Lateral Earth Pressure

9.1 Introduction

Vertical or near-vertical slopes of soil are supported by retaining walls, cantilever sheet pile walls, braced cuts, and other, similar structures. The proper design of those structures requires an estimation of lateral earth pressure, which is a function of several factors, such as:

(a) The type and amount of wall movement,

(b) The shear strength parameters of the soil,

(c) The unit weight of the soil, and

(d) The drainage conditions in the backfill.

Three types of lateral earth pressure can be discussed below,

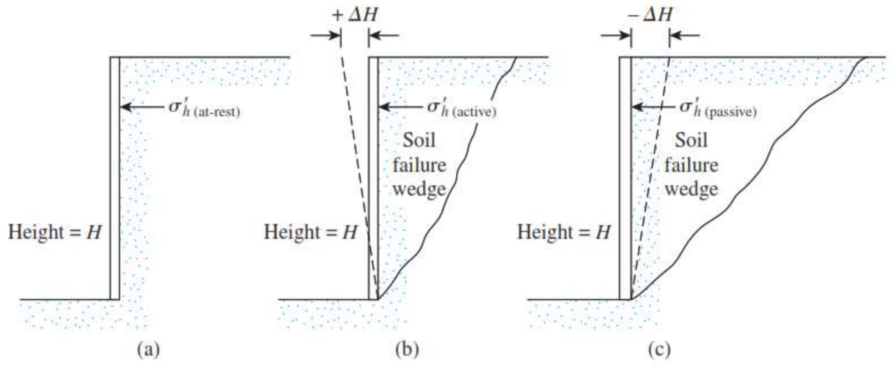


Figure 9.1 Nature of lateral earth pressure on a retaining wall

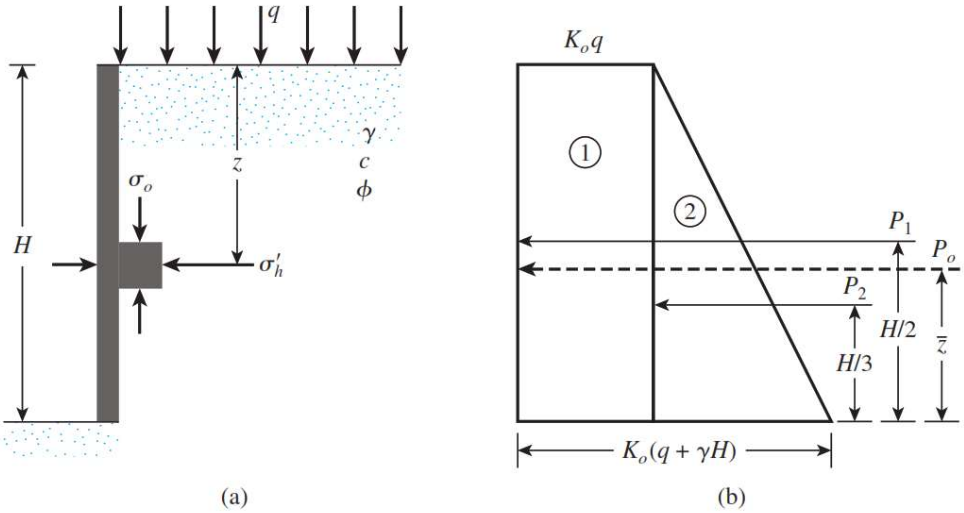
a. The wall may be restrained from moving (Figure 9.1a). The lateral earth pressure on the wall at any depth is called the *at-rest earth pressure*.

b. The wall may tilt away from the soil that is retained (Figure 9.1b). With sufficient wall tilt, a triangular soil wedge behind the wall will fail. The lateral pressure for this condition is referred to as *active earth pressure.*

c. The wall may be pushed into the soil that is retained (Figure 9.1c). With sufficient wall movement, a soil wedge will fail. The lateral pressure for this condition is referred to *as passive earth pressure.*

9.2 Lateral Earth Pressure at Rest

Consider a vertical wall of height H, as shown in Figure 9.2, retaining a soil having a unit weight of, γ. A uniformly distributed load, q/unit area, is also applied at the ground surface.



At-rest earth pressure

The shear strength of the soil is

$$s = c + \sigma \tan \emptyset$$

Where:

c = Cohesion

$\emptyset$ = Effective angle of friction

σ = Effective normal stress

At any depth (z) below the ground surface, the vertical subsurface stress is

$$\sigma_0 = q + \gamma z$$

If the wall is at rest and is not allowed to move at all, either away from the soil mass or into the soil mass, the lateral pressure at a depth (z) is

$$\sigma_h = K_0 \sigma_0 + u$$

<u>Where</u>

u = pore water pressure

K_0 = coefficient of at-rest earth pressure

For *<u>normally consolidated soil</u>*, the relation for K_0 (Jaky, 1944) is

$$K_o = 1 - \sin \emptyset$$

For *<u>overconsolidated soil</u>*, the at-rest earth pressure coefficient may be expressed as (Mayne and Kulhawy, 1982)

$$K_o = (1 - \sin \emptyset) OCR^{\sin \emptyset}$$

Where OCR= overconsolidated ratio

The lateral earth pressure effect on cantilever wall shown above can be calculated as follow.

