Reinforced Concrete Design of Shallow Foundations

Load Factors

According to ACI Code Section 9.2, depending on the type, the ultimate load-carrying capacity of a structural member should be one of the following:

$U = 1.4D$

$U = 1.2D + 1.6L$

where

U = ultimate load-carrying capacity of a member

D = dead loads

L = live loads

Strength Reduction Factor

The design strength provided by a structural member is equal to the nominal strength times a strength reduction factor, ϕ or

Design strength = ϕ (nominal strength)

Following are some of the recommended values of f (ACI Code Section 9.3):

Condition	Value of ϕ
a. Axial tension; flexure with or without axial tension	0.9
b. Shear or torsion	0.75
c. Axial compression with spiral reinforcement	0.75
d. Axial compression without spiral reinforcement	0.65
e. Bearing on concrete	0.65
f. Flexure in plain concrete	0.65

Design Concepts for a Rectangular Section in Bending

Figure A.1a shows a section of a concrete beam having a width b and a depth h. The assumed stress distribution across the section at ultimate load is shown in Figure A.1b.

The following notations have been used in this figure:

f_c' = compressive strength of concrete at 28 days

A_s = area of steel tension reinforcement

f_y = yield stress of reinforcement in tension

d = effective depth

l = location of the neutral axis measured from the top of the compression face $a = \beta l$

$\beta = 0.85$ for f_c' of 28 MN/m^2 of less and decreases at the rate of 0.05 for every 7 MN/m^2 increase of f_c'

However, it cannot be less than 0.65 in any case (ACI Code Section 10.2.7).

