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Determining the Limits of Integration:-

If we want to evaluate $\iint_R f(x,y)\,dA$ over the region R shown in Fig.(1), integrating first with respect to (y) and then with respect to (x). we take the following steps:

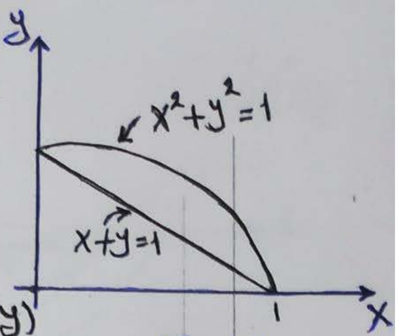


Fig.(1)

(1) we imagine a vertical line L cutting through R in the direction of increasing (y) as shown in Fig.(2).

(2) we integrate from the y-value where L enters R to the y-value where L leave R.

(3) we choose x-limits that include all the vertical lines that pass through R.

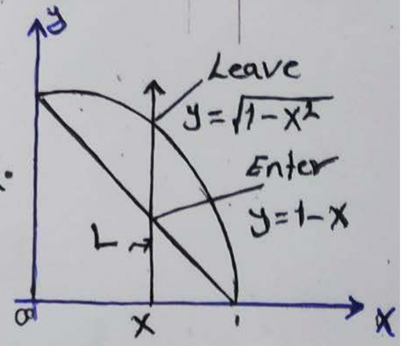
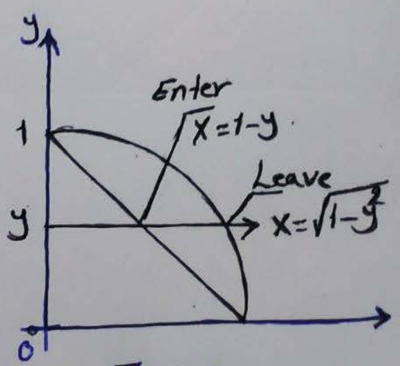


Fig.(2)

$$\therefore \iint_R f(x,y)\,dA = \int_{x=0}^{x=1}\int_{y=1-x}^{y=\sqrt{1-x^2}} f(x,y)\cdot dy\cdot dx$$

To calculate the same double integral as an integral with the order of integration reversed the procedure uses horizontal line as shown in Fig(3)

$$\therefore \iint_R f(x,y)\,dA = \int_{y=0}^{y=1}\int_{x=1-y}^{x=\sqrt{1-y^2}} f(x,y)\,dx\cdot dy.$$



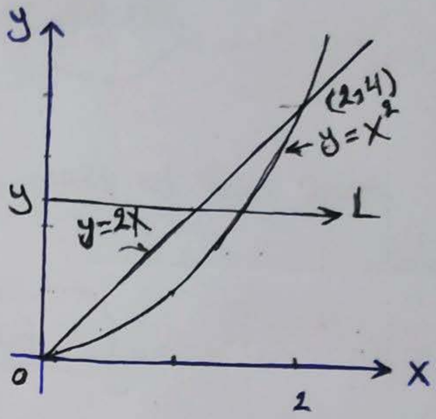
Fig(3)

Ex:- Find an equivalent integral to the $\int_0^2\int_{x^2}^{2x}(4x+2)\,dy\,dx$ by the order of the integral reversed.

Solution:-

R: $x=0$ to $x=2$

$y=x^2$ to $y=2x$



$$\therefore \int_0^2\int_{x^2}^{2x}(4x+2)\,dy\,dx = \int_0^4\int_{\frac{y}{2}}^{\sqrt{y}}(4x+2)\,dx\,dy$$

$$\int_0^4\int_{y/2}^{\sqrt{y}}(4x+2)\,dx\,dy = \int_0^4 \left(\frac{4x^2}{2}+2x\right)\Big]_{y/2}^{\sqrt{y}} \cdot dy$$

$$= \int_0^4 \left[\left(2(\sqrt{y})^2+2\sqrt{y}\right)-\left(2\left(\frac{y}{2}\right)^2+2*\frac{y}{2}\right)\right]dy$$

$$= \int_0^4\left(2y+2\sqrt{y}-\frac{y^2}{2}-y\right)dy = \int_0^4\left(y+2\sqrt{y}-\frac{y^2}{2}\right)dy$$

$$= \left[\frac{y^2}{2}+\frac{2(y)^{3/2}}{3/2}-\frac{y^3}{2*3}\right]_0^4$$

$$= \left(\frac{4^2}{2}+\frac{4}{3}(4)^{3/2}-\frac{(4)^3}{6}\right)$$

$$= 8+\frac{4}{3}(8)-\frac{32}{3}$$

$$= \boxed{8}$$