$$P_o = P_1 + P_2 = qK_oH + \frac{1}{2}\gamma H^2 K_o$$

Where

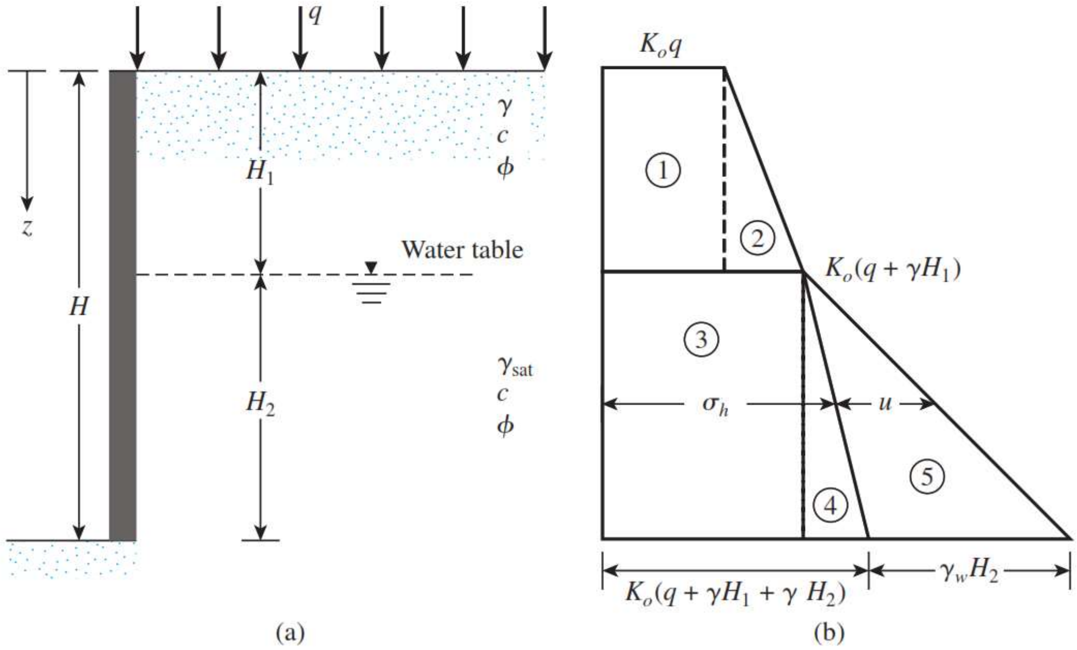
P1 =area of rectangle 1

P2= area of triangle 2

The location of the line of action of the resultant force, P_o, can be obtained by taking the moment about the bottom of the wall. Thus,

$$\bar{z} = \frac{P_1\left(\frac{H}{2}\right) + P_2\left(\frac{H}{3}\right)}{P_o}$$

If the water table is located at a depth $z < H$, the at-rest pressure diagram shown in Figure 9.3a will have to be somewhat modified, as shown in Figure 9.3 b



At-rest earth pressure with water table located at a depth $z < H$

$at\ z = 0\ ,$

$\sigma_h = K_o\sigma_o = K_o q$

$at\ z = H_1\ , \qquad \sigma_h = K_o\sigma_o = K_o(q + \gamma H_1)$

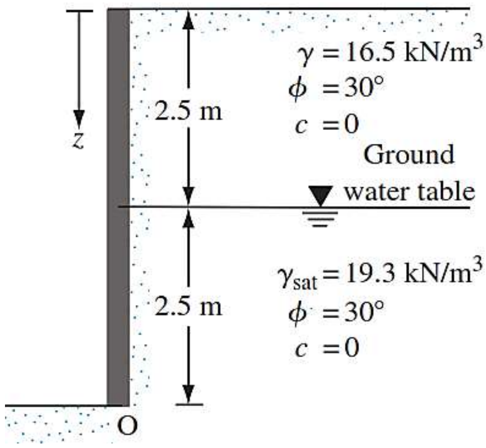
$at\ z = H_2\ , \qquad \sigma_h = K_o\sigma_o = K_o(q + \gamma H_1 + \gamma' H_2)$

$P_o = A_1 + A_2 + A_3 + A_4 + A_5$

Where A= area of the pressure diagram

$$P_o = K_o q H_1 + \frac{1}{2}\gamma H_1^2 K_o + K_o(q + \gamma H_1) + \frac{1}{2}\gamma' H_2^2 K_o + \frac{1}{2}\gamma_w H_2^2$$

Example: For the retaining wall shown in Figure, determine the lateral earth force at rest per unit length of the wall. Also, determine the location of the resultant force. Assume OCR = 1.