$$0.85 f_c' ab = A_s f_y$$

or

$$a = \frac{A_s f_y}{0.85 f_c' b} \qquad (A.2)$$

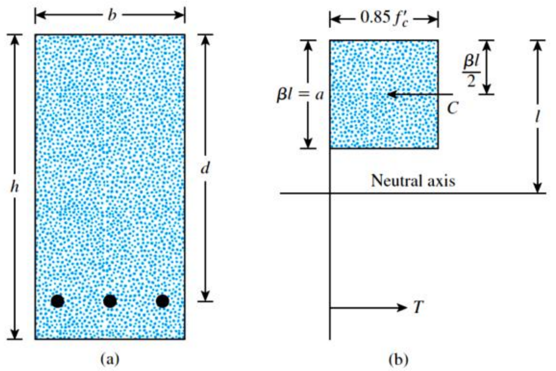


Figure A.1 Rectangular section in heading

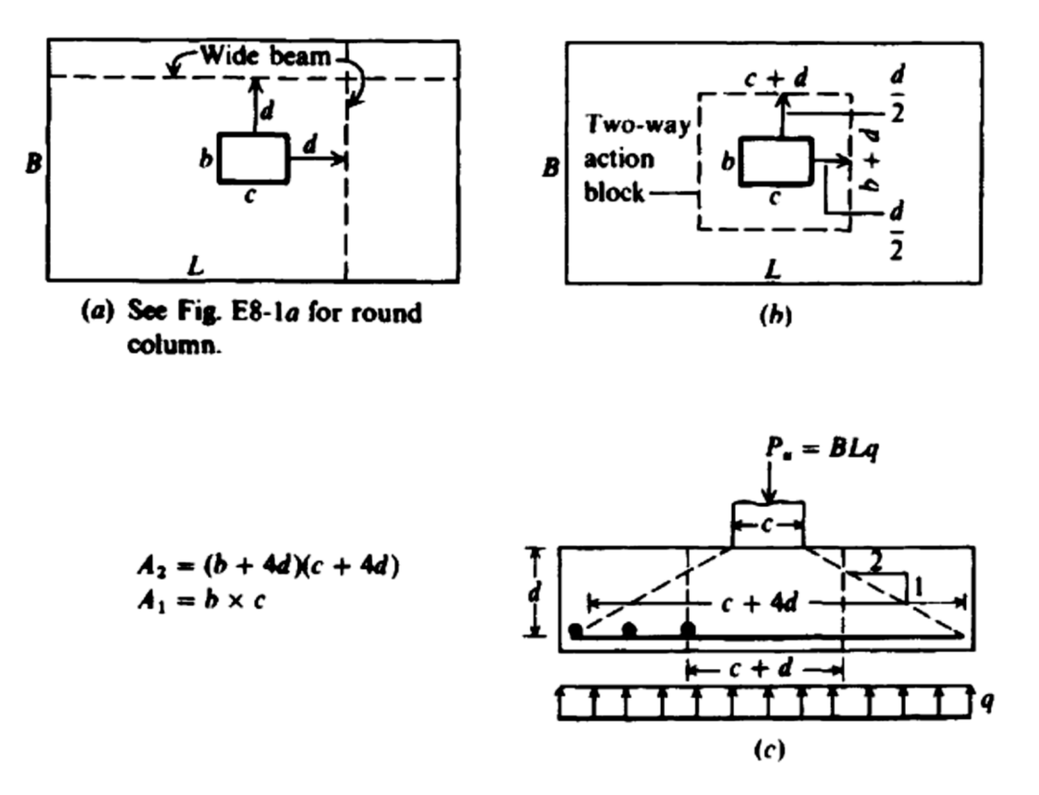


Figure 8-4 *(a) Section for wide-beam shear; (b) section for diagonal-tension shear; (c) method of computing area A2 for allowable column bearing stress.*

Also, for the beam section, the nominal ultimate moment can be given as

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) \qquad \text{(A.3)}$$

where M_n = theoretical maximum resisting moment (Nominal moment).

The steel percentage is defined by the equation

$$\rho = \frac{A_s}{bd}$$

The nominal or theoretical shear strength of a section, V_n, can be given as

$$V_n = V_c + V_s \qquad \text{(A.9)}$$

where V_c = nominal shear strength of concrete

V_s = nominal shear strength of reinforcement

The permissible shear strength, V_u, can be given by

$$V_u = \emptyset V_n = \emptyset\,(V_c + V_s) \qquad \text{(A.10)}$$

The values of V_c can be given by the following equations (ACI Code Sections 11.2 and 11.11).

$$V_c = 0.17\sqrt{f_c'}\ bd \qquad \text{(for member subjected to shear and flexure)} \qquad \text{(A.11a)}$$

And

$$V_c = 0.33\sqrt{f_c'}\ bd \qquad \text{(for member subjected to diagonal tension (punching shear))} \qquad \text{(A.11b)}$$

where f_c' is in MN/m^2 (MPa) , V_c is in MN, b and d are in m, and λ = 1 for normal weight concrete.

Dowels are required if the column load exceeds the following:

$$0.85\,\phi\,f_c'\,A_1\sqrt{A_2/A_1}$$

The ratio $A_2/A_1 < 2$ and the ϕ *factor* is 0.65. The area A_1 is the column contact area *(b* x *c)*; the area A_2 is the base of the frustum that can be placed entirely in the footing as shown in Fig. 8-4c

Design Example of a Square Foundation for a Column

Figure A.3a shows a square column foundation with the following conditions:

Live load = L = 675 kN

Dead load = D = 1125 kN

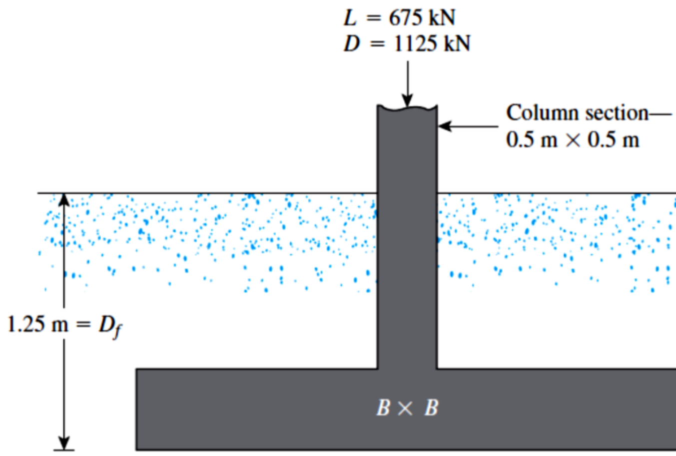
Allowable gross soil-bearing capacity = q_{all} = 145 kN/m^2

