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Lagrange Multipliers :-

The extreme values of a function $f(x,y,z)$ whose variable are subject to a constraint of form $g(x,y,z)=0$ are to be found on the surface $g=0$ at the points where

$$\nabla f = \lambda \nabla g$$

λ called a Lagrange multiplier.

Ex:- Find the greatest and smallest values that the function $f(x,y) = xy$ takes on the ellipes

$$\frac{x^2}{8} + \frac{y^2}{2} = 1$$

Solution:-

$$g(x) = \frac{x^2}{8} + \frac{y^2}{2} - 1 = 0$$

$$\nabla f = \lambda \nabla g$$

$$\frac{\partial f}{\partial x} i + \frac{\partial f}{\partial y} j = \lambda \left(\frac{\partial g}{\partial x} i + \frac{\partial g}{\partial y} j \right)$$

$$y i + x j = \lambda \left(\frac{2x}{8} i + \frac{2y}{2} j \right)$$

$$y = \lambda \frac{x}{4} \quad \text{— } ①$$

$$x = \lambda y \quad \text{— } ② \quad \text{sub in eq (1)}$$

$$y = \lambda * \frac{\lambda y}{4} = \frac{\lambda^2 y}{4}$$

$$\frac{\lambda^2}{4} = 1 \Rightarrow \lambda^2 = 4 \Rightarrow \boxed{\lambda = \mp 2}$$

or $y = 0$

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Case I :- If $y=0$ then $x=y=0$

The point $(0,0)$ is not on the ellipes

Case II :- If $y\neq 0$ then $\lambda=\mp 2$

$$x=\mp 2y$$

Sub. this $(x=\mp 2y)$ in the eq $g(x,y)=0$ gives

$$\frac{(\mp 2y)^2}{8}+\frac{y^2}{2}=1$$

$$\frac{4y^2+4y^2}{8}=1 \Rightarrow 8y^2=8$$

$$y^2=1 \Rightarrow y=\mp 1$$

$f(x,y)=f(\mp 2,\mp 1)$.

$f(\mp 2,1)=\mp 2$

$f(\mp 2,-1)=\mp 2$