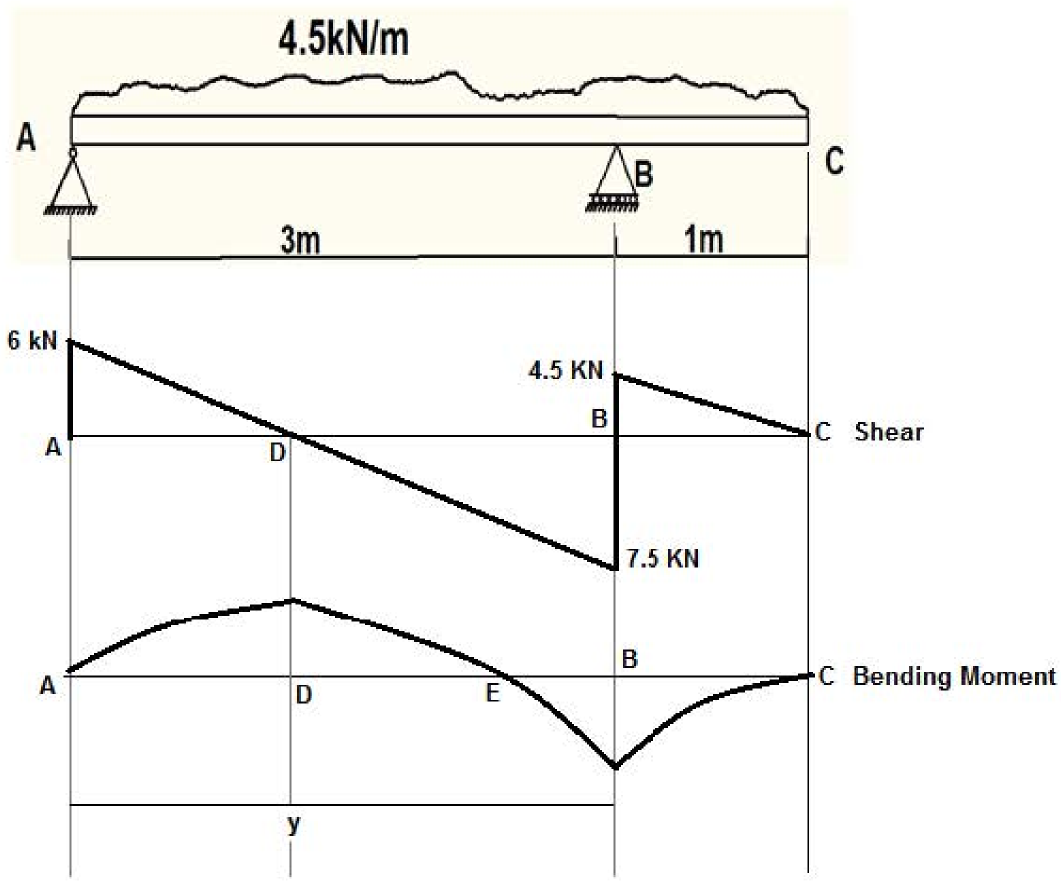
$\therefore$ x = 1.33 m (point of zero shear and maximum bending moment)

M_D = (6x1.33) – 4.5 x 1.33x $\frac{1.33}{2}$ = 4 kN.m

M_E = 6 x y- 4.5 x y $\frac{y}{2}$ = 0

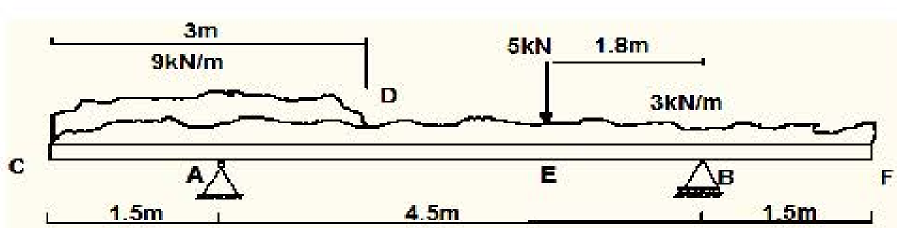
2.25 y^2 - 6y = 0, or 2.25y = 6

Therefore y= 2.67m



3-11. An overhanging beam is under uniformly distributed load of different intensities over its span as illustrated in Fig. 3-19 below. Draw the shear force and bending moment diagrams. Show the maximum positive and negative bending moment.

Fig. 3-19



Ans:

Taking the moment about A

N.B. the 9kN/ m will have zero moment about A

R_B x 4.5 = (3 x 4.5) x 3.75+(5 x 2.7) = 64.125

$R_B = \frac{\mathbf{64.125}}{\mathbf{4.5}}$ = 14.25 kN ,

and

R_A = 9 x 3+ 5 + (3 x 4.5) – 14.25

= 31.25 kN

The shearing force diagram is

F_c = 0

F_A = 0- 9 x 1.5+ 31.25 = +17.45 kN

F_D = + 17.75 – 0 x 1.5 = + 4.25 kN

F_E = + 4.25 – 3 x 1.2 - 5 = -4.35 kN

F_B = - 4.35 – 3x 1.8 + 14.25 = + 4.5 kN

F_F = + 4.5 – 3 x 1.5 = 0

The bending moment diagram is,

$M_{C=0}$

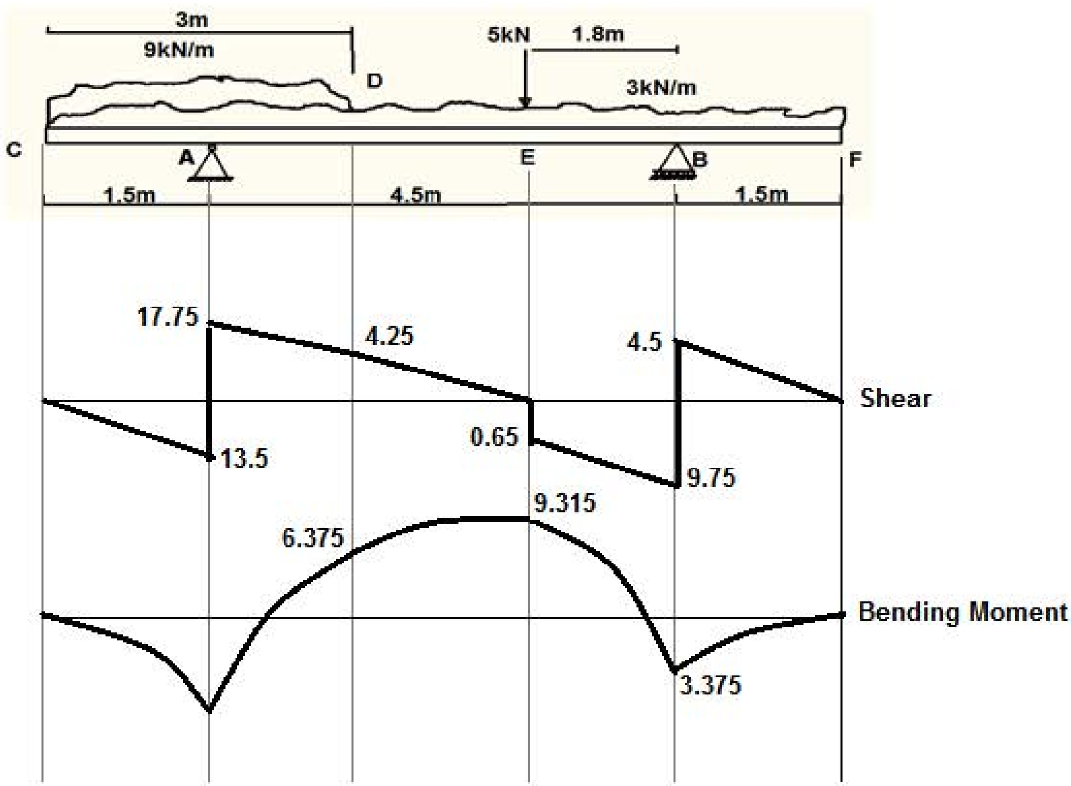
$M_{A=}$ - 9 $\frac{(1.5)^2}{2}$ = - 10.125 kN

M_D = - 9 $\frac{(3)^2}{2}$ + 31.125 x 1.5 = 6.375 kN.m

$M_B = \frac{3\,x\,(1.5)^2}{2}$ = - 3.375 kN.m

$M_E = \frac{3\,(\,3.3)^2}{2}$ + 14.25 x 1.8 = 9.315 kN.m

$M_F = 0$



3-12. Find the loading on the beam AD from its shear force diagram shown in Fig. 3-20 below, and draw its bending moment diagram.

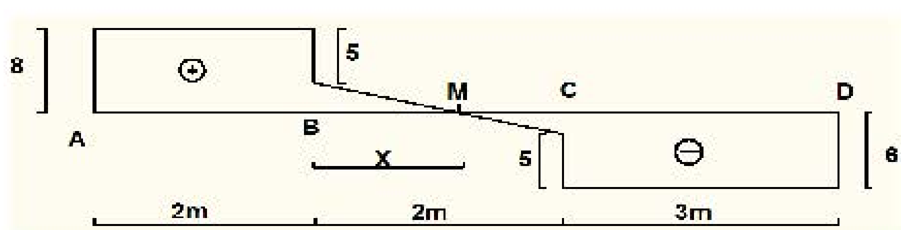


Fig. 3-20

Ans:

Given L= 7 m shear force at A= 8kN at A. The reaction is equal to shear force. Therefore the reaction is 8 kN.

Between A and B :

There is a straight line between A and B. This means that there is no load between A and B, at point B there is a sudden reduction of 5kN this means that there is a concentrated load of 5 kN.

Between B and C:

The shear force has an inclined straight line between B and C this means that there is a uniformly distributed load between B and C>

Now:

8-5 = 3 kN

6-5 = 1 kN ,

then the decrease is 3+1= 4 kN at a distance 2 m, therefore the uniformly distributed load is ;

$$\frac{3+1}{2} = \frac{4}{2} = 2 \text{ kN/m}$$

At C: the shear force has a sudden decrease of 6 - 1 = 5 kN , therefore the point load is 5 kN at C .

Between C and D:

There is a straight horizontal line between C and D. Therefore there is no point load between C and D. At point D the shear force increase from -6 to zero. Therefore is a reaction of 6kN at D.