Column size = 0.5 m x 0.5 m

f_c' = 20.68 MN/m^2

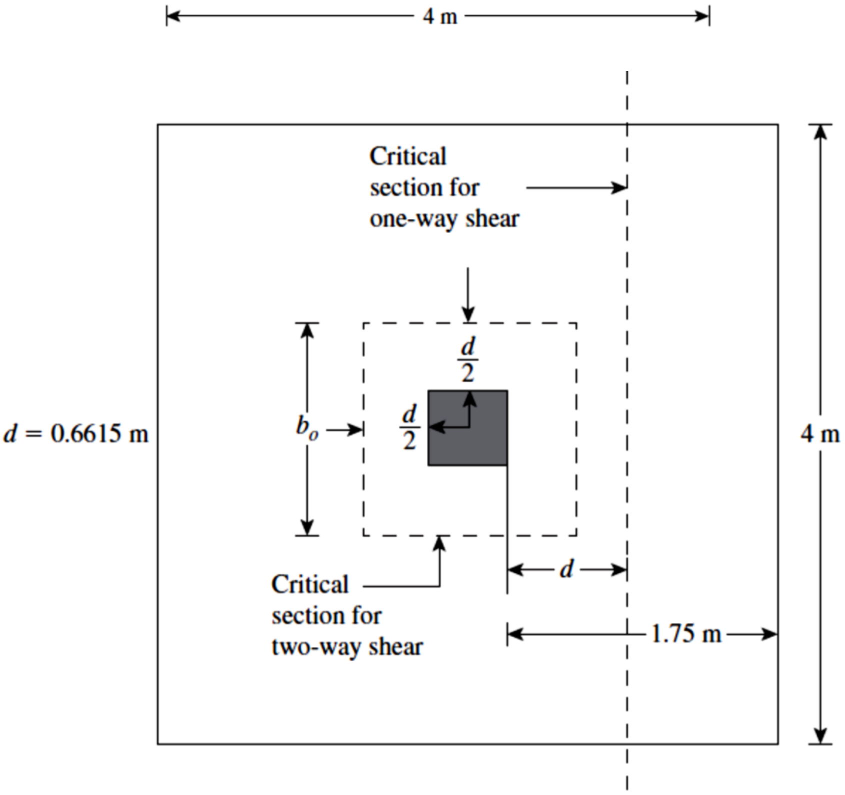
f_y = 413.7 MN/m

Let it be required to design the column foundation.

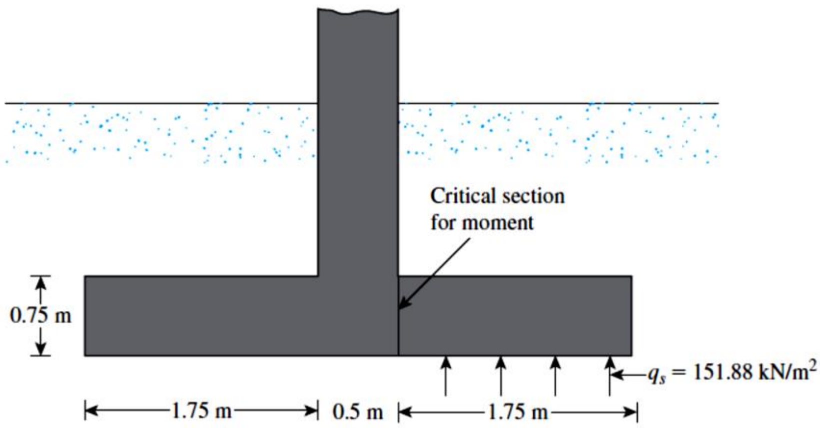


L = 675 kN
D = 1125 kN
Column section—
0.5 m × 0.5 m
1.25 m = D_f
$B \times B$

(a)

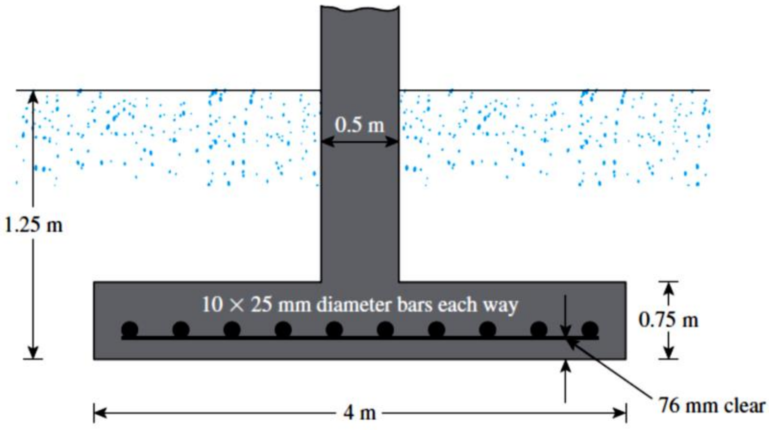


4 m
Critical
section for
one-way shear
$\frac{d}{2}$
$\frac{d}{2}$
b_o
d = 0.6615 m
4 m
d
1.75 m
Critical
section for
two-way shear



