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M.Sc. Murtadha Al-Masoudy
Mathematics.2-
Second stage

Polar Coordinates

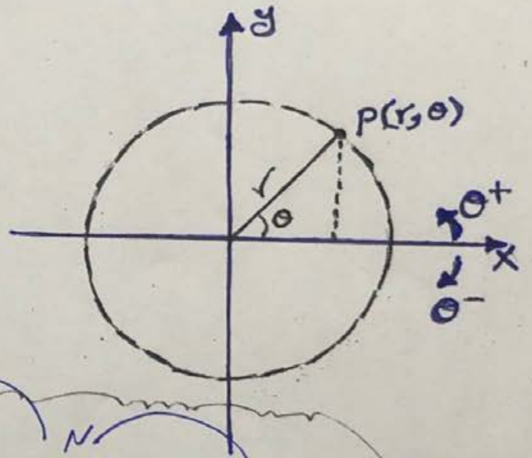
we Can define the polar coordinates for each point in the plane as the pair (r, θ), where r is the distance of point from origin and θ is the angle between the line segment from the origin the point and the X-axis.

$x = r\cos\theta$

$y = r\sin\theta$

$r = \sqrt{x^2 + y^2}$

$\theta = \tan^{-1}\frac{y}{x}$



Polar Coordinates

$P(r, \theta)$

Directed distance from O to P

Directed angle from initial ray to OP

There are many choices for the representation of any point in Polar Coordinates $P(r, \theta)$

For $r = r$ then $\theta = \theta + 2\pi n$ where $n = 0, \mp 1, \mp 2, \ldots$

If $r = -r$ then $\theta = (\pi - \theta) + 2\pi n$ where $n = 0, \mp 1, \mp 2, \ldots$

i.e $P(r, \theta) = P(r, \theta + 2\pi n)$

$P(-r, \theta) = P(-r, (\pi - \theta) + 2\pi n)$

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EX:- Find all the polar coordinates of the point $P(2, \pi/6)$?

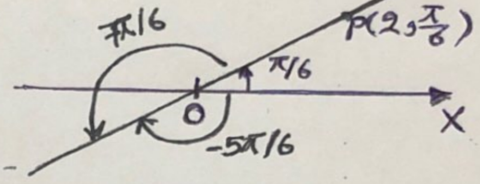
Solution:-

For $r = 2$

$\theta = \theta + 2\pi n \qquad n = 0, \mp 1, \mp 2, \dots$

$n = 0 \Rightarrow \theta = \frac{\pi}{6}$

$n = 1 \Rightarrow \theta = \frac{\pi}{6} + 2\pi = 13\pi/6 \quad \dots$



For $r = -2$

$\theta = (\pi - \theta) + 2\pi n \qquad n = 0, \mp 1, \mp 2, \dots$

$n = 0 \Rightarrow \theta = \pi - \frac{\pi}{6} = -5\pi/6$

$n = 1 \Rightarrow \theta = (\pi - \frac{\pi}{6}) + 2\pi = 17\pi/6 \quad \dots$

$\therefore P(2, \frac{\pi}{6}) = P(2, \frac{13\pi}{6}) = P(-2, \frac{-5\pi}{6}) = P(-2, \frac{17\pi}{6}) \dots$

EX:- Convert $P(-1, -1)$ into polar coordinate.

Solution:-

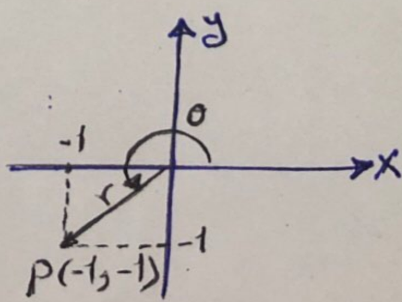
$r = \sqrt{(-1)^2 + (-1)^2} = \sqrt{2}$

$\theta = \tan^{-1} \frac{-1}{-1} \Rightarrow \theta = \pi/4$ This is not correct angle

because $P(-1, -1)$ is in the third quadrant So,

$\theta = \frac{\pi}{4} + \pi = 5\pi/4$

$\therefore P(-1, -1) = P(\sqrt{2}, \frac{5\pi}{4})$



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Ex:- Convert $2x - 5x^3 = 1 + xy$ into polar coordinates

Solution:-

$x = r\cos\theta$

$y = r\sin\theta$

$\theta = \tan^{-1}\frac{y}{x}$

$2x - 5x^3 = 1 + xy$

$2r\cos\theta - 5(r\cos\theta)^3 = 1 + (r\sin\theta)(r\cos\theta)$

$= 2r\cos\theta - 5r^3\cos^3\theta = 1 + r^2\sin\theta\cos\theta$

Ex:- Convert the polar equation $4r\cos\theta + r\sin\theta = 8$ into cartesian coordinates equation that express y in term of x.

Solution:-

$x = r\cos\theta$ & $y = r\sin\theta$

$\theta = \tan^{-1}\frac{y}{x}$

$4(r\cos\theta) + r\sin\theta = 8$

$4x + y = 8$

$y = 8 - 4x$

Ex:- Convert $r = -8\cos\theta$ into Cartesian coordinate.