Solution:

$K_\circ = 1 - \sin \emptyset = 1 - \sin 30 = 0.5$

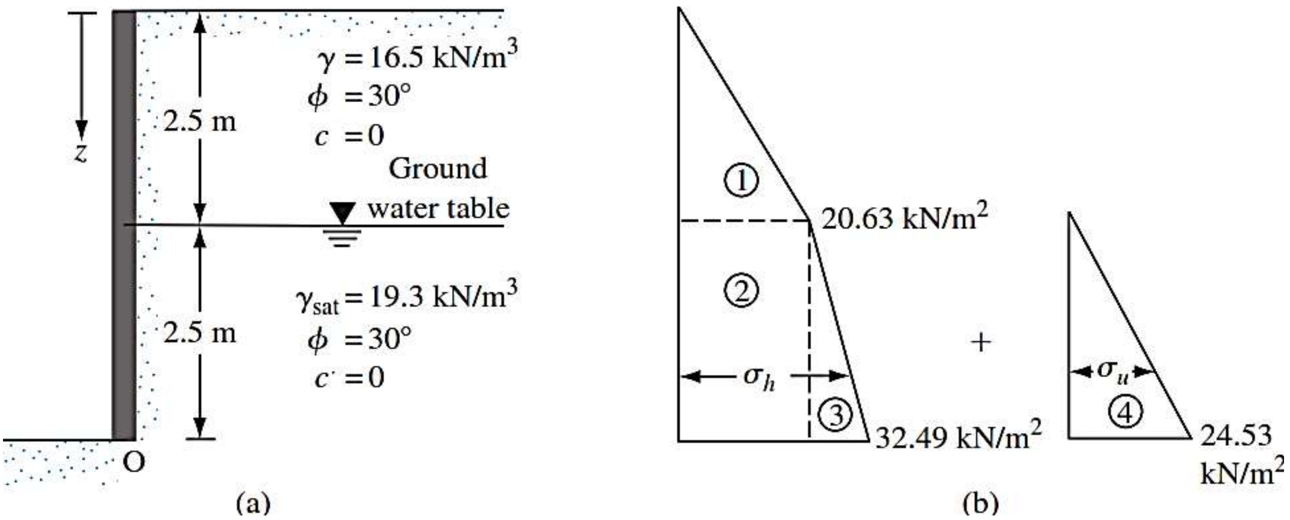
Z (m)	$\sigma_\circ$ (kN/m^2)	σ_h (kN/m^2)
0	0	0
2.5	$\gamma H = 16.5 \times 2.5 = 41.25$	$K_0 \gamma H = 0.5 \times 16.5 \times 2.5 = 20.63$
5	$16.5 \times 2.5 + (19.3 - 9.81) \times 2.5 = 64.98$	$0.5 \times 64.98 = 32.49$

The hydraulic pressure can be calculated as follow.

At Z= 0.0 to 2.5 m u=0.0

From Z= 2.5m to 5 m

$u = H\gamma_w = 2.5 \times 9.81 = 24.53\, kN/m^2$



The total force per unit length of wall can be determined from the area of pressure diagram,

$$P_o = A_1 + A_2 + A_3 + A_4$$

$$= \frac{1}{2} \times 2.5 \times 20.63 + 2.5 \times 20.63 + \frac{1}{2} \times 2.5 \times (32.49 - 20.63) + \frac{1}{2} \times 2.5 \times 24.53$$

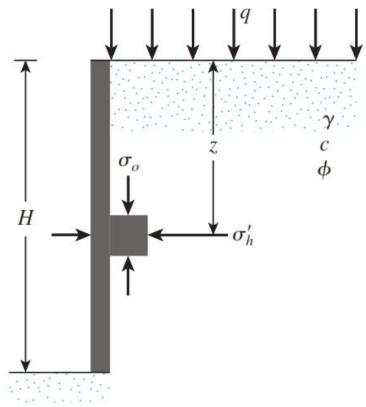
$$= 122.85\ kN/m$$

The location of center of pressure measured from the bottom of the wall (o)

$$\bar{z} = \frac{A_1\left(2.5 + \frac{2.5}{3}\right) + A_2\left(\frac{2.5}{2}\right) + (A_3 + A_4)\left(\frac{2.5}{3}\right)}{P_o}$$

$$\bar{z} = \frac{25.788\left(2.5 + \frac{2.5}{3}\right) + 51.575\left(\frac{2.5}{2}\right) + (14.825 + 30.663)\left(\frac{2.5}{3}\right)}{122.85} = 1.53\ m$$

Example: For the retaning wall shown in Figure.



Given: $H = 3.6m, q = 0, \gamma = 17.5\ kN/m^3, c = 0\ and\ \emptyset = 30^\circ$

Determine the at-rest lateral earth force per meter length of the wall. Also, find the location of the resultant. Use OCR= 2

Solution

$$K_o = (1 - \sin\emptyset)\text{OCR}^{\sin\emptyset} = (1 - \sin 30)\, 2^{\sin 30} = 0.707$$

$$@\ z = 0 \ \rightarrow \sigma_o = 0\ and\ \sigma_h = 0$$

$$@\ z = 3.6 \ \rightarrow \sigma_o = \gamma H = 17.5 \times 3.6 = 63\ \ kN/m^2$$

$$and\ \sigma_h = K_o \sigma_o = 0.707 \times 63 = 44.54\ \ kN/m^2$$

$$P_o = \frac{1}{2} \times K_o \gamma H^2 = \frac{1}{2} \times 0.707 \times 17.5 \times 3.6^2 = 80.17\ kN/m$$

9.3 Active Pressure

For design purposes, two classical lateral earth pressure theories are commonly used to estimate active and passive earth pressures:

- Rankine's theory (1857) relies on calculating the earth pressure coefficients based on the Mohr–Coulomb shear strength of the backfill soil.
- Coulomb's theory, on the other hand, dates back to the 18th century (Coulomb, 1776)and considers the stability of a soil wedge behind a retaining wall.

