



Class: 2st

Subject: Mathematics

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De Moivre's Theorem :-

If $Z = r(\cos\theta + i\sin\theta)$ and (n) is a positive integer number then:

$$Z^n = [r(\cos\theta + i\sin\theta)]^n = r^n (\cos n\theta + i\sin n\theta)$$

$$Z^{1/n} = [r(\cos\theta + i\sin\theta)]^{1/n}$$

$$= r^{1/n} \left[\cos\left(\frac{\theta + 2\pi k}{n}\right) + i\sin\left(\frac{\theta + 2\pi k}{n}\right) \right]$$

Where $k = 1, 2, 3, \dots, n-1$

Ex: Find $\left(\frac{1}{2} + \frac{1}{2}i\right)^{10}$

Solution:

$$r = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \boxed{\frac{1}{\sqrt{2}}}$$

$$\theta = \tan^{-1} \frac{1/2}{1/2} \Rightarrow \theta = 45^\circ = \boxed{\frac{\pi}{4}}$$

$$Z^n = r^n (\cos n\theta + i\sin n\theta)$$

$$\therefore \left(\frac{1}{2} + i\frac{1}{2}\right)^{10} = \left(\frac{1}{\sqrt{2}}\right)^{10} \left(\cos \frac{10\pi}{4} + i\sin \frac{10\pi}{4}\right)$$

$$= \frac{1}{(\sqrt{2})^{10}} \left(\cos \frac{5\pi}{2} + i\sin \frac{5\pi}{2}\right)$$

$$= \frac{1}{32} (0 + i)$$

$$= \boxed{\frac{1}{32} i}$$



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Ex: Find the value of $(-1+i)^{1/3}$.

Solution:

$$r = \sqrt{(-1)^2 + 1^2} = \sqrt{2}$$

$$\theta = \tan^{-1} \frac{1}{-1} = -45^\circ$$

$$\theta = 180 - 45 = 135^\circ$$

$$z^{1/n} = r^{1/n} \left[\cos\left(\frac{\theta + 2\pi k}{n}\right) + i \sin\left(\frac{\theta + 2\pi k}{n}\right) \right]$$

$$(-1+i)^{1/3} = (\sqrt{2})^{1/3} \left[\cos\left(\frac{135 + 2\pi k}{3}\right) + i \sin\left(\frac{135 + 2\pi k}{3}\right) \right]$$

$$k = 0, 1, 2 \quad (n=3 \Rightarrow n-1=2)$$

$$\begin{aligned} \text{at } k=0 \Rightarrow (-1+i)^{1/3} &= (\sqrt{2})^{1/3} \left[\cos\left(\frac{135}{3}\right) + i \sin\left(\frac{135}{3}\right) \right] \\ &= (\sqrt{2})^{1/3} [\cos 45 + i \sin 45] \end{aligned}$$

$$\begin{aligned} \text{at } k=1 \Rightarrow (-1+i)^{1/3} &= (\sqrt{2})^{1/3} \left[\cos\left(\frac{135+2\pi}{3}\right) + i \sin\left(\frac{135+2\pi}{3}\right) \right] \\ &= (\sqrt{2})^{1/3} [\cos 165 + i \sin 165] \end{aligned}$$

$$\begin{aligned} \text{at } k=2 \Rightarrow (-1+i)^{1/3} &= (\sqrt{2})^{1/3} \left[\cos\left(\frac{135+4\pi}{3}\right) + i \sin\left(\frac{135+4\pi}{3}\right) \right] \\ &= (\sqrt{2})^{1/3} [\cos 285 + i \sin 285] \end{aligned}$$



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Roots of Equations :

If $Z = r e^{i\theta}$ is a complex number different from zero and (n) , then there are precisely (n) different complex numbers w_0, w_1, \dots, w_{n-1} , that are $(n$ th) roots of Z .

Let $w = \rho e^{i\alpha}$ is $(n$ th) root of $Z = r e^{i\theta}$

$$\text{So } w^n = Z \Rightarrow \rho^n e^{i n \alpha} = r e^{i \theta}$$

$\therefore \rho = \sqrt[n]{r}$ is the real positive $(n$ th) root of $(r$

$$n \alpha \neq \theta$$

We can say they may differ only by an integer multiple of (2π) that is

$$n \alpha = \theta + 2\pi k$$

Where $k = 0, \pm 1, \pm 2, \dots$

$$\alpha = \frac{\theta}{n} + \frac{2\pi k}{n}$$

The $(n$ th) roots of $Z = r e^{i\theta}$ are given by

$$\sqrt[n]{r} e^{i\left(\frac{\theta}{n} + \frac{2\pi k}{n}\right)}$$

Where $k = 0, \pm 1, \pm 2, \dots, n-1$



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Ex: - Find the sixth roots of $Z = -8$