Solution:-

$x = r\cos\theta$ & $y = r\sin\theta$

$r = \sqrt{x^2 + y^2} \Rightarrow r^2 = x^2 + y^2$

$r = -8\cos\theta] * r \Rightarrow r^2 = -8r\cos\theta$

$x^2 + y^2 = -8x$

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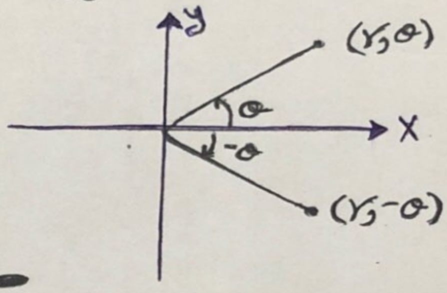
* Symmetry of Polar Coordinates :-

(1) About x-axis (polar axis) :-

If θ is replaced by $-\theta$ and the equation is unchanged then the curve is Symmetric about the x-axis.

Ex:- $\cos\theta = \cos(-\theta)$

i.e $\theta \to -\theta$

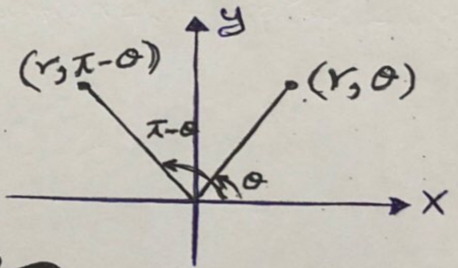


(2) About y-axis (90-axis) :-

If θ is replaced by $(\pi-\theta)$ and the equation is unchanged then the curve is Symmetric about y-axis.

Ex:- $\sin(\pi-\theta) = \sin\theta$

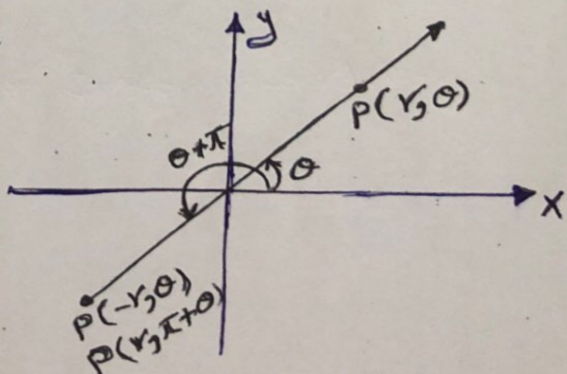
i.e $\theta \longrightarrow \pi-\theta$



(3) About the origin (Pole) :-

If (r) is replaced by $(-r)$ and θ is replaced by $(\pi+\theta)$ and the equation is unchanged, then the curve is Symmetric about the origin.

Ex:- $r^2 = (-r)^2$



Note:-

Any two Symmetries yield the other

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Ex:- Show that the equations below are equivalent

(1) $r = 1 - \sin(\theta - \frac{\pi}{3})$

(2) $r = 1 + \cos(\theta + \frac{\pi}{6})$ (3) $r = \cos(\theta + \frac{\pi}{6}) - 1$

Solution:-

(1) $r = 1 - \sin(\theta - \frac{\pi}{3})$

$= 1 - (\sin\theta \cos\frac{\pi}{3} - \cos\theta \sin\frac{\pi}{3})$

$= 1 - (\frac{1}{2}\sin\theta - \frac{\sqrt{3}}{2}\cos\theta)$

(2) $r = 1 + \cos(\theta + \frac{\pi}{6})$

$= 1 + (\cos\theta \cos\frac{\pi}{6} - \sin\theta \sin\frac{\pi}{6})$

$= 1 + (\frac{\sqrt{3}}{2}\cos\theta - \frac{1}{2}\sin\theta)$

$= 1 - (\frac{1}{2}\sin\theta - \frac{\sqrt{3}}{2}\cos\theta)$

(3) $r = \cos(\theta + \frac{\pi}{6}) - 1$

let $r \rightarrow -r$ & $\theta = \theta + \pi$

$-r = \cos(\theta + \pi + \frac{\pi}{6}) - 1$

$-r = \cos(\theta + \frac{7\pi}{6}) - 1$

$-r = \cos\theta \cos\frac{7\pi}{6} - \sin\theta \sin\frac{7\pi}{6} - 1$

$-r = \frac{-\sqrt{3}}{2}\cos\theta - (\frac{-1}{2}\sin\theta) - 1$

$[-r = -\frac{\sqrt{3}}{2}\cos\theta + \frac{1}{2}\sin\theta - 1] * -1$

$r = 1 - (\frac{1}{2}\sin\theta - \frac{\sqrt{3}}{2}\cos\theta)$

$\therefore$ The equations are equivalent.

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Ex:- Sketch $r = 1 + \cos\theta$

Solution:-

① $r = -r \Rightarrow -r = 1 + \cos\theta$ Not symmetric about the origin

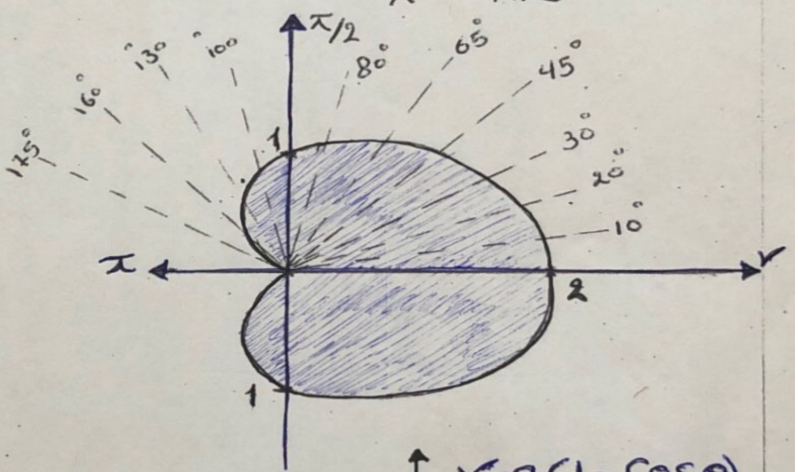
② $\theta = \pi - \theta \Rightarrow r = 1 + \cos(\pi - \theta)$