9.3.1 Rankine Active Earth Pressure

The lateral earth pressure described in Section 9.2 involves walls that do not yield at all. However, if a wall tends to move away from the soil a distanceΔx, as shown in Figure 9.3 a, the soil pressure on the wall at any depth will decrease. For a wall that is frictionless, the horizontal stress, σ_h, at depth z will equal $K_o\sigma_o$ when Δx is zero. However, with $\Delta x > 0$, σ_h, will be less than $K_o\sigma_o$.

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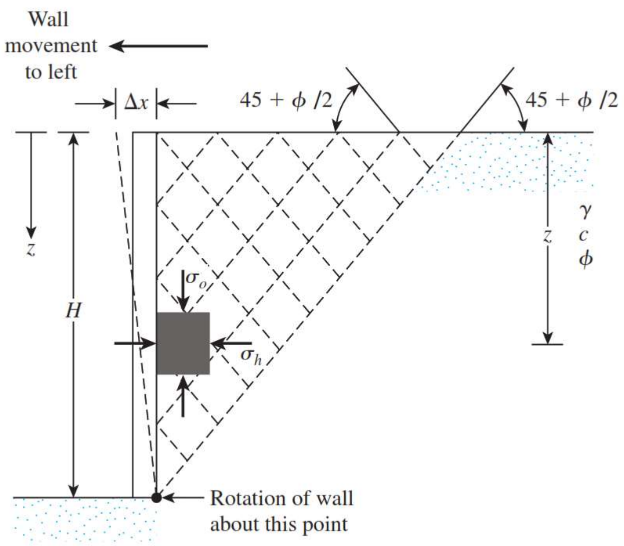


(a)

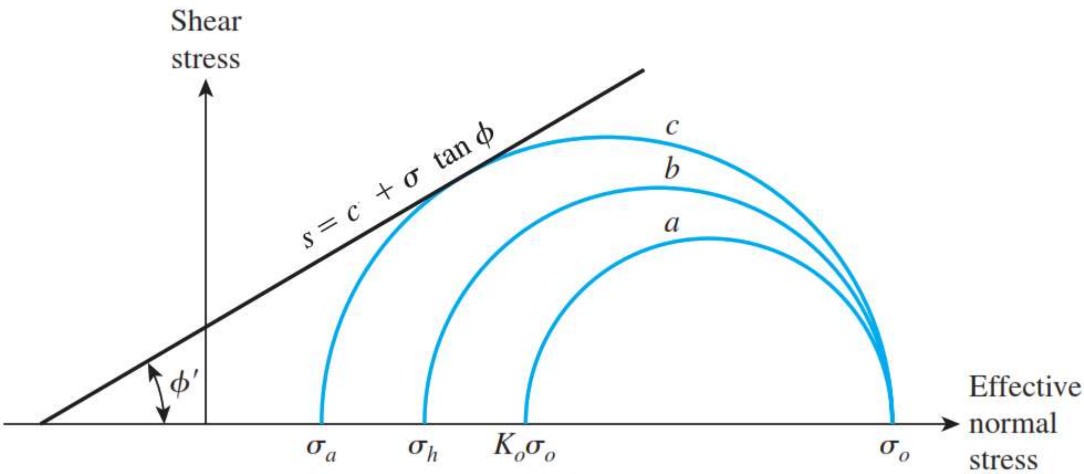
The Mohr's circles corresponding to wall displacements where:

Circles (a) for Δx = 0

and circles (b) for Δx > 0 as shown in Figure 9.3 b.

If the displacement of the wall, Δx, continues to increase, the corresponding Mohr's circle eventually will just touch the Mohr–Coulomb failure envelope defined by the equation.

$$s = c + \sigma \tan \emptyset$$



(b)

This circle, marked c in the figure, represents the failure condition in the soil mass; the horizontal stress then equals σ_a , referred to as the ***Rankine active pressure***. The slip lines (failure planes) in the soil mass will then make angles of $\left(45 \mp \frac{\emptyset}{2}\right)$with the horizontal, as shown in Figure 9.3 a.

Rankine active pressure can be calculated as follow based on principal stresses for a Mohr's circle that touches the Mohr–Coulomb failure envelope:

$$\sigma_o = \sigma_a\, tan^2\left(45 + \frac{\emptyset}{2}\right) + 2c\tan\left(45 + \frac{\emptyset}{2}\right)$$

$$\sigma_a = \frac{\sigma_o}{tan^2\left(45 + \frac{\emptyset}{2}\right)} + \frac{2c}{\left(45 + \frac{\emptyset}{2}\right)}$$

$$\sigma_a = \sigma_o\, tan^2\left(45 - \frac{\emptyset}{2}\right) - 2c\tan\left(45 - \frac{\emptyset}{2}\right)$$

$$\sigma_a = \sigma_o\, K_a - 2c\sqrt{K_a}$$

Where: $K_a = tan^2\left(45 - \frac{\emptyset}{2}\right) = Rankine\ active - pressure\ coefficient.$

The variation of the active pressure with depth for the wall shown in is given in Figure 9.3 c.

Note that $\sigma_o = 0$ at z $= 0$ and $\sigma_o = \gamma H$ at $z = H$.

