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Mathematics-2-
$2^{nd}$ Year.

# Function of Several Variables

The equation of the Surface is :-

$$F(x,y,z) = 0$$

- $z = f_1(x,y)$
  - $\dfrac{\partial z}{\partial x}$
  - $\dfrac{\partial z}{\partial y}$
- $y = f_2(x,z)$
  - $\dfrac{\partial y}{\partial x}$
  - $\dfrac{\partial y}{\partial z}$
- $x = f_3(y,z)$
  - $\dfrac{\partial x}{\partial z}$
  - $\dfrac{\partial x}{\partial y}$

The case where $z = f(x,y)$ it regards

$x, y$ : is independent variable

$z$ : is a function ($z$ is a dependent variable)

The first partial derivatives are write :-

$$\frac{\partial z}{\partial x} = z_x = f_x = \frac{\partial f}{\partial x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x, y) - f(x,y)}{\Delta x}$$

$$\frac{\partial z}{\partial y} = z_y = f_y = \frac{\partial f}{\partial y} = \lim_{\Delta y \to 0} \frac{f(x, y+\Delta y) - f(x,y)}{\Delta y}$$

And for the Second Partial derivatives :-

$$\frac{\partial^2 z}{\partial x^2} = z_{xx} = f_{xx} = \frac{\partial}{\partial x}(z_x)$$

$$\frac{\partial^2 z}{\partial y^2} = z_{yy} = f_{yy} = \frac{\partial}{\partial y}(z_y)$$

$$\frac{\partial^2 z}{\partial x \partial y} = z_{yx} = f_{yx} = \frac{\partial}{\partial y}(z_x)$$

$$\frac{\partial^2 z}{\partial y \partial x} = z_{xy} = f_{xy} = \frac{\partial}{\partial x}(z_y)$$

**Ex:-** If $Z = x^3 + y^4 + x\sin y + y\cos x$ then find $Z_{xy}$, $Z_{yx}$

**Solution:-**

$$Z_x = 3x^2 + \sin y - y\sin x$$

$$Z_y = 4y^3 + x\cos y + \cos x$$

$$Z_{xy} = \frac{\partial}{\partial x}(Z_y) = \cos y - \sin x$$

$$Z_{yx} = \frac{\partial}{\partial y}(Z_x) = \cos y - \sin x$$

**Ex:-** If $f(x,y) = y\cdot\sin(xy)$ then find $f_x$, $f_y$

**Solution:-**

$$f_x = y * \cos(xy) * y + \sin(xy) * 0 \quad = y^2\cos(xy)$$

$$f_y = y\cdot\cos(xy) * x + \sin(xy) * 1$$

$$= yx\cos(xy) + \sin(xy)$$

**Ex:-** If $Z = \tan^{-1}\left(\frac{y}{x}\right)$ then show that $x\frac{\partial Z}{\partial x} + y\frac{\partial Z}{\partial y} = 0$

**Solution:-**

$$\frac{\partial Z}{\partial x} = \frac{1}{1+(y/x)^2} * \frac{x*0 - y*1}{x^2} = \frac{-y}{x^2+y^2}$$

$$\frac{\partial Z}{\partial y} = \frac{1}{1+(y/x)^2} * \frac{x*1 - y*0}{x^2} = \frac{x}{x^2+y^2}$$

$$\therefore x\frac{\partial Z}{\partial x} + y\frac{\partial Z}{\partial y} = x\left(\frac{-y}{x^2+y^2}\right) + y\left(\frac{x}{x^2+y^2}\right) = 0$$