$= 1 + \cos\pi\cos\theta + \sin\theta\sin\pi$

$= 1 - \cos\theta$ Not symmetric about y-axis

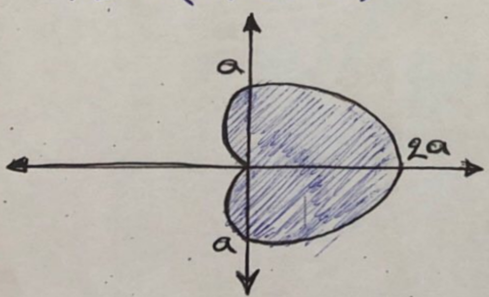
③ $\theta = -\theta \Rightarrow r = 1 + \cos(\bar{\theta}) = 1 + \cos\theta$ Symmetric about X-axis

θ	r
0	2
$\pi/4$	$1/\sqrt{2}$
$\pi/2$	1
$3\pi/4$	$-1/\sqrt{2}$
π	0

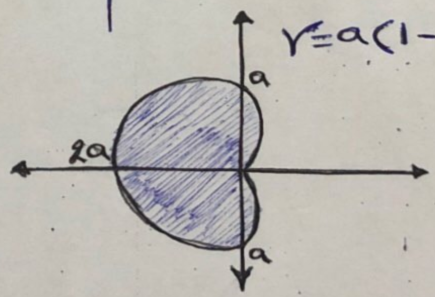


Note:-

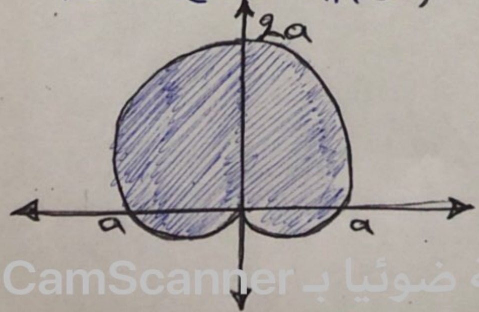
$r = a(1 + \cos\theta)$



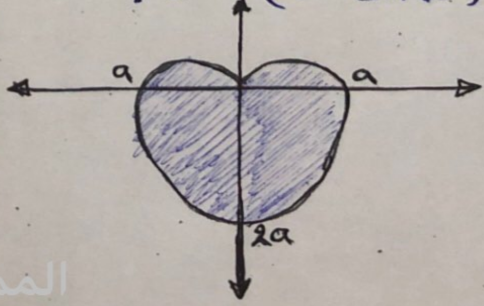
$r = a(1 - \cos\theta)$



$r = a(1 + \sin\theta)$



$r = a(1 - \sin\theta)$



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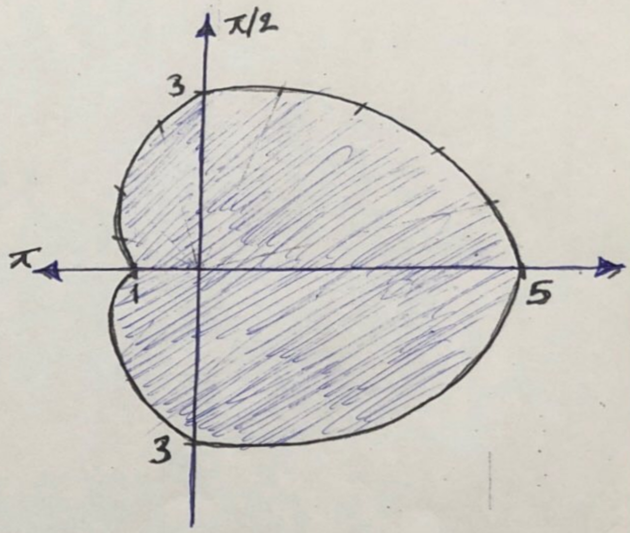
Ex:- Sketch $r = 3 + 2\cos\theta$

Solution:-

$\theta = -\theta \Rightarrow r = 3 + 2\cos(-\theta) = 3 + 2\cos\theta$

$\therefore$ Symmetric about x-axis

θ	r
0	5
$\pi/2$	3
π	1



Ex:- Sketch $r = 2 + 3\cos\theta$

Solution:-

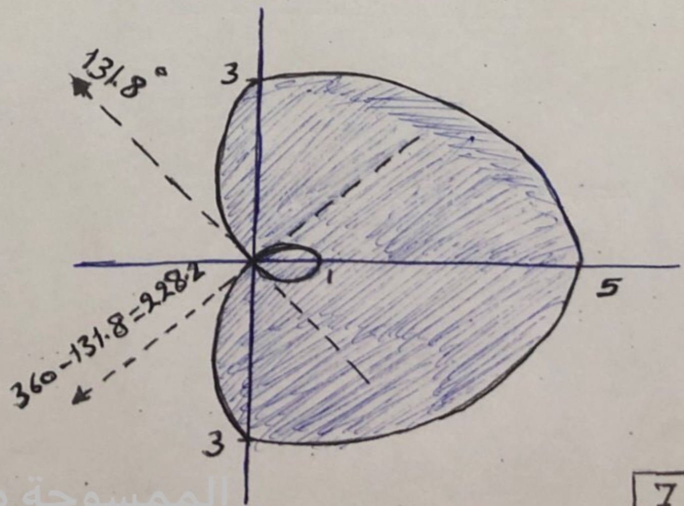
$\theta = -\theta \Rightarrow r = 2 + 3\cos(-\theta) = 2 + 3\cos\theta$