The bending moment

$M_A = 0$

$M_B = 8 \times 2 = 16$ kN.m

$M_c = 6 \times 3 = 18$ kN .m

$M_D = 0$

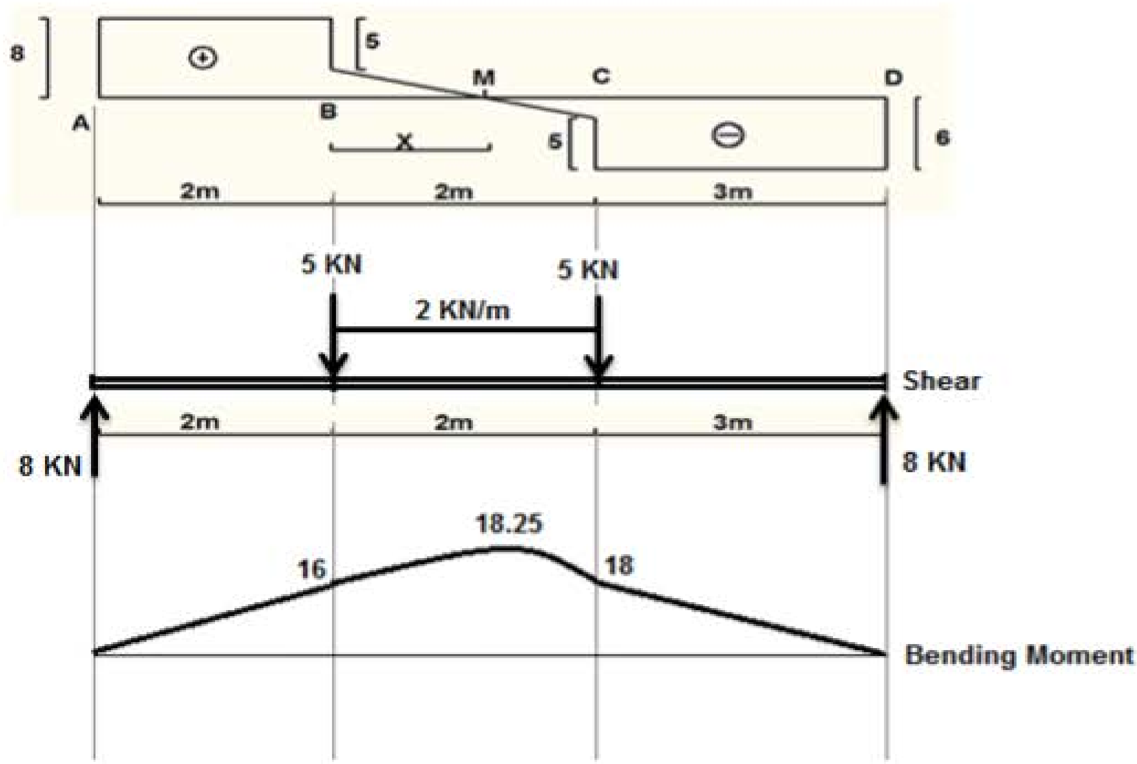
The maximum bending moment will occur at

$$\frac{x}{3} = \frac{2-x}{1} \quad \text{(from the shear force diagram)}$$

$\therefore\ x = 1.5\ m$

$$M_A = (8 \times 3.5) - (5 \times 1.5) - (2 \times 1.5) \times \frac{1.5}{2} = 18.25\ kN$$

The plot of the forces on this beam are shown below.



3-13. A beam ABC is 9 m long and supported at B and C, 6 m apart as shown in Fig. 3-21 below. The beam carries a triangular distribution of load over the portion BC together with an applied counterclock wise couple of a moment 80 kN.m at B and uniformly distributed load (u.d.l.) 0f 10kN/m over AB, as shown. Draw the shear force and bending moment diagrams of the beam.

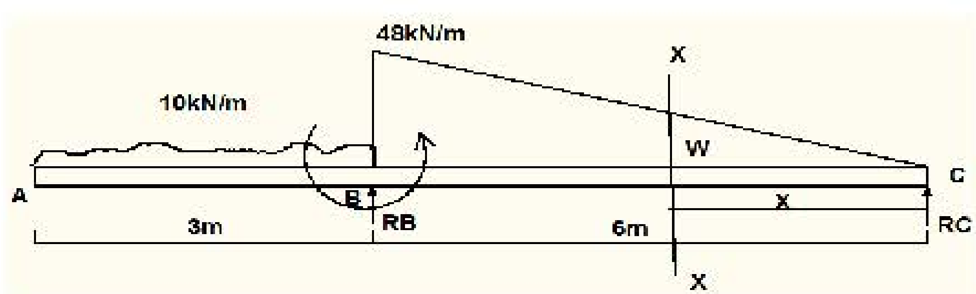


Fig. 3-21

Solution:

Taking the moments about B

$(R_C \text{ x } 6) + (10 \text{ x } 3 \text{ x } 1.5) + 80 = (\frac{1}{2} \text{ x } 6 \text{ x } 48) \text{ x } \frac{1}{3} \text{ x } 6$

$6 R_C + 45 = 80 = 288$

$R_C = 27.2$ kN

$R_C + R_B = (10 \text{ x } 3 + (\frac{1}{2} 6 \text{ x } 48)$

$= 30 + 144 = 174$

$\therefore \quad R_B = 174 – 27.2 = 146.8$ kN

At any distance from C between C and B the shear force is given by :