Solution: -

$$Z = -8 + 0i \Rightarrow r = \sqrt{(-8)^2 + (0)^2} = \boxed{8}$$

$$\theta = \tan^{-1} \frac{0}{-8} \Rightarrow \boxed{\theta = \pi}$$

$$Z = r e^{i\theta} \Rightarrow \boxed{Z = 8 e^{i\pi}}$$

$$\omega_k = \sqrt[n]{r} e^{i \frac{(\theta + 2\pi k)}{n}} = \sqrt[n]{r} \left[\cos\left(\frac{\theta + 2\pi k}{n}\right) + i \sin\left(\frac{\theta + 2\pi k}{n}\right) \right]$$

$$n = 6 \Rightarrow k = 0, 1, 2, 3, 4, 5$$

$$\text{at } \underline{k=0} \Rightarrow \omega_0 = \sqrt[6]{8} \left[\cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right] = \sqrt{2} \left(\frac{\sqrt{3}}{2} + \frac{1}{2} i \right)$$

$$\text{at } \underline{k=1} \Rightarrow \omega_1 = \sqrt[6]{8} \left[\cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \right] = \sqrt{2} i$$

$$\text{at } \underline{k=2} \Rightarrow \omega_2 = \sqrt[6]{8} \left[\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6} \right] = \sqrt{2} \left(-\frac{\sqrt{3}}{2} + \frac{1}{2} i \right)$$

$$\text{at } \underline{k=3} \Rightarrow \omega_3 = \sqrt[6]{8} \left[\cos \frac{7\pi}{6} + i \sin \frac{7\pi}{6} \right] = \sqrt{2} \left(-\frac{\sqrt{3}}{2} - \frac{1}{2} i \right)$$

$$\text{at } \underline{k=4} \Rightarrow \omega_4 = \sqrt[6]{8} \left[\cos \frac{9\pi}{6} + i \sin \frac{9\pi}{6} \right] = -\sqrt{2} i$$

$$\text{at } \underline{k=5} \Rightarrow \omega_5 = \sqrt[6]{8} \left[\cos \frac{11\pi}{6} + i \sin \frac{11\pi}{6} \right] = \sqrt{2} \left(\frac{\sqrt{3}}{2} - \frac{1}{2} i \right)$$



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Ex: → Find the roots of the equation: $Z^3 = -1 + i$

Solution: —

$$-1 + i = r(\cos \theta + i \sin \theta)$$

$$r = \sqrt{(-1)^2 + (1)^2} = \sqrt{2}$$

$$\theta = \tan^{-1} \frac{1}{-1} \Rightarrow \theta = \frac{3\pi}{4}$$

$$Z^3 = -1 + i \Rightarrow Z = \sqrt[3]{-1 + i} = (-1 + i)^{1/3}$$

$$\therefore (-1 + i)^{1/3} = \left[\sqrt{2} \left(\cos \frac{3\pi}{4} + 2\pi k + i \sin \frac{3\pi}{4} + 2\pi k \right) \right]^{1/3}$$

$$= (\sqrt{2})^{1/3} \left[\cos \left(\frac{3\pi}{4n} + \frac{2\pi k}{n} \right) + i \sin \left(\frac{3\pi}{4n} + \frac{2\pi k}{n} \right) \right]$$

$$k = 0, 1, 2 \quad \& \quad n = 3$$

$$\text{at } k=0 \Rightarrow Z_0 = (\sqrt{2})^{1/3} \left[\cos \frac{3\pi}{12} + i \sin \frac{3\pi}{12} \right] = (\sqrt{2})^{1/3} (1 + i)$$

$$\text{at } k=1 \Rightarrow Z_1 = (\sqrt{2})^{1/3} \left[\cos \frac{11\pi}{12} + i \sin \frac{11\pi}{12} \right]$$

$$\text{at } k=2 \Rightarrow Z_2 = (\sqrt{2})^{1/3} \left[\cos \frac{19\pi}{12} + i \sin \frac{19\pi}{12} \right]$$



H.W

Complex Numbers

① Find
a/ i^2

b/ i^3

c/ $(4-2i) + (-6+5i)$

d/ $\frac{-5+5i}{4-3i}$

② Prove that $|z_1 - z_2|^2 + |z_1 + z_2|^2 = 2|z_1|^2 + 2|z_2|^2$

③ Write the following in polar forms

a/ $4-3i$

b/ $\sqrt{-i}$

c/ z^4

d/ z/\bar{z}

④ Find the value of

a/ $(1+i)^8$

b/ $(i)^{1/4}$

c/ \sqrt{z}

⑤ Find X and y from the

$$(X^2y - 2) + i(X + 2Xy - 5) = 0$$

⑥ Find the fourth roots of (-1) .