$\therefore$ Symmetric about the x-axis

put $r = 0 \Rightarrow 2 + 3\cos\theta = 0 \Rightarrow \cos\theta = \frac{-2}{3}$

$\theta = 131.8^\circ$

θ	r
0	5
$\frac{\pi}{2}$	2
131.8°	0
150°	-0.7
π	-1



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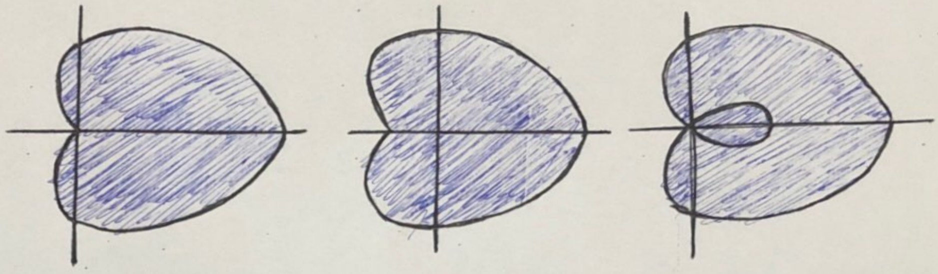
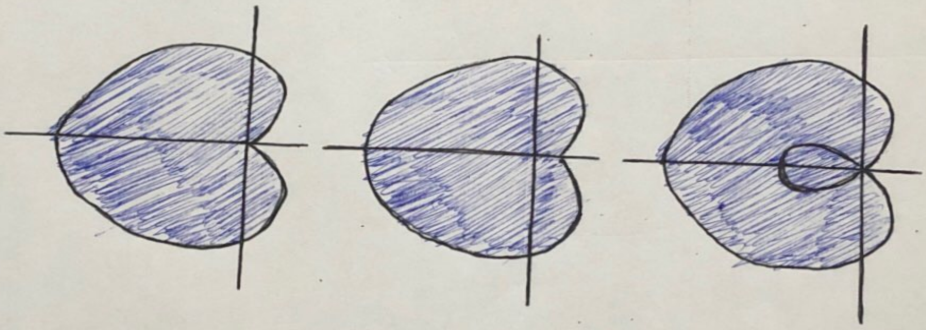
Note:-

	$a=b$	$a>b$	$a<b$

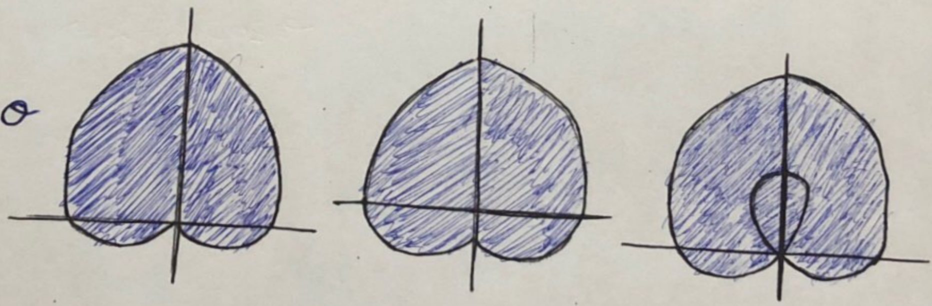
(1) $r = a + b\cos\theta$



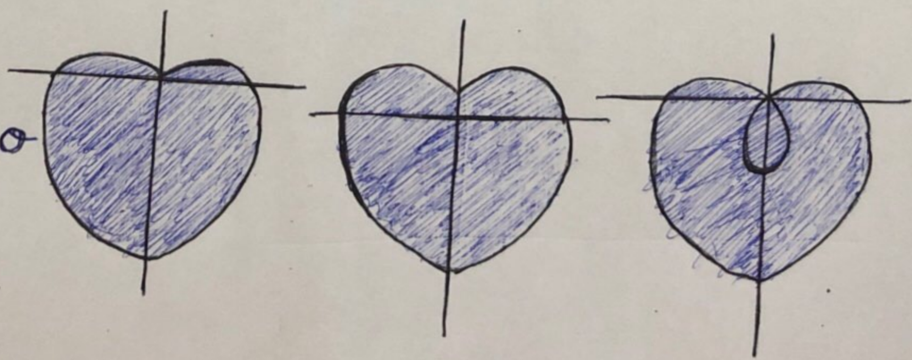
(2) $r = a - b\cos\theta$



(3) $r = a + b\sin\theta$



(4) $r = a - b\sin\theta$



EX:- Sketch $r = -2 + 3\sin\theta$ find the equivelant equation.

Solution:-

at $r = -r$ and $\theta = \theta + \pi$

$-r = -2 + 3\sin(\theta + \pi)$

$-r = -2 + 3(\sin\theta\cos\pi + \sin\pi\cos\theta)$

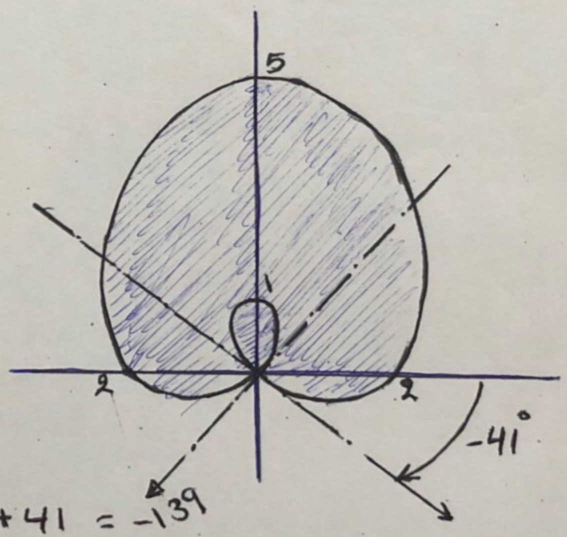
$-r = -2 - 3\sin\theta \Rightarrow r = 2 + 3\sin\theta$