At x – x shear force (S.F.) = $- \frac{1}{2} \text{ w x} + R_c$, and by proportions

$\frac{w}{x} = \frac{48}{6} = 8$

i.e w = 8 x kN. m

$\therefore$ S.F. at x- x $= - (R_C - \frac{1}{2} \text{x8x . x})$

$= - R_C + 4 \text{ x}^2$

$= - 27.2 + 4 \text{ x}^2$ (see Fig. below)

The B.M at x-x

$= -(\frac{1}{2} \text{ wx}) \frac{X}{3} + R_C. \text{ X}$

$= 27.2\text{x} - \frac{4x^3}{3}$

From a maximum value of

$d \frac{(B.M)}{dx} = \text{S.F}=0$

∴ $4x^2 = 272$

∴ $x = 2.61$ m from C

or

$$B.M_{max} = 27.2(2.61) - \frac{4}{3}(2.61)^3$$

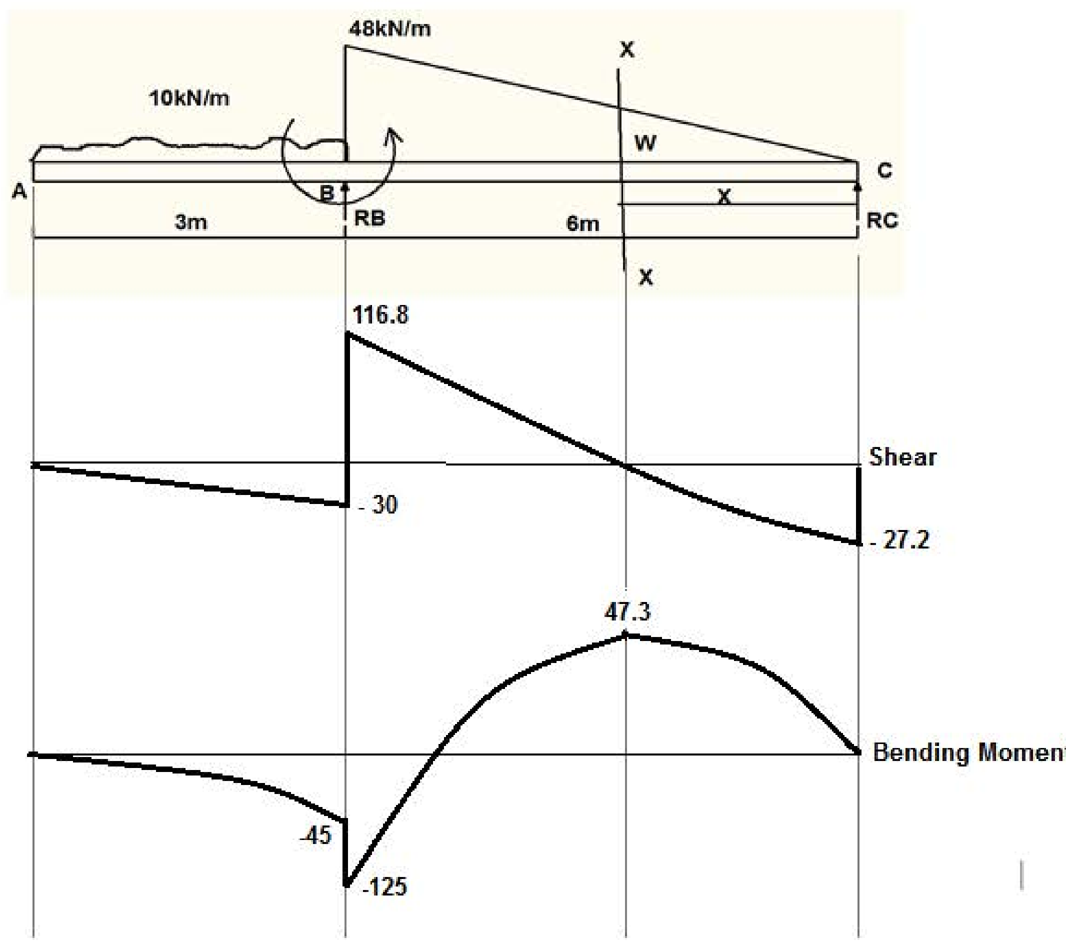
$$= 47.3 \text{ kN.m}$$

B.M at A and C = 0

B.M immediately at left of B

$$= -(10 \times 3 \times 1.5) = -45 \text{ kN.m}$$

N.B. There will be no discontinuity in S.F. diagram, because the effect of 80 kN is reflected in value of reaction.