Critical section
for moment
0.75 m
q_s = 151.88 kN/m^2
1.75 m
0.5 m
1.75 m

(c)



(d)

General Considerations

Let the average unit weight of concrete and soil above the base of the foundation be 21.97 kN/m^3. So, the net allowable soil-bearing capacity

$q_{\text{all(net)}} = 145 - (D_f)(21.97) = 145 - (1.25)(21.97) = 117.54$ kN/m^2

Hence, the required foundation area is

$$A = B^2 = \frac{D + L}{q_{\text{all(net)}}} = \frac{675 + 1125}{117.54} = 15.31\ \text{m}^2$$

$B = \sqrt{A} = \sqrt{15.31} = 3.91$ m

Use a foundation with dimensions (B) of 4 m x 4 m.

The factored load for the foundation is

$U = 1.2D + 1.6L = (1.2)\ (1125) + (1.6)\ (675) = 2430$ kN

Hence, the factored soil pressure is

$$q_s = \frac{U}{B^2} = \frac{2430}{16} = 151.88\,\text{kN/m}^2$$

Assume the thickness of the foundation to be equal to 0.75 m. With a clear cover of 76 m over the steel bars and an assumed bar diameter of 25 mm, we have

$$d = 0.75 - 0.076 - \frac{0.025}{2} = 0.6615\,\text{m}$$

Check for Shear

V_u should be equal to or less than ϕVc

$$V_u \leq \phi(0.17)\sqrt{f'_c}\,bd$$

The critical section for one-way shear is located at a distance d from the edge of the column as shown in Figure A.3b. So

$V_u = q_s$ x critical area = (151.88) (4) (1.75 - 0.6615) = 661.3 kN

Also

$\phi V_c = (0.75)(0.17)\sqrt{20.68}\ (4)\ (0.6615)\ (1000) = 1534.2$ kN

So,

$V_u = 661.3$ kN $< \phi Vc = 1534.2$ kN —O.K.

For two-way shear, the critical section is located at a distance of $d/2$ from the edge of the column. This is shown in Figure A.3b. For this case,

$$\phi V_c = \phi(0.33)\sqrt{f_c'}b_o d$$

The term b_o is the perimeter of the critical section for two-way shear. Or for this design,

$b_o = 4[0.5 + 2(d/2)] = 4[0.5 + 2(0.3308)] = 4.65$ m

Hence,

$\phi Vc = (0.75)\ (0.33)\sqrt{20.68}\ (4.65)\ (0.6615) = 3.462$ MN = 3462 kN

Also,

$V_u = (q_s)$(critical area)

Critical area = (4 x 4) - $(0.5 + 0.6615)^2 = 14.65$ m^2

So,

$V_u = (151.88)\ (14.65) = 2225.18$ kN

$V_u = 2225.18$ kN $< \phi Vc = 3462$ kN — O.K.

The assumed depth of foundation is more than adequate.

Flexural Reinforcement

According to Figure A.3c, the moment at critical section (ACI Code Section 15.4.2) is

$$M_u = (q_s B)\left(\frac{1.75}{2}\right)^2 = \frac{[(151.88)(4)](1.75)^2}{2} = 930.27 \text{ kN-m}$$

$\phi M_n \geq M_u$

$M_n = \dfrac{M_u}{\emptyset}$

$M_n = 930.27/\ 0.9 = 1033.6$ kN.m = 1033x 10^6 N.mm

Try $\alpha = 0.2$ d = 0.2 x 660= 132 mm

$M_n = \text{As}\ f_y\ (d - \frac{a}{2})$

1033 x 10^6 = A_s x 413.7 x (660 – 132/2)

A_s = 4203 mm^2

$$\rho = \frac{A_s}{bd} = \frac{4203}{4000x\ 660} = 0.00152$$

ρ_{min} for footing = 0.0018 > 0.00152

∴ use ρ_{min}

Note: Use total footing thickness (h) when using ρ_{min}

∴ $A_s = \rho_{min}\ x\ b\ x\ h$ = 0.0018 x 4000 x 750 =5400 mm^2

Provide 11 X 25-mm diameter bars each way [As = (491) (11) = 5401 mm^2].

Check for Development Length (L_d)

From ACI Code Section 12.2.2, for 25 mm diameter bars, $L_d \approx 50\phi \approx 1250$ mm. Actual L_d provided is (4 - 0.5/2) - 0.076 (cover) = 1.674 m > 1.25 m — O.K.