$\therefore$ The curve is symmetric about the y-axis

at $r = 0 \Rightarrow 0 = 2 + 3\sin\theta$

$\sin\theta = \frac{-2}{3} \Rightarrow \theta = -41$

θ	r
0	2
-41	0
$-\pi/2$	-1
$\pi/2$	5



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EX :- Sketch $r^2 = a^2 \cos\theta$

Solution :-

(1) at $r = -r \Rightarrow (-r)^2 = a^2 \cos\theta \Rightarrow r^2 = a^2 \cos\theta$

$\therefore$ It is symmetric about the origin

(2) at $\theta = -\theta \Rightarrow r^2 = a^2 \cos(-\theta) = a^2 \cos\theta$

$\therefore$ It is symmetric about the x-axis

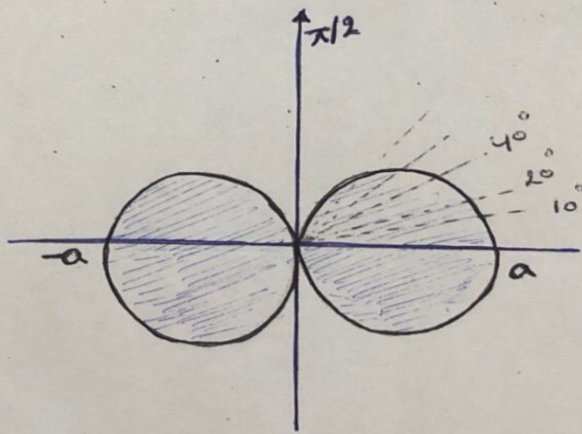
(3) at $r = -r$ and $\theta = -\theta$

$(-r)^2 = a^2 \cos(-\theta) \Rightarrow r^2 = a^2 \cos\theta$

$\therefore$ It is symmetric about y-axis

If $r = 0 \Rightarrow 0 = a^2 \cos\theta \Rightarrow \theta = \pi/2$

θ	r
0	$\mp a$
10	$\mp 0.99a$
20	$\mp 0.96a$
⋮	⋮
$\pi/2$	0



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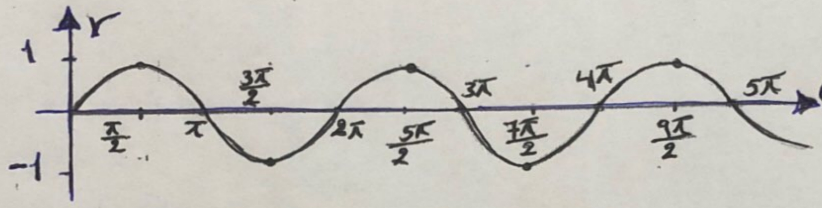
Note:-

The curve $r = a\sin(n\theta)$; $r = a\cos(n\theta)$, $n > 1$
n is integer number represents

(1) Rose of (n) leaves if (n) is an odd number

(2) Rose of (2n) leaves if (n) is an even number



$y = \sin x$

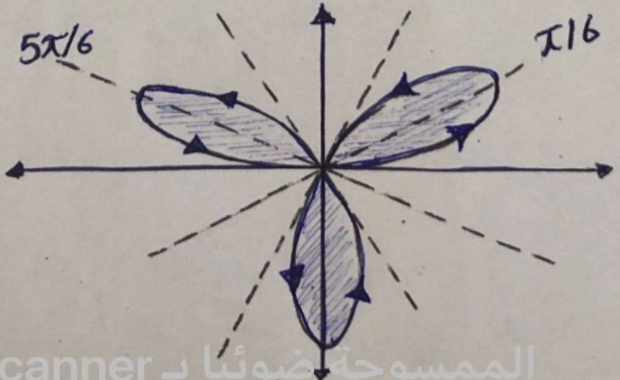
Ex:- Sketch $r = \sin 3\theta$

Solution:-

(1) put $\sin 3\theta = 0 \Rightarrow 3\theta = 0, \pi, 2\pi, \dots$
$\theta = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \dots \Rightarrow r = 0$

(2) Put $\sin 3\theta = 1 \rightarrow 3\theta = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \dots$
$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{9\pi}{6}, \dots \Rightarrow r = 1$

(3) Put $\sin 3\theta = -1 \Rightarrow 3\theta = \frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2}, \dots$
$\theta = \frac{3\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}, \dots \Rightarrow r = -1$



Ex:- Sketch $r = \sin 2\theta$

Solution:-

1- put $\sin 2\theta = 0 \Rightarrow 2\theta = 0, \pi, 2\pi, \dots$

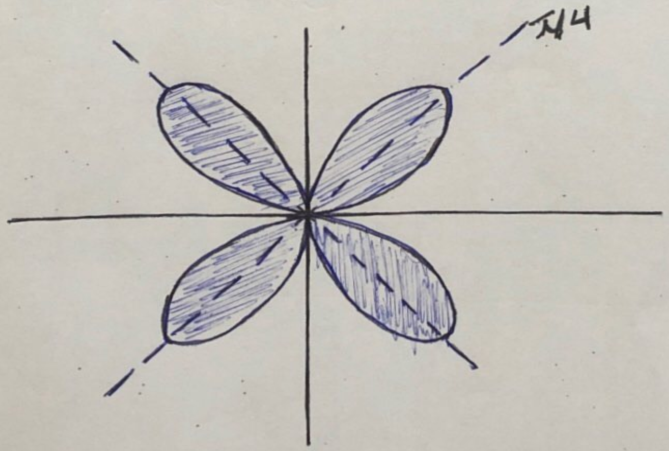
$$\theta = 0, \frac{\pi}{2}, \pi, \dots \Rightarrow r = 0$$