3-14. A simply supported beam 5 m long carries a load of 10 kN on a bracket welded to the beam as shown in Fig. 3-22 be

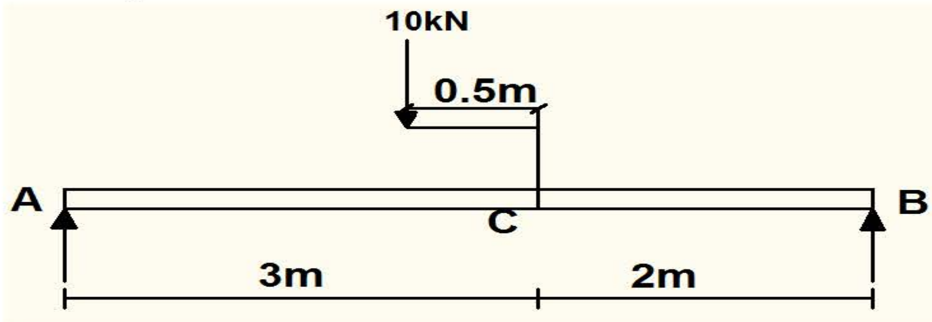


Fig. 3-22

Solution:

The 10 kN load applied on the bracket will have the following two effects:

1. Vertical load of 10 kN at C
2. A conterclock wise couple of moment = 10x0.5= 5kN.m at C.

Now :

Taking the moment about A

R_B x 5 = (10x3) – 5 = 25

R_B = 10-5 =5 kN

R_A = 10 – 5 = 5 kN

The shear force diagram :

F_A = + R_A = 5 kN

F_C = + 5-10 = 5 kN

F_B = - 5 kN

Bending moment diagram :

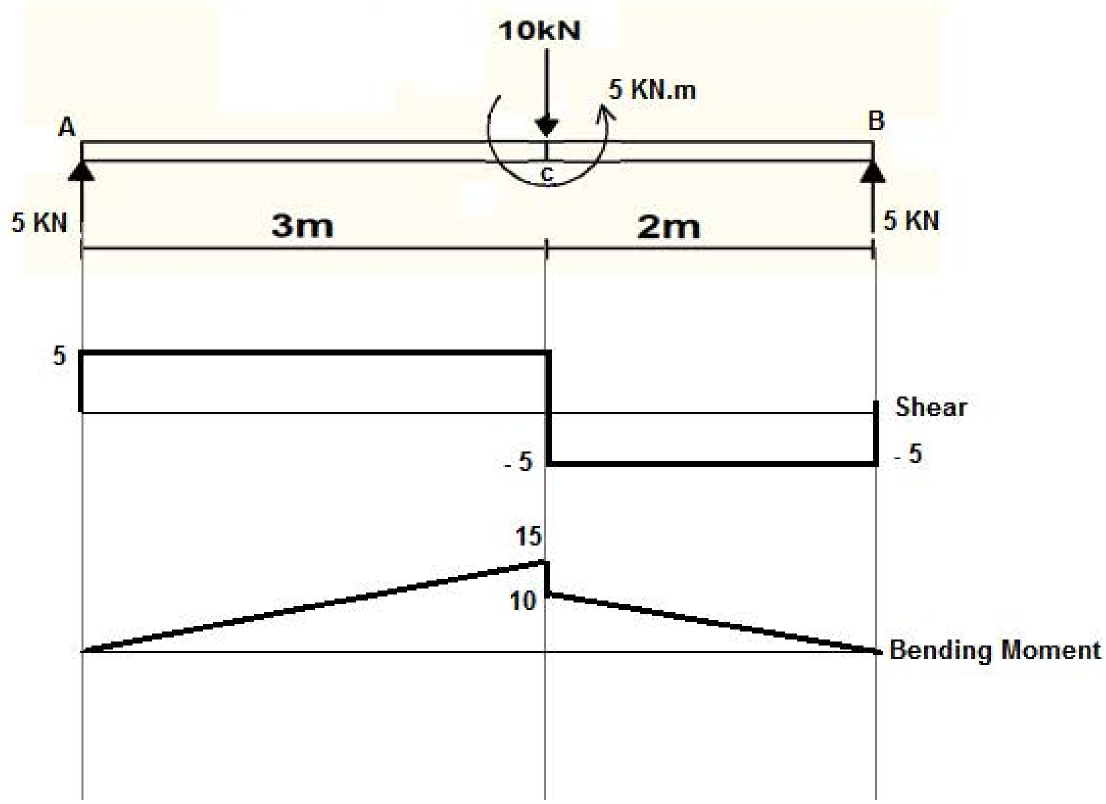
$M_{A=}$ 0

M_C = 5 x 3 = 15 kN.m (with help of R_A)

M_C =5x2 = 10 kN

$M_B = 0$

The decrease in M_C from 15 kN.m due to conterclock wise moment of 5 kN.



3-15 A horizontal beam AB of length 4m is hinged at A and supported on roller at B . The beam carries inclined loads of 100N, 200N and 300N inclined at 60^0, 45^0 and 30^0 to the horizontal as shown in Fig. 3-23 below. Draw the S.F. and B.M. and thrust diagrams for the beam.

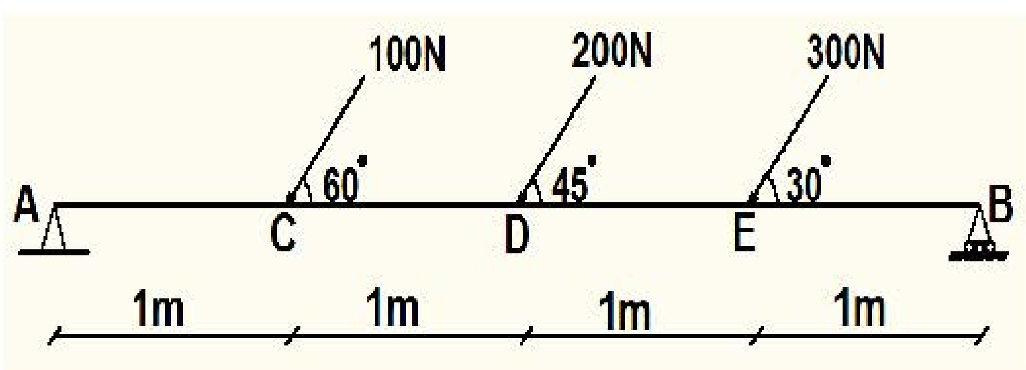


Fig. 3-23

Solution:

Solving the inclined for horizontal and vertical components

H_C = 100 x cos 60 = 100 x 0.5 = 50 N

V_C = 100 x sin 60 = 100 x 0.866 = 86.6 N

H_D = 200 x cos 45 = 141.4 N

V_D = 200 x sin 45 =141.4 N

H_E = 300 x cos 30 = 300 x 0.866 = 259.8 N

V_E = 300 x sin 30 = 300 x 0.5 = 150 N

H_A = 50 + 114.4 + 259.8 = 451.20 N

The reactions ; Take the moments of all forces about A then

R_B x 4 = 86.6 x 1+ 141.4 x 2 + 150 x 3

= 819.4

$R_B = \frac{819.4}{4}$ = 204.85 N

R_A = The total vertical load - R_B

or R_A = (86.6 + 141.4 + 150) = - 204.85

= 173.15 N

The shear force diagram

S.F at A = + R_A = 173.15 N

Between A and C remain constant

At C S.F = 173.1-86.6 = 86.55N

At D S.F = 86.55- 141.4 = - 54.85 N

At E S.F = 54.85 + 150 = 204.85 N

At B is equal to the reaction = 204.85 N

The B.M diagram:

The moment is only due to vertical loads

The B.M. at A = 0

B.M. at C = R_A x 1 = 173.15 x 1 = 173.15 N.m

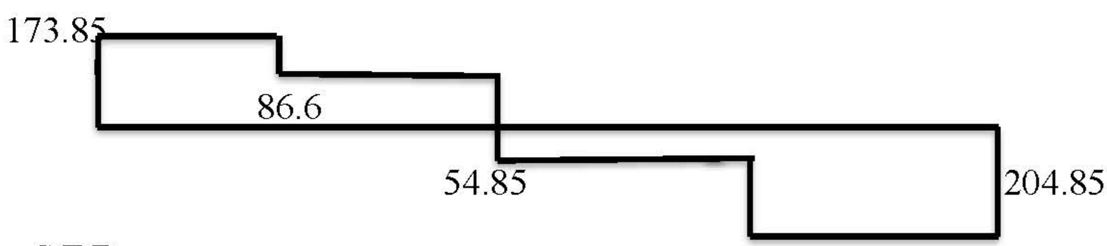
B.M. at D = R_A x 2 – 86.6 x 1

= 173.15 x 2 – 86.6 x 1 = 259.7 N.m

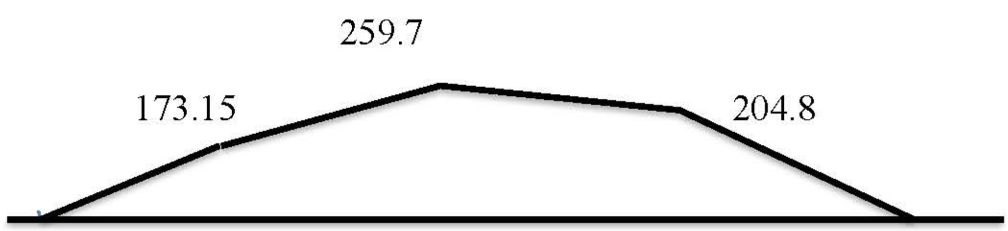
B.M. at E = R_A x 3 – 86.6 x2 – 141.4 x 1

= + 204.85 N.m

B.M. at B = 0



S.F.D



B.M.D

The thrust:

Axial forces through the X- axis

H_A = 451.2 N

At C : $H_A - 50 = 401.2$ N

At D : 401.2-141.40 = 259.8 N

At E : 259.8- 259.8 = 0

Axial force between E and B is zero

3-16 A simply supported beam of 6m span shown in Fig. 3-24 carries a triangular load of 25 kN. Draw a shear force and bending moment diagrams for the beam.