The pressure distribution shows that at $z = 0$ the active pressure equals

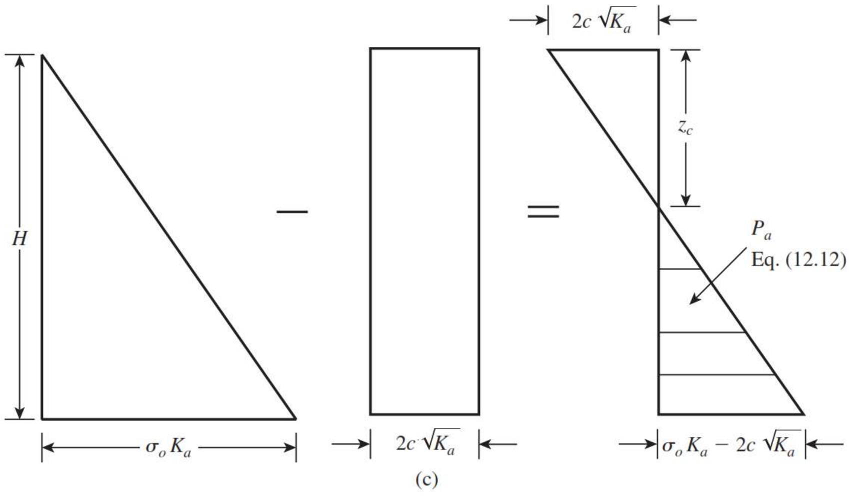
$$-2c\sqrt{K_a}$$

, indicating a tensile stress that decreases with depth and becomes zero at a depth $z = zc$, or

$$\gamma z_c\, K_a - 2c\sqrt{K_a} = 0 \qquad \rightarrow z_c = \frac{2c\sqrt{K_a}}{\gamma\, K_a}$$

Where:

$The\ depth\ zc\ is\ usually\ referred\ to\ as\ the\ depth\ of\ tensile\ crack, because\ the\ ter$
$stress\ in\ the\ soil\ will\ eventually\ cause\ a\ crack\ along\ the\ soil–wall\ interface$



Rankine active pressure

Thus, the total Rankine active force per unit length of the wall before the tensile crack occurs is:

$$P_a = \frac{1}{2}(H - z_c)\left(\gamma H K_a - 2c\sqrt{K_a}\right)$$

Or

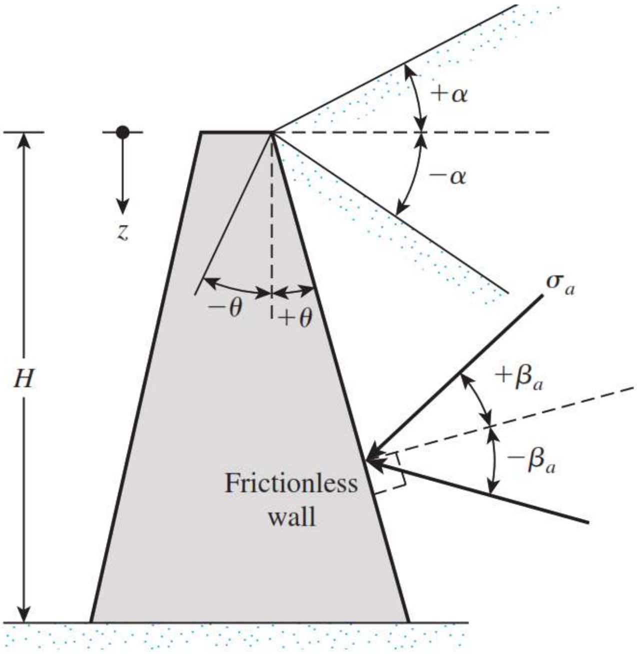
$$P_a = \frac{1}{2}\left(H - \frac{2c}{\gamma\, K_a}\right)\left(\gamma H K_a - 2c\sqrt{K_a}\right)$$

However, it is important to realize that the active earth pressure condition will be reached only if the wall is allowed to “yield” sufficiently. The necessary amount of outward displacement of the wall is about 0.001H to 0.004H for granular soil backfills and about 0.01H to 0.04H for cohesive soil backfills

9.3.2 A Generalized Case for Rankine Active Pressure—Granular Backfill

In previous section, the relationship was developed for Rankine active pressure for a retaining wall with a vertical back and a horizontal backfill.

Figure 9.4 shows a retaining wall whose back is inclined at an angle θ with the vertical. The granular backfill is inclined at an angle α with the horizontal.