2- put $\sin 2\theta = 1 \Rightarrow 2\theta = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \dots$

$$\theta = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \dots \Rightarrow r = 1$$

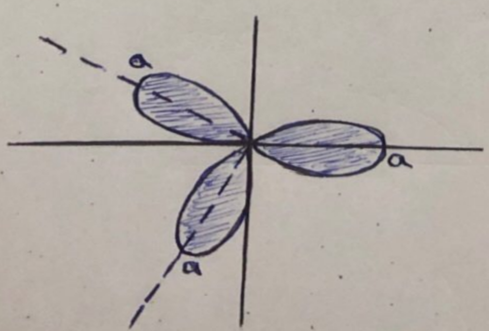
3- put $\sin 2\theta = -1 \Rightarrow 2\theta = \frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2}, \dots$

$$\theta = \frac{3\pi}{4}, \frac{7\pi}{4}, \frac{11\pi}{4}, \dots \Rightarrow r = -1$$

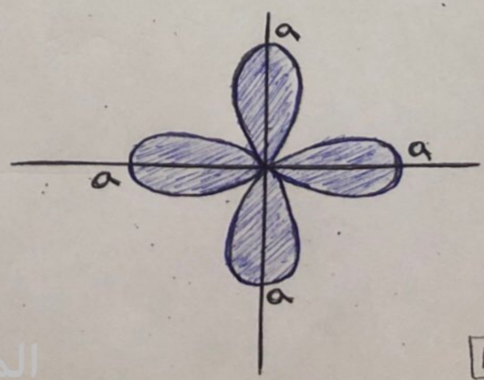


Notes:-

$r = a\cos 3\theta$



$r = a\cos 2\theta$



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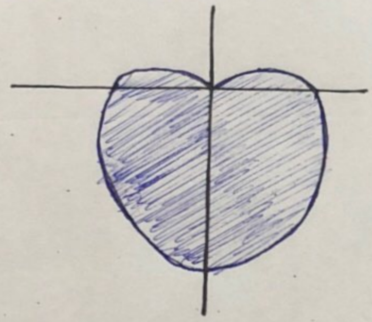
Note:—

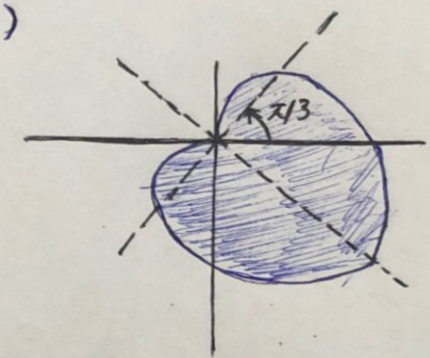
The curve $r=f(\theta-\alpha)$ is a result of the curve $r=f(\theta)$ rotating by the angle (α) measured in a counter clockwise direction.

But the curve $r=f(\theta+\alpha)$ is a result of the rotation in a clockwise direction

Ex:— Sketch $r=1-\sin(\theta-\frac{\pi}{3})$

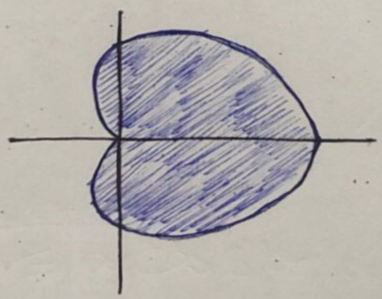


$r=1-\sin\theta$

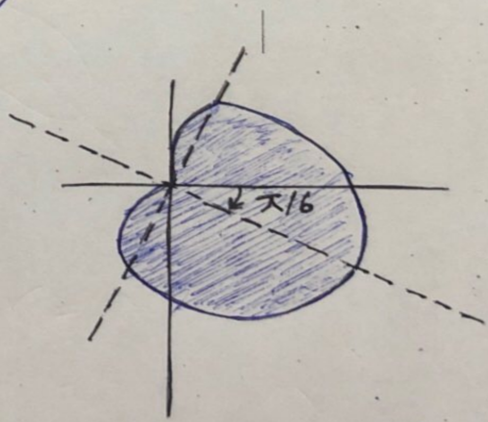


$r=1-\sin(\theta-\frac{\pi}{3})$

Ex:— Sketch $r=1+\cos(\theta+\frac{\pi}{6})$



$r=1+\cos\theta$



$r=1+\cos(\theta+\frac{\pi}{6})$

* Intersection of two Curves:—

To find the points of intersection of two curves, the equations of curves must be Solved simultaneously for variable (r) or (θ) and to find the other points we must graph (sckech) this curves.

