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M.Sc. Murtadha Al-Masoudy
Math. 2
2^{nd} Stage

Matrices

A rectangular array of number like

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ b_{21} & b_{22} & b_{23} \end{bmatrix}$$

is called a matrix. We call A a "2 by 3" matrix because it has two rows and three colums

Matrix Addition and Multiplication :-

Matrices with same shape can be added by adding corresponding elements.

EX:- If $A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 0 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 & 2 \\ 2 & 3 & -1 \end{bmatrix}$

then find $A + B$.

Solution:-

$$A + B = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 0 & -3 \end{bmatrix} + \begin{bmatrix} 1 & -2 & 2 \\ 2 & 3 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 2+1 & 1+(-2) & 3+2 \\ 1+2 & 0+3 & -3+(-1) \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -1 & 5 \\ 3 & 3 & -4 \end{bmatrix}$$

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* To multiply a matrix by a number (c), we multiply each element by (c)

Ex:- If $A=\begin{bmatrix} 2 & 1 & 3 \\ 1 & 0 & -3 \end{bmatrix}$ then, find $7A$.

Sol.

$$7A = 7\begin{bmatrix} 2 & 1 & 3 \\ 1 & 0 & -3 \end{bmatrix} = \begin{bmatrix} 7*2 & 7*1 & 7*3 \\ 7*1 & 7*0 & 7*-3 \end{bmatrix}$$

$$7A = \begin{bmatrix} 14 & 7 & 21 \\ 7 & 0 & -21 \end{bmatrix}$$

A system of Simultaneous linear equations

$$a_{11}x + a_{12}y + a_{13}z = b_1$$
$$a_{21}x + a_{22}y + a_{23}z = b_2$$

can be written as matrix form

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

or $AX = B$

where

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$x = \frac{\Delta x}{\Delta} \; ; \; y = \frac{\Delta y}{\Delta}$$
$$z = \frac{\Delta z}{\Delta}$$

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EX:- Solve the linear equations by using matrix to find x, y & z for the following equations.

$$3x-2y+5z=7 \quad —① $$
$$2x+y-z=-6 \quad —② $$
$$4x-3y+2z=-5 \quad —③ $$

Solution:-

$$\begin{bmatrix} 3 & -2 & 5 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 7 \\ -6 \\ -5 \end{bmatrix} \Leftrightarrow AX=B$$

$$\Delta=\begin{vmatrix} 3 & -2 & 5 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{vmatrix}$$

$$=3\begin{vmatrix} 1 & -1 \\ -3 & 2 \end{vmatrix}-(-2)\begin{vmatrix} 2 & -1 \\ 4 & 2 \end{vmatrix}+5\begin{vmatrix} 2 & 1 \\ 4 & -3 \end{vmatrix}$$

$$=3[(1*2)-(-3*-1)]+2[(2*2)-(4*-1)]+5[(2*-3)-(4*1)]$$

$$=\boxed{-37}.$$

$$\Delta x=\begin{vmatrix} 7 & -2 & 5 \\ -6 & 1 & -1 \\ -5 & -3 & 2 \end{vmatrix}$$

$$=7\begin{vmatrix} 1 & -1 \\ -3 & 2 \end{vmatrix}-(-2)\begin{vmatrix} -6 & -1 \\ -5 & 2 \end{vmatrix}+5\begin{vmatrix} -6 & 1 \\ -5 & -3 \end{vmatrix}$$

$$=7[(1*2)-(-3*-1)]+2[(-6*2)-(-5*-1)]+5[(-6*-3)-(-5*1)]$$

$$=\boxed{74}.$$

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$$\Delta y = \begin{vmatrix} 3 & 7 & 5 \\ 2 & -6 & -1 \\ 4 & -5 & 2 \end{vmatrix}$$

$$= 3\begin{vmatrix} -6 & -1 \\ -5 & 2 \end{vmatrix} - 7\begin{vmatrix} 2 & -1 \\ 4 & 2 \end{vmatrix} + 5\begin{vmatrix} 2 & -6 \\ 4 & -5 \end{vmatrix}$$

$$= 3\left[(-6*2)-(-5*-1)\right] - 7\left[(2*2)-(4*-1)\right] + 5\left[(2*-5)-(4*-6)\right]$$

$$= \boxed{-37}.$$

$$\Delta z = \begin{vmatrix} 3 & -2 & 7 \\ 2 & 1 & -6 \\ 4 & -3 & -5 \end{vmatrix}$$

$$= 3\begin{vmatrix} 1 & -6 \\ -3 & -5 \end{vmatrix} - (-2)\begin{vmatrix} 2 & -6 \\ 4 & -5 \end{vmatrix} + 7\begin{vmatrix} 2 & 1 \\ 4 & -3 \end{vmatrix}$$

$$= 3\left[(1*-5)-(-3*-6)\right] + 2\left[(2*-5)-(4*-6)\right] + 7\left[(2*-3)-(4*1)\right]$$

$$= \boxed{-111}.$$

$$\therefore\ x = \frac{\Delta x}{\Delta} = \frac{74}{-37} = \boxed{-2}.$$

$$y = \frac{\Delta y}{\Delta} = \frac{-37}{-37} = \boxed{1}.$$

$$z = \frac{\Delta z}{\Delta} = \frac{-111}{-37} = \boxed{3}.$$

$\boxed{4}$

Inverse of Matrix :—

To find the inverse of matrix, we following these Steps if the determinant not zero ($\Delta \neq 0$)

① Construct the matrix of cofactors of A

$$\text{cof } A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$

② Construct the transposed matrix of Cofactors (adjoint of A)

$$\text{adj } A = (\text{cof } A)^T = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$$

