Total differential :-

$$df(x,y) = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy$$

Ex:- If $z = x^3 - xy^2 + 3$ then find dz.

Solution:-

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$$

$$= (3x^2 - y^2)\cdot dx + (-2xy)\cdot dy = (3x^2 - y^2)\cdot dx - 2xy\cdot dy$$

Chain Rule :-

① If $w = f(x,y)$, $x = g(u)$ and $y = h(u)$ then

$$\frac{dw}{du} = \frac{\partial w}{\partial x}\cdot\frac{dx}{du} + \frac{\partial w}{\partial y}\cdot\frac{dy}{du}$$

② If $w = f(x,y)$, $x = g_1(u,v)$ and $y = g_2(u,v)$ then

$$\frac{dw}{du} = \frac{\partial w}{\partial x}\cdot\frac{dx}{du} + \frac{\partial w}{\partial y}\cdot\frac{dy}{du}$$

$$\frac{dw}{dv} = \frac{\partial w}{\partial x}\cdot\frac{dx}{dv} + \frac{\partial w}{\partial y} + \frac{dy}{dv}$$

Ex:- Use the chain rule to find the derivative of

$$f(x,y) = xy$$

with respect to(t) along the path $x = \cos t$, $y = \sin t$

Solution :-

$$\frac{\partial f}{\partial x} = y = \sin t$$

$$\frac{\partial f}{\partial y} = x = \cos t$$

$$\frac{dx}{dt} = -\sin t \quad \& \quad \frac{dy}{dt} = \cos t$$

$$\frac{df}{dt} = \frac{\partial f}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$$

$$= (\sin t)(-\sin t) + (\cos t)(\cos t)$$

$$= -\sin^2 t + \cos^2 t$$

$$= \boxed{\cos 2t}$$

Ex:- Find the value of df/dt at $t=0$ if $f(x,y,z) = xy + z$ and $x = \cos t$, $y = \sin t$, $z = t$

Solution :-

$$\frac{df}{dt} = \frac{\partial f}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial f}{\partial y}\cdot\frac{dy}{dt} + \frac{\partial f}{\partial z}\cdot\frac{dz}{dt}$$

$$= (y)(-\sin t) + (x)(\cos t) + 1\times(1)$$

$$= -\sin^2 t + \cos^2 t + 1$$

$$= 1 + \cos 2t$$

$$\left.\frac{df}{dt}\right)_{t=0} = 1 + \cos(0) = \boxed{2}$$

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