Ex:— Find the intersection of the curves
$r^2 = 4a^2\cos\theta$ and $r = a(1-\cos\theta)$, $a>0$

Solution:—

$r^2 = 4a^2\cos\theta \Rightarrow \cos\theta = \dfrac{r^2}{4a^2}$ — ①

Sub. eq(1) in eq.(2)

$r = a(1-\cos\theta)$ —— ②

$\therefore r = a(1-\dfrac{r^2}{4a^2}) \Rightarrow r = a(\dfrac{4a^2-r^2}{4a^2})$

$r = \dfrac{4a^2-r^2}{4a} \Rightarrow 4ar = 4a^2 - r^2$

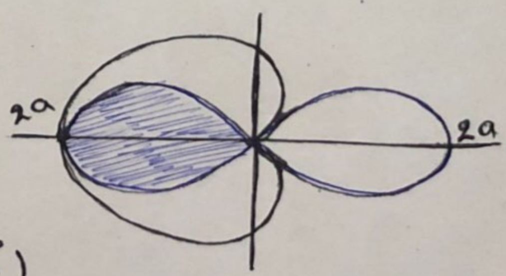
$r^2 + 4ar - 4a^2 = 0$

$r = \dfrac{-4a \mp \sqrt{(4a)^2 - 4*1*(-4a^2)}}{2(1)} \Rightarrow r = -2a \mp 2a\sqrt{2}$

at $r = -2a \mp 2a\sqrt{2}$ Sub in eq (1)

$r = -2a + 2a\sqrt{2} \Rightarrow \theta = 80.2^\circ$

$r = -2a - 2a\sqrt{2} \Rightarrow \theta = \infty$



The points of intersection are:-

$(-2a+2a\sqrt{2}, 80.2^\circ)$, $(-2a+2a\sqrt{2}, -80.2^\circ)$

$(0,0)$; $(2a, \pi)$

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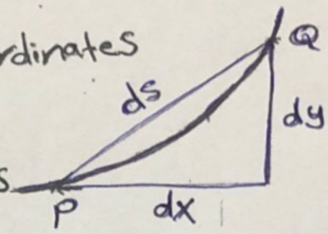
* Arc length :–

$(ds)^2 = (dx)^2 + (dy)^2$ Cartesian Coordinates



$(ds)^2 = r^2 (d\theta)^2 + (dr)^2$. Polar coordinates

$$ds = \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

Thus, the length of arc PQ is:

$$L_P^Q = \int_P^Q ds = \int_P^Q \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

EX:– Find the length of the curve $r = \theta^2$ for $0 \leqslant \theta \leqslant \pi$

Solution:–

$$r = \theta^2 \Rightarrow \frac{dr}{d\theta} = 2\theta$$

$$ds = \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta \Rightarrow ds = \sqrt{(\theta^2)^2 + (2\theta)^2}\, d\theta$$

$$ds = \sqrt{\theta^4 + 4\theta^2}\, d\theta = \sqrt{\theta^2(\theta^2+4)}\, d\theta$$

$$ds = \theta\sqrt{\theta^2+4}\, d\theta$$

$$\therefore L_0^\pi = \int_0^\pi \theta\sqrt{\theta^2+4}\, d\theta = \frac{1}{2}\int_0^\pi (\theta^2+4)^{1/2} \cdot (2\theta)\, d\theta$$

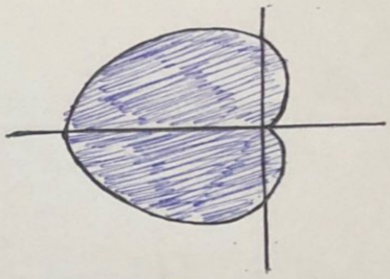
$$L_0^\pi = \frac{1}{2} \frac{(\theta^2+4)^{3/2}}{3/2}\Bigg]_0^\pi = \frac{1}{3}\left[(\pi^2+4)^{3/2} - (0^2+4)^{3/2}\right]$$

$$= \frac{1}{3}\left[51.597 - 8\right]$$

$$= 14.532$$

EX:- Find the length of the curve $r = 1-\cos\theta$

Solution:-



From the sketch $0 \le \theta \le 2\pi$

$r = 1-\cos\theta \Rightarrow \frac{dr}{d\theta} = \sin\theta$

$$L = \int_0^{2\pi} \sqrt{(1-\cos\theta)^2 + (\sin\theta)^2}\, d\theta$$

$$= \int_0^{2\pi} \sqrt{1 - 2\cos\theta + \cos^2\theta + \sin^2\theta}\, d\theta \quad , \quad \cos^2\theta + \sin^2\theta = 1$$

$$= \int_0^{2\pi} \sqrt{2 - 2\cos\theta}\, d\theta \quad , \quad 2\sin^2\frac{\theta}{2} = 1-\cos\theta$$

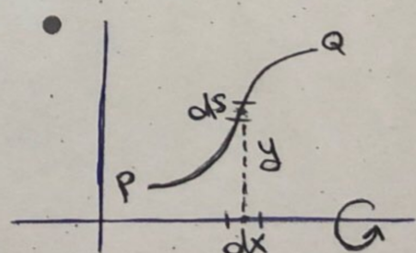
$$= \int_0^{2\pi} \sqrt{2(2\sin^2\frac{\theta}{2})}\, d\theta = \int_0^{2\pi} 2\sin\frac{\theta}{2}\, d\theta$$

$$= 2\int_0^{2\pi} 2\sin\frac{\theta}{2}\left(\frac{1}{2}d\theta\right) = -4\cos\frac{\theta}{2}\Big]_0^{2\pi}$$

$$= -4\left[\cos\frac{2\pi}{2} - \cos\frac{0}{2}\right] = -4(-1-1) = 8$$

* Surface area of rotation:-

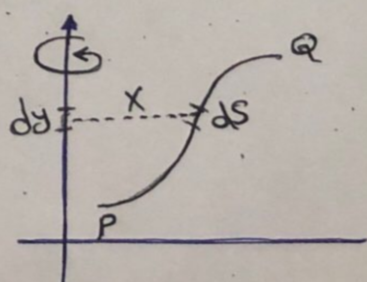
① About the X-axis:-



$$S = 2\pi \int_P^Q (r\sin\theta)\sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