General case for a retaining wall with granular backfill

For a Rankine active case, the lateral earth pressure σ_a at a depth z can be given as (Chu, 1991)

$$\sigma_a = \frac{\gamma z \cos\alpha \sqrt{1 + sin^2\emptyset - 2\sin\emptyset\cos\psi_a}}{\cos\alpha + \sqrt{sin^2\emptyset - sin^2\alpha}}$$

$$\psi_a = sin^{-1}\left(\frac{\sin\alpha}{\sin\phi}\right) - \alpha + 2\theta$$

$$\beta_a = tan^{-1}\left(\frac{\sin\phi\ \sin\psi_a}{1 - \sin\phi\ \cos\psi_a}\right)$$

$$P_a = \frac{1}{2}\gamma H^2 K_a$$

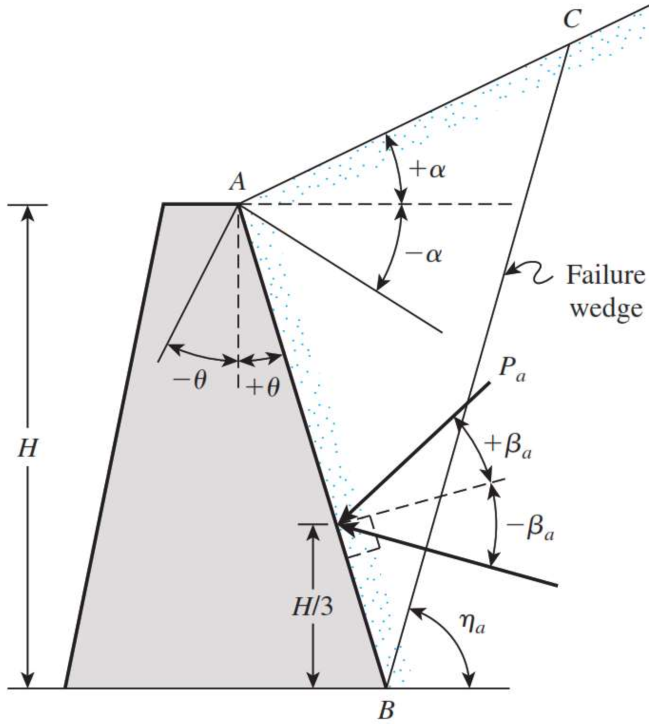
Where

$$K_a = \frac{\cos(\alpha-\theta)\sqrt{1+sin^2\emptyset - 2\sin\emptyset\cos\psi_a}}{cos^2\theta(\cos\alpha + \sqrt{sin^2\emptyset - sin^2\alpha})}$$

= rankine aktive earth pressure coefficent for generalized case

The location and direction of the resultant force Pa is shown in Figure 9.5. Also shown in this figure is the failure wedge, ABC. Note that BC will be inclined at an angle(η). Or

$$\eta_a = \frac{\pi}{4} + \frac{\emptyset}{2} + \frac{\alpha}{2} - \frac{1}{2} sin^{-1}\left(\frac{\sin\alpha}{\sin\emptyset}\right)$$



$$\eta_a = \frac{\pi}{4} + \frac{\phi}{2} + \frac{\alpha}{2} - \frac{1}{2}\sin^{-1}\left(\frac{\sin\alpha}{\sin\phi}\right)$$

Tables 9.1 and 9.2 give the variations of Ka and β_a for various values of α, θ and $\emptyset$

Variation of $K_{a(R)}$

α (deg)	θ (deg)	$K_{a(R)}$ ϕ (deg) 28	30	32	34	36	38	40
0	0	0.361	0.333	0.307	0.283	0.260	0.238	0.217
	2	0.363	0.335	0.309	0.285	0.262	0.240	0.220
	4	0.368	0.341	0.315	0.291	0.269	0.248	0.228
	6	0.376	0.350	0.325	0.302	0.280	0.260	0.242
	8	0.387	0.362	0.338	0.316	0.295	0.276	0.259
	10	0.402	0.377	0.354	0.333	0.314	0.296	0.280
	15	0.450	0.428	0.408	0.390	0.373	0.358	0.345
5	0	0.366	0.337	0.311	0.286	0.262	0.240	0.219
	2	0.373	0.344	0.317	0.292	0.269	0.247	0.226
	4	0.383	0.354	0.328	0.303	0.280	0.259	0.239
	6	0.396	0.368	0.342	0.318	0.296	0.275	0.255
	8	0.412	0.385	0.360	0.336	0.315	0.295	0.276
	10	0.431	0.405	0.380	0.358	0.337	0.318	0.300
	15	0.490	0.466	0.443	0.423	0.405	0.388	0.373
10	0	0.380	0.350	0.321	0.294	0.270	0.246	0.225
	2	0.393	0.362	0.333	0.306	0.281	0.258	0.236
	4	0.408	0.377	0.348	0.322	0.297	0.274	0.252
	6	0.426	0.395	0.367	0.341	0.316	0.294	0.273
	8	0.447	0.417	0.389	0.363	0.339	0.317	0.297
	10	0.471	0.441	0.414	0.388	0.365	0.344	0.324
	15	0.542	0.513	0.487	0.463	0.442	0.422	0.404
15	0	0.409	0.373	0.341	0.311	0.283	0.258	0.235
	2	0.427	0.391	0.358	0.328	0.300	0.274	0.250
	4	0.448	0.411	0.378	0.348	0.320	0.294	0.271
	6	0.472	0.435	0.402	0.371	0.344	0.318	0.295
	8	0.498	0.461	0.428	0.398	0.371	0.346	0.323
	10	0.527	0.490	0.457	0.428	0.400	0.376	0.353
	15	0.610	0.574	0.542	0.513	0.487	0.463	0.442
20	0	0.461	0.414	0.374	0.338	0.306	0.277	0.250
	2	0.486	0.438	0.397	0.360	0.328	0.298	0.271
	4	0.513	0.465	0.423	0.386	0.353	0.323	0.296
	6	0.543	0.495	0.452	0.415	0.381	0.351	0.324
	8	0.576	0.527	0.484	0.446	0.413	0.383	0.355