Check for Bearing Strength

ACI Code Section 10.14 indicates that the bearing strength should be at least $0.85\ \phi\ f_c'\ A_1 \sqrt{A_2/A_1}$ with a limit of $\sqrt{A_2/A_1} \leq 2$

for this problem,

$A_2 = (b + 4d)(c + 4d)$ = (0.5 + 4x0.66) (0.5+4x0.66) = 9.86

$A_1 = (b\ x\ c)$ = 0.5 x 0.5 = 0.25 m^2

$\sqrt{A_2/A_1} = \sqrt{9.86/0.25} = 6.28 > 2$

Take $\sqrt{A_2/A_1} = 2$

Bearing strength = $0.85\ \phi\ f_c'\ A_1 \sqrt{A_2/A_1}$ = (0.85)(0.65)(20.68)(0.5)^2 (2) = 5.71 MN = 5712 kN

The factored column load U = 2430 kN < 5712 kN — O.K.

Design Example of a Rectangular Foundation for a Column

This section describes the design of a rectangular foundation to support a column having dimensions of 0.4 m x 0.4 m in cross section. Other details are as follows:

Dead load = D = 290 kN

Live load = L = 110 kN

Depth from the ground surface to the top of the foundation = 1.2 m

Allowable gross soil-bearing capacity = 120 kN/m^2

Maximum width of foundation = B = 1.5 m

f_y = 413.7 MN/m^2

f_c' = 20.68 MN/m^2

Unit weight of soil = γ = 17.27 kN/m^3

Unit weight of concrete = γ_c = 22.97 kN/m^3

General Considerations

For this design, let us assume a foundation thickness of 0.45 m (Figure A.4a). The weight of foundation/m^2 = 0.45 γ_c = (0.45) (22.97) = 10.34 kN/m^2,

and the weight of soil above the foundation/m^2 = (1.2)γ = (1.2) (17.27) = 20.72 kN/m^2.

Hence, the net allowable soil bearing capacity [$q_{net(all)}$] = 120 - 10.34 - 20.72 = 88.94 kN/m^2.

The required area of the foundation A = (D + L)/ $q_{net(all)}$ = (290 + 110) / 88.94 = 4.5 m^2.

Hence, the length of the foundation is A / B = 4.5 m^2/ B = 4.5/1.5 = 3 m.

The factored column load = 1.2D + 1.6L = 1.2(290) + 1.6(110) = 524 kN.

The factored soil-bearing capacity, q_s = factored load/foundation area = 524/4.5 = 116.44 kN/m^2.

Shear Strength of Foundation

Assume that the steel bars to be used have a diameter of 16 mm. So, the effective depth

d = 450 - 76 – 16/2 = 366 mm. (Note that the assumed clear cover is 76 mm.)

Figure A.4a shows the critical section for one-way shear. According to this figure

$$V_u = \left(1.5 - \frac{0.4}{2} - 0.366\right)Bq_s = (0.934)(1.5)(116.44) = 163.13 \text{ kN}$$

The nominal shear capacity of concrete for one-way beam action

$V_c = 0.17 \sqrt{f_c'}\, Bd = 0.17 \sqrt{20.68}$ (1.5) (0.366) = 0.4244 MN = 424.4 kN

Now

V_u = 163.13 < ϕ V_c = (0.75) (424.4) = 318.3 kN — O.K.

The critical section for two-way shear is also shown in Figure A.4a. This is based on the recommendations given by ACI Code. For this section

$V_u = q_s[(1.5)(3) – 0.766^2]$ = 455.66 kN

The nominal shear capacity of the foundation can be given as

$V_c = 0.33 \sqrt{f_c'}\, b_o d$

where b_o = perimeter of the critical section

or

$V_c = (0.33) \sqrt{20.68}$ (4x 0.766) (0.366) = 1.683 MN

So, for two-way shear condition

V_u = 455.66 kN < ϕ Vc = (0.75) (1683) = 1262.25 kN

Therefore, the section is adequate.