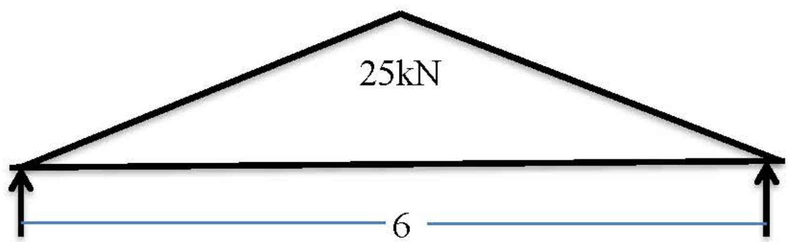


Fig. 3-24

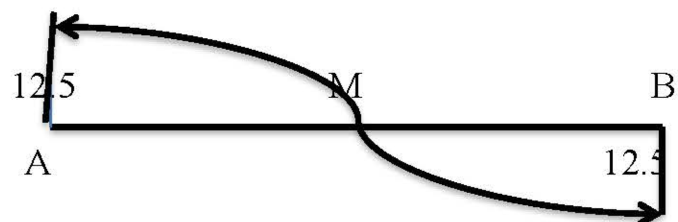
Solution

$R_A = R_B = \frac{25}{2} = 12.5$ kN

Shear force diagram

$F_A = + R_A = 12.5$ kN

$F_B = - R_B = -12.5$ kN

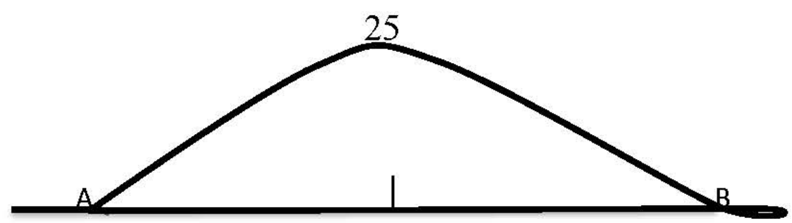


S.F.D

Bending moment diagram

$M_m = \frac{Wl}{6} = \frac{25 \ x\ 6}{6} = 25$ kN.m

.



B.M.D

3-17 Draw the shear force and the bending moment diagram for a hinged beam loaded as shown in the Fig.3-25 below.

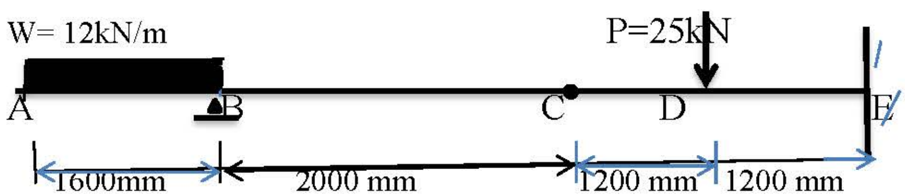
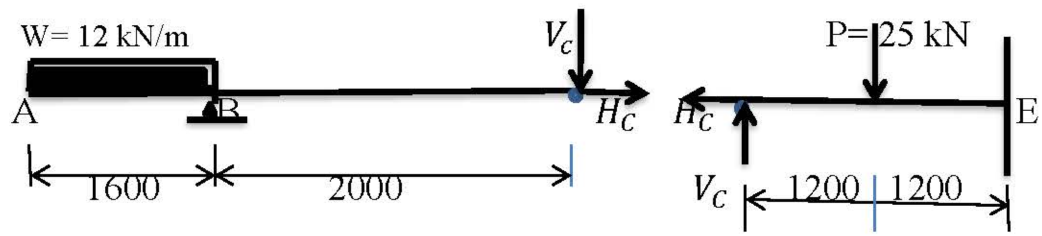


Fig. 3-25



Solution

$\sum M_C = 0$

$= 12\text{x } 1.6\ (\frac{1.6}{2} + 2)\ - R_B \text{ x } 2 = 0$

$R_B = \frac{12\ x\ 1.6}{2} (\frac{1.6}{2} + 2\ ) = 26.88$ kN

$\sum M_B = 0$

$= 12 \quad \frac{(1.6)^2}{2} - V_B \text{ x } 2 = 0$

$V_B = 12 \frac{(1.6)^2}{2x2} = 7.68$ kN

Shear force :

$V_{A=0}$

$V_{B=-}$(12x 1.6) + 26.88 = - 19.2+26.88 = 7.66 kN

$V_{C=} V_{B=}$7.66 kN

$V_{D=} V_{C-}$ P = 7.66 – 25 = -17.32

$V_{E=} V_D$ = - 17.32 KN

Bending moment :

$M_A = 0$

$M_B = - 12 \frac{(1.6)^2}{2} = - 15.36$

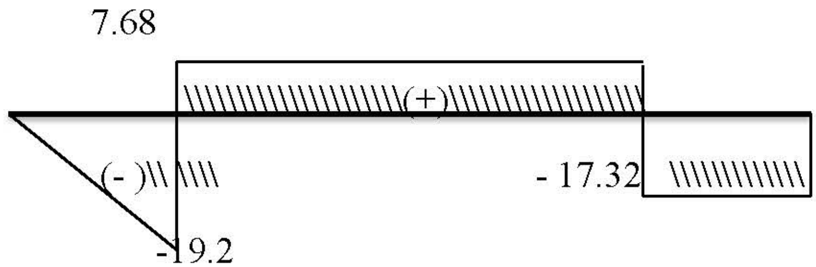
$M_C = 0$ (hinge)

$M_D = V_C$ x 1.2 = 7.68 x 1.2 = 9.216 kN

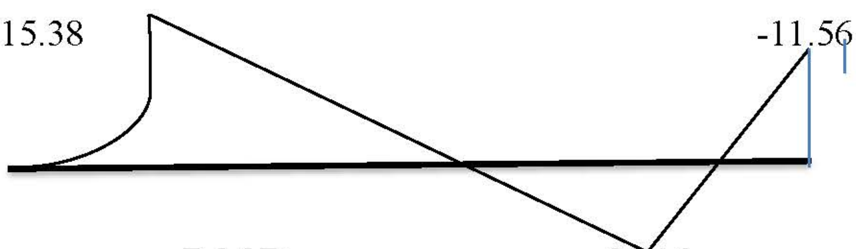
$M_E = V_C\ (1.2 + 1.2) – 25 \text{ x } 1.2$

$= 7.68 \text{ x } 2.4 – 25\text{x } 1.2$

$= - 11.568$ kN.m



S.F.D



B.M.D

3-18. The intensity of loading on a simply supported beam of 6 m span increases gradually from 800 N/m run at one end to 2000 N/m run at the other as shown in Fig. 3.26 below. Find the position and amount of maximum bending moment. Also draw the shear force and bending moment diagram.