Variation of β_a'

α (deg)	θ (deg)	β_a ϕ (deg) 28	30	32	34	36	38	40
	0	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	2	3.525	3.981	4.484	5.041	5.661	6.351	7.124
	4	6.962	7.848	8.821	9.893	11.075	12.381	13.827
0	6	10.231	11.501	12.884	14.394	16.040	17.837	19.797
	8	13.270	14.861	16.579	18.432	20.428	22.575	24.876
	10	16.031	17.878	19.850	21.951	24.184	26.547	29.039
	15	21.582	23.794	26.091	28.464	30.905	33.402	35.940
	0	5.000	5.000	5.000	5.000	5.000	5.000	5.000
	2	8.375	8.820	9.311	9.854	10.455	11.123	11.870
	4	11.553	12.404	13.336	14.358	15.482	16.719	18.085
5	6	14.478	15.679	16.983	18.401	19.942	21.618	23.441
	8	17.112	18.601	20.203	21.924	23.773	25.755	27.876
	10	19.435	21.150	22.975	24.915	26.971	29.144	31.434
	15	23.881	25.922	28.039	30.227	32.479	34.787	37.140
	0	10.000	10.000	10.000	10.000	10.000	10.000	10.000
	2	13.057	13.491	13.967	14.491	15.070	15.712	16.426
	4	15.839	16.657	17.547	18.519	19.583	20.751	22.034
10	6	18.319	19.460	20.693	22.026	23.469	25.032	26.726
	8	20.483	21.888	23.391	24.999	26.720	28.559	30.522
	10	22.335	23.946	25.653	27.460	29.370	31.385	33.504
	15	25.683	27.603	29.589	31.639	33.747	35.908	38.114
	0	15.000	15.000	15.000	15.000	15.000	15.000	15.000
	2	17.576	18.001	18.463	18.967	19.522	20.134	20.812
	4	19.840	20.631	21.485	22.410	23.417	24.516	25.719
15	6	21.788	22.886	24.060	25.321	26.677	28.139	29.716
	8	23.431	24.778	26.206	27.722	29.335	31.052	32.878
	10	24.783	26.328	27.950	29.654	31.447	33.332	35.310
	15	27.032	28.888	30.793	32.747	34.751	36.802	38.894
	0	20.000	20.000	20.000	20.000	20.000	20.000	20.000
	2	21.925	22.350	22.803	23.291	23.822	24.404	25.045
	4	23.545	24.332	25.164	26.054	27.011	28.048	29.175
20	6	24.876	25.966	27.109	28.317	29.604	30.980	32.455
	8	25.938	27.279	28.669	30.124	31.657	33.276	34.989
	10	26.755	28.297	29.882	31.524	33.235	35.021	36.886
	15	27.866	29.747	31.638	33.552	35.498	37.478	39.491

9.4 Rankine Active Pressure with Vertical Wall Back face and Inclined c– Ø Soil Backfill

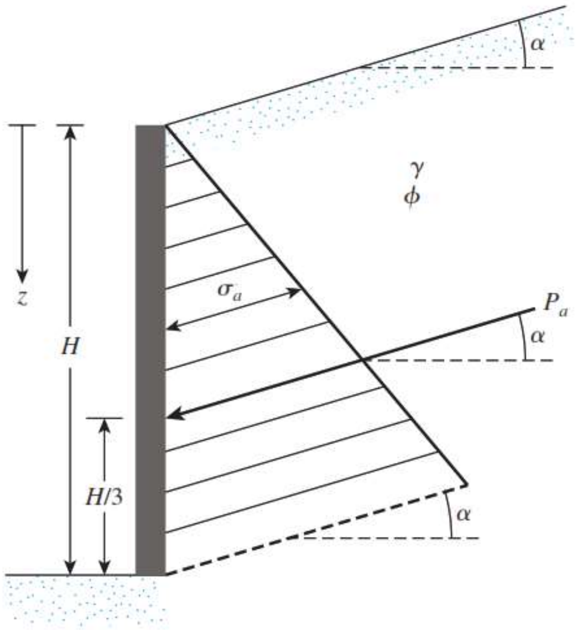
For a frictionless retaining wall with a vertical back face ($\theta = 0$) and inclined backfill of c– Ø soil (see Figure below), the active pressure at any depth z can be given as:

$$\sigma_a = \gamma z K_a = \gamma z K_a \cos\alpha$$

where

$$K_a = \frac{1}{\cos^2\phi}\left\{\begin{array}{l} 2\cos^2\alpha + 2\left(\dfrac{c}{\gamma z}\right)\cos\phi\sin\phi \\ -\sqrt{\left[4\cos^2\alpha(\cos^2\alpha - \cos^2\phi) + 4\left(\dfrac{c}{\gamma z}\right)^2\cos^2\phi + 8\left(\dfrac{c}{\gamma z}\right)\cos^2\alpha\sin\phi\cos\phi\right]}\end{array}\right\} - 1$$

Some values of Ka are given in Table 9.3.