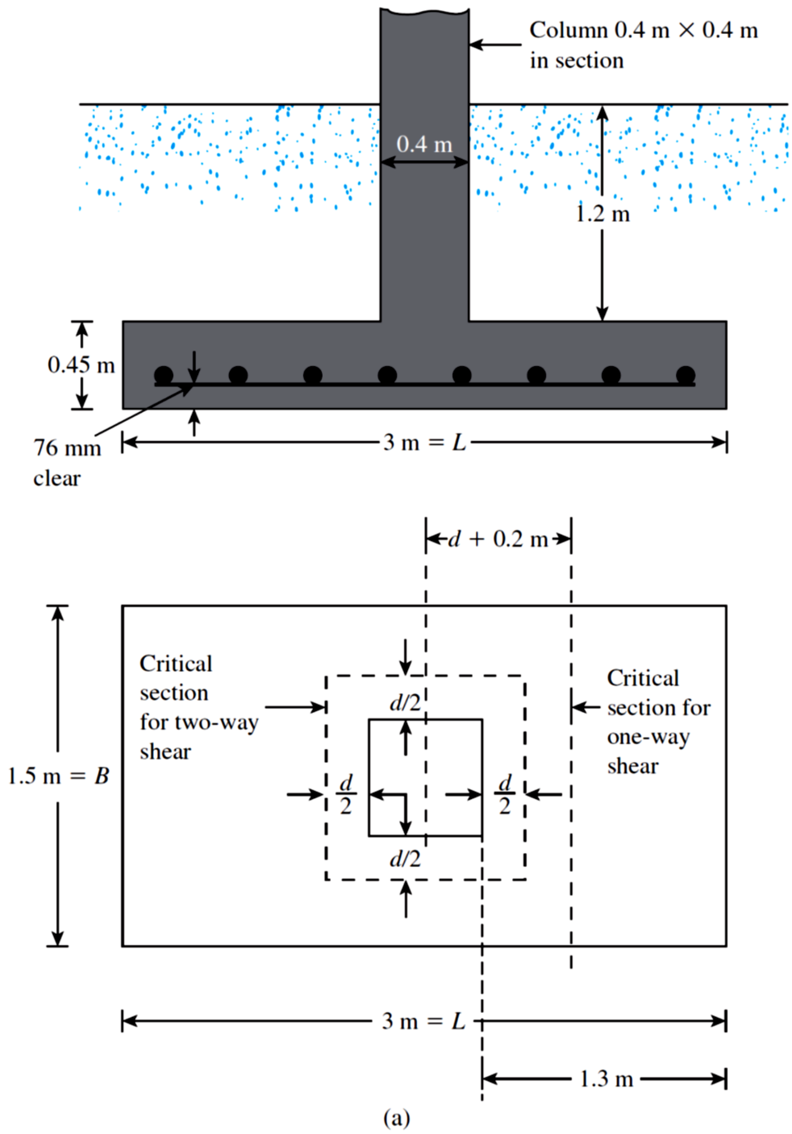


Figure A.4 Rectangular foundation for a column

According to Figure A.4a, the design moment about the column face is

$$M_u = \frac{(q_s B)1.3^2}{2} = \frac{(116.44)(1.5)(1.3)^2}{2} = 147.59 \text{ kN-m} \qquad = 147.59 \times 10^6 \text{ N.mm}$$

$$M_u = \phi M_n = \phi A_s f_y \left(d - \frac{a}{2}\right)$$

Try $a = 0.2\, d = 0.2 \times 0.366 = 0.073$ m = 73 mm

$147.59 \times 10^6 = 0.9 \times A_s \times 413.7 \times (366 - 73/2)$

$A_s = 1203 \text{ mm}^2$

$\rho = A_s / (B\, d) = 1203/(1500 \times 366) = 0.0219 > \rho_{min}$ O.K

use 6 No. 16 mm bars.

Flexural Reinforcement in the Short Direction

According to Figure A.4a, the moment at the face of the column is

$$M_u = \frac{(q_s L)(0.55)^2}{2} = \frac{(116.44)(3)(0.55)^2}{2} = \mathbf{52.83\,kN\text{-}m} \quad = 52.83\text{x } 10^6 \text{ N.mm}$$

d in short direction = 450 -76 -16 -16/2 = 350 mm

$$M_u = \phi M_n = \phi A_s f_y \left(d - \frac{a}{2}\right)$$

Try a = 0.2 d = 0.2 x 0.35 = 0.07 m = 70 mm

52.83 x10^6 = 0.9 x A_s x 413.7 x (350 – 70/2)

A_s = 450 mm^2

ρ = A_s / (B d) = 450/ (3000 x 350) = 0.0004< ρ_{min}

use ρ_{min}

Important note: Use gross area when using ρ_{min}

A_s = 0.0018 x3000x 450 =2430 mm^2

Use 12 No. 16 mm bars