



Class: 2st

Subject: Mathematics

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Total differential :-

$$df(x,y) = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy$$

Ex:- If $z = x^3 - xy^2 + 3$ then find dz .

Solution:-

$$dz = \frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy$$

$$= (3x^2 - y^2) dx + (-2xy) dy = (3x^2 - y^2) dx - 2xy dy$$

Chain Rule:-

① If $w = f(x,y)$, $x = g(u)$ and $y = h(u)$ then

$$\frac{dw}{du} = \frac{\partial w}{\partial x} \cdot \frac{dx}{du} + \frac{\partial w}{\partial y} \cdot \frac{dy}{du}$$

② If $w = f(x,y)$, $x = g_1(u,v)$ and $y = g_2(u,v)$ then

$$\frac{dw}{du} = \frac{\partial w}{\partial x} \cdot \frac{\partial x}{\partial u} + \frac{\partial w}{\partial y} \cdot \frac{\partial y}{\partial u}$$

$$\frac{dw}{dv} = \frac{\partial w}{\partial x} \cdot \frac{\partial x}{\partial v} + \frac{\partial w}{\partial y} \cdot \frac{\partial y}{\partial v}$$



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Ex: - Use the chain rule to find the derivative of $f(x,y) = xy$

with respect to (t) along the path $x = \cos t$, $y = \sin t$

Solution: -

$$\frac{\partial f}{\partial x} = y = \sin t$$

$$\frac{\partial f}{\partial y} = x = \cos t$$

$$\frac{dx}{dt} = -\sin t \quad \& \quad \frac{dy}{dt} = \cos t$$

$$\frac{df}{dt} = \frac{\partial f}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial f}{\partial y} \frac{dy}{dt}$$

$$= (\sin t)(-\sin t) + (\cos t)(\cos t)$$

$$= -\sin^2 t + \cos^2 t$$

$$= \boxed{\cos 2t}$$

Ex: - Find the value of df/dt at $t=0$ if $f(x,y,z) = xy + z$ and $x = \cos t$, $y = \sin t$, $z = t$

Solution: -

$$\frac{df}{dt} = \frac{\partial f}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial f}{\partial y} \cdot \frac{dy}{dt} + \frac{\partial f}{\partial z} \cdot \frac{dz}{dt}$$

$$= (y)(-\sin t) + (x)(\cos t) + 1 \cdot (1)$$

$$= -\sin^2 t + \cos^2 t + 1$$

$$= 1 + \cos 2t$$

$$\left. \frac{df}{dt} \right|_{t=0} = 1 + \cos(0) = \boxed{2}$$

$\boxed{4}$