For a problem of this type, the depth of tensile crack is given as: $z_c = \frac{2c}{\gamma}\sqrt{\frac{1+sin\emptyset}{1-sin\emptyset}}$

Table 9.4 Values of K_a

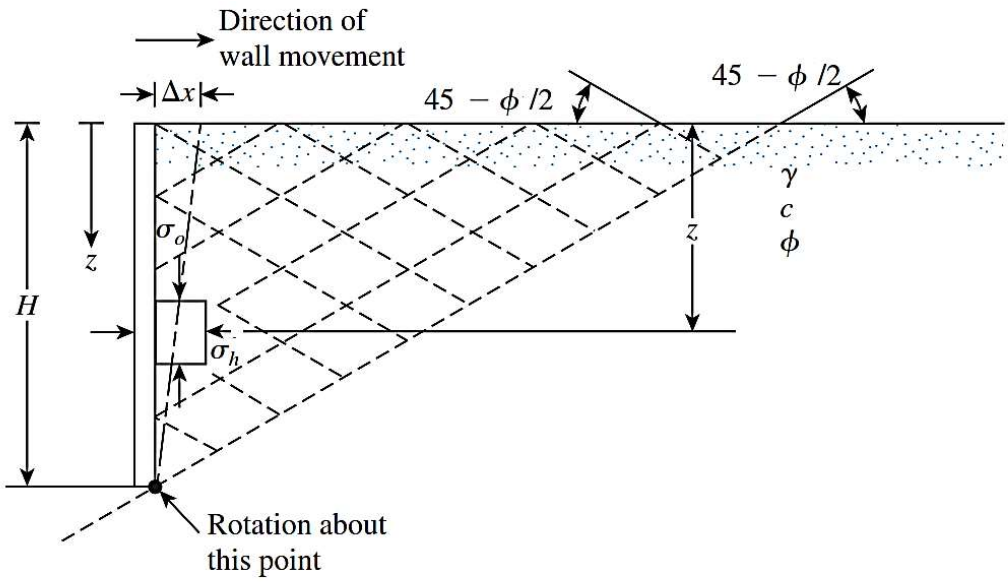
ϕ (deg)	α (deg)	$\frac{c}{\gamma z}$ = 0.025	0.05	0.1	0.5
15	0	0.550	0.512	0.435	−0.179
	5	0.566	0.525	0.445	−0.184
	10	0.621	0.571	0.477	−0.186
	15	0.776	0.683	0.546	−0.196
20	0	0.455	0.420	0.350	−0.210
	5	0.465	0.429	0.357	−0.212
	10	0.497	0.456	0.377	−0.218
	15	0.567	0.514	0.417	−0.229
25	0	0.374	0.342	0.278	−0.231
	5	0.381	0.348	0.283	−0.233
	10	0.402	0.366	0.296	−0.239
	15	0.443	0.401	0.321	−0.250
30	0	0.305	0.276	0.218	−0.244
	5	0.309	0.280	0.221	−0.246
	10	0.323	0.292	0.230	−0.252
	15	0.350	0.315	0.246	−0.263

9.5 Passive Pressure

9.5.1 Rankine Passive Earth Pressure

If the wall is pushed into the soil mass by an amount Δx, as shown in Figure below, the vertical stress at depth z will stay the same; however, the horizontal stress will increase. Thus, $\sigma_h$will be greater than $K_o\sigma_o$.

The horizontal stress, σ_h, at this point is referred to as the Rankine passive pressure, or $\sigma_h = \sigma_P$.



For a Rankine passive case, the lateral earth pressure σ_p at a depth z can be given as

$$\sigma_p = \sigma_o\, tan^2\left(45 + \frac{\emptyset}{2}\right) + 2c\tan\left(45 + \frac{\emptyset}{2}\right)$$

$$\sigma_p = \sigma_o\, K_p + 2c\sqrt{K_a}$$

Where: $K_p = tan^2\left(45 + \frac{\emptyset}{2}\right) = Rankine\ passive - pressure\ coefficient.$

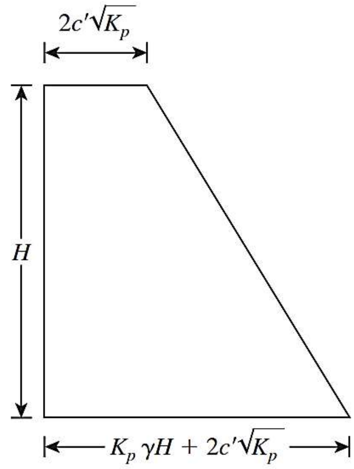
Note that $\sigma_o = 0$ at $z = 0$

The pressure distribution shows that at $z = 0$ the active pressure equals

$$2c\sqrt{K_p}$$

And at z=H $\sigma_o = \gamma H$ and $\sigma_p = K_p\gamma H + 2c\sqrt{K_p}$

.



Thus, the total Rankine active force per unit length of the wall before the tensile crack occurs is:

$$P_a = \frac{1}{2}\gamma H^2 K_a + 2c\, H\sqrt{K_p}$$

The approximate magnitudes of the wall movements, Δ_x, required to develop failure under passive conditions are as follows:

Soil type	Wall movement for passive condition, Δx
Dense sand	$0.005H$
Loose sand	$0.01H$
Stiff clay	$0.01H$
Soft clay	$0.05H$