Transpose means to write the rows as columns

③ Then

$$A^{-1} = \frac{1}{\Delta} \text{adj } A$$

$$= \frac{1}{\Delta} \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$$

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Ex:– Use the determinant formula to find the inverse of the matrix

$$A = \begin{bmatrix} 2 & 3 & -4 \\ 1 & 2 & 3 \\ 3 & -1 & -1 \end{bmatrix}$$

Solution:–

$$\text{Cof } A = \begin{bmatrix} +\begin{vmatrix} 2 & 3 \\ -1 & -1 \end{vmatrix} & -\begin{vmatrix} 1 & 3 \\ 3 & -1 \end{vmatrix} & +\begin{vmatrix} 1 & 2 \\ 3 & -1 \end{vmatrix} \\ -\begin{vmatrix} 3 & -4 \\ -1 & -1 \end{vmatrix} & +\begin{vmatrix} 2 & -4 \\ 3 & -1 \end{vmatrix} & -\begin{vmatrix} 2 & 3 \\ 3 & -1 \end{vmatrix} \\ +\begin{vmatrix} 3 & -4 \\ 2 & 3 \end{vmatrix} & -\begin{vmatrix} 2 & -4 \\ 1 & 3 \end{vmatrix} & +\begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 10 & -7 \\ 7 & 10 & 11 \\ 17 & -10 & 1 \end{bmatrix}$$

$$\text{adj } A = \begin{bmatrix} 1 & 7 & 17 \\ 10 & 10 & -10 \\ -7 & 11 & 1 \end{bmatrix}$$

$$\Delta = 2\begin{vmatrix} 2 & 3 \\ -1 & -1 \end{vmatrix} - 3\begin{vmatrix} 1 & 3 \\ 3 & -1 \end{vmatrix} + (-4)\begin{vmatrix} 1 & 2 \\ 3 & -1 \end{vmatrix}$$

$$= 2[(2*-1)-(-1*3)] - 3[(1*-1)-(3*3)] - 4[(1*-1)-(2*3)]$$

$$= \boxed{60}$$

$$\therefore A^{-1} = \frac{1}{\Delta}\,\text{adj}\,A$$

$$= \frac{1}{60}\begin{bmatrix} 1 & 7 & 17 \\ 10 & 10 & -10 \\ -7 & 11 & 1 \end{bmatrix}$$

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* To Solve a system of Simultaneous linear equations by using matrix inverse method.

$$[A][X]=[B]$$

$$[X]=[A]^{-1}[B]$$

EX:— Solve the following system of equations by matrix method.

$$2X_1-2X_2+4X_3=2 \quad — \ ①$$

$$4X_1-2X_3=4 \quad — \ ②$$

$$X_1+X_2+6X_3=12 \quad — \ ③$$

Solution:—

$$[X]=\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} \qquad B=\begin{bmatrix} 2 \\ 4 \\ 12 \end{bmatrix}$$

$$[A]=\begin{bmatrix} 2 & -2 & 4 \\ 4 & 0 & -2 \\ 1 & 1 & 6 \end{bmatrix}$$

$$\Delta = 2\begin{vmatrix} 0 & -2 \\ 1 & 6 \end{vmatrix} - (-2)\begin{vmatrix} 4 & -2 \\ 1 & 6 \end{vmatrix} + 4\begin{vmatrix} 4 & 0 \\ 1 & 1 \end{vmatrix}$$

$$=2[(0*6)-(1*-2)]+2[(4*6)-(1*-2)]+4[(4*1)-(1*0)]$$

$$=\boxed{72}.$$

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$$\text{Cof } A = \begin{bmatrix} 2 & -26 & 4 \\ 16 & 8 & -4 \\ 4 & 20 & 8 \end{bmatrix}$$

$$\text{adj } A = \begin{bmatrix} 2 & 16 & 4 \\ -26 & 8 & 20 \\ 4 & -4 & 8 \end{bmatrix}$$

$$[A]^{-1} = \frac{1}{\Delta} \text{adj } A =$$

$$= \frac{1}{72} \begin{bmatrix} 2 & 16 & 4 \\ -26 & 8 & 20 \\ 4 & -4 & 8 \end{bmatrix}$$

$$[X] = [A]^{-1} [B]$$

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \frac{1}{72} \begin{bmatrix} 2 & 16 & 4 \\ -26 & 8 & 20 \\ 4 & -4 & 8 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 12 \end{bmatrix}$$

$$= \frac{1}{72} \begin{bmatrix} 2*2 + 16*4 + 4*12 \\ -26*2 + 8*4 + 20*12 \\ 4*2 + (-4)*4 + 8*12 \end{bmatrix}$$

$$= \frac{1}{72} \begin{bmatrix} 116 \\ 220 \\ 86 \end{bmatrix} = \begin{bmatrix} 116/72 \\ 220/72 \\ 86/72 \end{bmatrix}$$

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H.W

(1) Solve the linear equations by using matrix form

$AX=B$

$2X-3y+4Z=-19$ — (1)

$6X+4y-2Z=8$ — (2)

$X+5y+4Z=23$ — (3)

(2) Find the inverse of matrix

$$A=\begin{bmatrix} 1 & 8 & 9 \\ 0 & 4 & 6 \\ 0 & 0 & 3 \end{bmatrix}$$

(3) Solve the following system of equations

$X+8y+9Z=10$ — (1)

$4y+6Z=10$ — (2)

$3Z=-10$ — (3)

by a/ Matrix method.

b/ Determinant method.

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