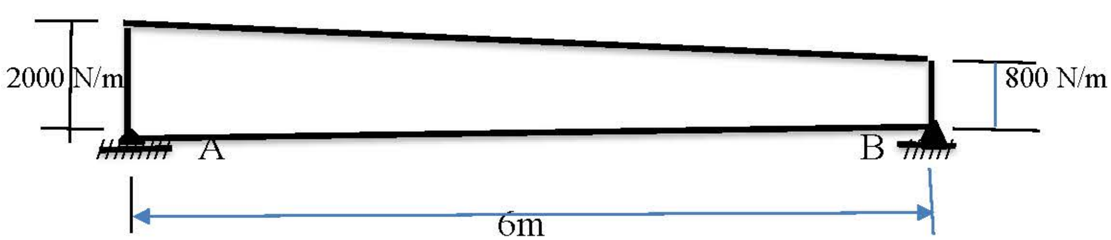


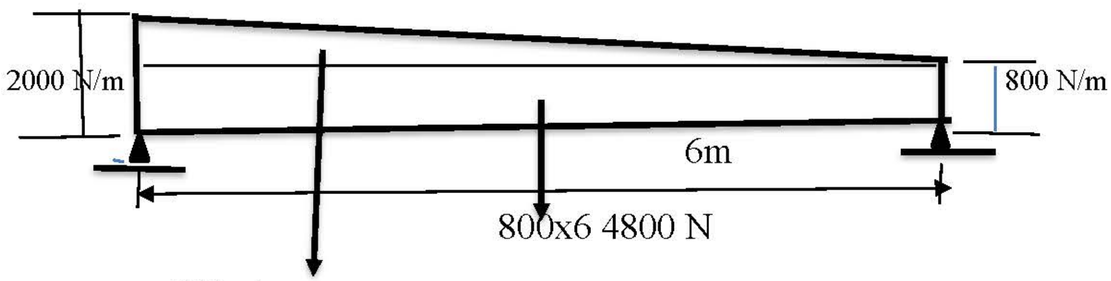
Fig. 2-26

Solution:

Take the moment at B

$R_A = 4800$ N

Take moment at A



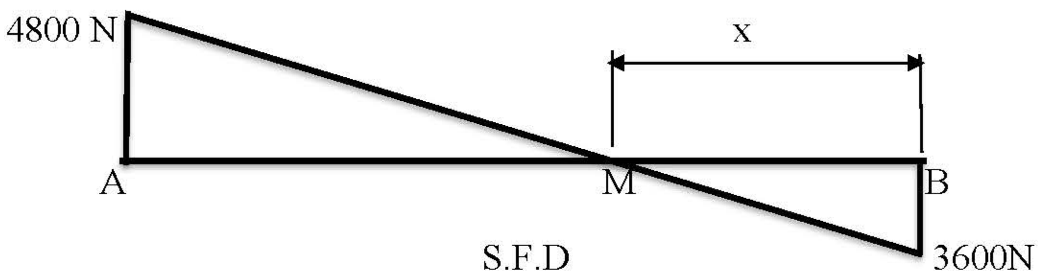
$\frac{1200x\,6}{2}$ = 3600 N

R_A =3600 N   (or by $\sum V = 0$ we get $R_A = 3600$ N)

The shear force diagram:

V_A = + R_A = 4800 N

V_B = - R_B = -3600 N



S.F.D

The bending moment diagram:

Exercises (to be solved):

3-19 Draw the shear force and the bending moment diagrams for the beam subjected to the loads as shown in Fig. 2-27 below.

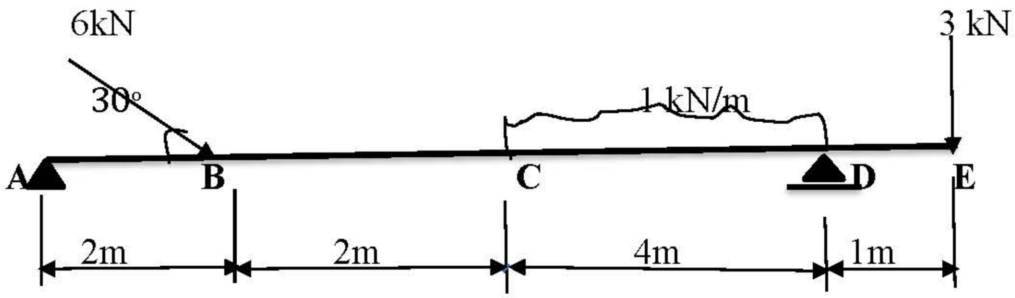


Fig. 2-27

3-20 The shear force diagram for a loaded beam is shown in Fig. 3-28 below. Determine the loading on the beam and draw the bending moment diagram. Also find the location of the point of contreflexur.