Lecturer: M.Sc Murtadha Mohsen Al-Masoudy
E-mail: Murtadha_Almasoody@mustaqbal-college.edu.iq

Triple Integrals :-

The integral $\iiint_D f(x,y,z)\, dV$ is called the volume integral and it is evaluated in the manner that

$$\iiint_D f(x,y,z)\cdot dV = \iint_R \left(\int_{z_1}^{z_2} f(x,y,z)\, dz \right)\cdot dA$$

The volume of a closed bounded region D in Space is

$$V_D = \iiint_D dV$$

Ex:- Evalute $\int_0^{\sqrt{2}}\int_0^{3y}\int_{x^2+3y^2}^{8-x^2-y^2} dz\cdot dx\cdot dy$

Solution:-

$$\int_0^{\sqrt{2}}\int_0^{3y} z\Big]_{x^2+3y^2}^{8-x^2-y^2}\cdot dx\cdot dy = \int_0^{\sqrt{2}}\int_0^{3y}\left[(8-x^2-y^2)-(x^2+3y^2)\right]\cdot dx\cdot dy$$

$$= \int_0^{\sqrt{2}}\int_0^{3y}(8-2x^2-4y^2)\cdot dx\cdot dy = \int_0^{\sqrt{2}} 8x - 2\frac{x^3}{3} - 4y^2x\Big]_0^{3y}\cdot dy$$

$$= \int_0^{\sqrt{2}}\left[\left(8*3y-\frac{2}{3}(3y)^3-4y^2*3y\right)\right]dy = \int_0^{\sqrt{2}}(24y-18y^3-12y^3)\cdot dy$$

$$= \int_0^{\sqrt{2}}(24y-30y^3)\cdot dy = \frac{24y^2}{2} - \frac{30y^4}{4}\Big]_0^{\sqrt{2}}$$

$$= 12(\sqrt{2})^2 - \frac{15}{2}(\sqrt{2})^4 = 24 - 30$$

$$= \boxed{-6}$$