$$= 2\pi \int_P^Q y \cdot dS$$

② About the y-axis:-

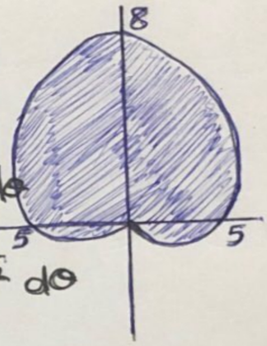


$$S = 2\pi \int_P^Q (r\cos\theta)\sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

$$= 2\pi \int_P^Q x\, dS$$

EX:- Find the surface area of $r = 4 + 4\sin\theta$ when the segment from $\theta = -\pi/2$ to $\pi/2$ is revolved around y-axis.

Solution:-



$$S = 2\pi \int_{P}^{Q} (r\cos\theta)\sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

$$S = 2\pi \int_{-\pi/2}^{\pi/2} (4 + 4\sin\theta)\cos\theta \sqrt{(4+4\sin\theta)^2 + (4\cos\theta)^2}\, d\theta$$

$$= 2\pi \int_{-\pi/2}^{\pi/2} 4(1+\sin\theta)\cos\theta \sqrt{16(1+\sin\theta)^2 + 16(\cos\theta)^2}\, d\theta$$

$$= 2\pi \int_{-\pi/2}^{\pi/2} 16(1+\sin\theta)\cos\theta \sqrt{1 + 2\sin\theta + \sin^2\theta + \cos^2\theta}\, d\theta$$

$$= 2\pi \int_{-\pi/2}^{\pi/2} 16(1+\sin\theta)\cos\theta \sqrt{1 + 2\sin\theta + 1}\, d\theta$$

$$= 2\pi \int_{-\pi/2}^{\pi/2} (4)(2)(2)(1+\sin\theta)\cos\theta \sqrt{2 + 2\sin\theta}\, d\theta$$

$$= 8\pi \int_{-\pi/2}^{\pi/2} (2 + 2\sin\theta)\, 2\cos\theta \sqrt{2 + 2\sin\theta}\, d\theta$$

$$= 8\pi \int_{-\pi/2}^{\pi/2} 2\cos\theta (2 + 2\sin\theta)^{3/2}\, d\theta$$

$$= 8\pi \left.\frac{(2 + 2\sin\theta)^{5/2}}{5/2}\right]_{-\pi/2}^{\pi/2} = \frac{16\pi}{5}\left[2 + 2\sin\theta\right]^{5/2}\Big]_{-\pi/2}^{\pi/2}$$

$$= \frac{16\pi}{5}\left[\left(2 + 2\sin\frac{\pi}{2}\right)^{5/2} - \left(2 + \sin\left(\frac{-\pi}{2}\right)\right)^{5/2}\right]$$

$$= \boxed{\frac{512\pi}{5}} \text{ units}^2$$

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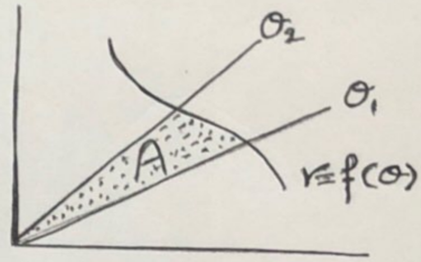
* plane area :-

$$A = \frac{1}{2}\int_{\theta_1}^{\theta_2} r^2 \, d\theta$$



EX:- Find the area of the region that enclosed by the curve $r = 5\sin\theta$ and radius ($\theta = 0$ and $\theta = \pi/3$).

Solution:-

$$A = \frac{1}{2}\int_{\theta_1}^{\theta_2} r^2 \, d\theta = \frac{1}{2}\int_0^{\pi/3} (5\sin\theta)^2 \, d\theta$$

$$= \frac{1}{2}\int_0^{\pi/3} 25\sin^2\theta \, d\theta = \frac{25}{2}\int_0^{\pi/3} \sin^2\theta \cdot d\theta$$

$$\because \sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$\therefore A = \frac{25}{2}\int_0^{\pi/3} \frac{1-\cos 2\theta}{2} \, d\theta = \frac{25}{4}\int_0^{\pi/3} (1-\cos 2\theta) \cdot d\theta$$

$$A = \frac{25}{4}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi/3}$$

$$= \frac{25}{4}\left[\left(\frac{\pi}{3} - \frac{\sin\frac{2\pi}{3}}{2}\right) - \left(0 - \frac{\sin 0}{2}\right)\right]$$

$$= \boxed{3.84} \text{ square unit}$$

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Polar Coordinates

H.W :-

1- Convert $P(0, \pi/2)$ to Cartesian coordinates.

2- Convert $x^2 - y^2 = 4$ into polar coordinates

3- Convert $r = 4\tan\theta\sec\theta$ into Cartesian Coordinates

4- Sketch $r = 1 - 2\cos\theta$

5- Sketch $r = 5 - 5\sin\theta$

6- Graph $r = 7 - 6\cos\theta$

7- Graph $r = 2 + 4\cos\theta$

8- Graph $r = \sin 4\theta$

9- Graph $r = \cos 5\theta$

10- Find the length of the cardiod $r = 1 + \sin\theta$ for $\theta = 0$ to $\theta = \pi/2$

11- Find the surface area of $r = 3 + 3\cos\theta$ around the Polar axis from 0 to π

12- Find the area of the region bounded by the polar curve $r = 3 + 2\cos\theta$