Lecture
12 first
stage

Linear Momentum And Collisions

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1- LinearMomentum

Definition of Linear Momentum, $\vec{\mathbf{p}}$

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}}$$

SI unit: $\text{kg} \cdot \text{m/s}$

Momentum is a vector; its direction is the same as the direction of the velocity

2- Momentum and Newton's SecondLaw

- Newton's second law, as we wrote it before:

$$\sum \vec{F} = m\vec{a}$$

is only valid for objects that have constant mass. Here is a more general form, also useful when the mass is changing:

Newton's Second Law

$$\sum \vec{F} = \frac{\Delta \vec{p}}{\Delta t}$$

3- Impulse

Definition of Impulse, $\vec{I}$

$$\vec{I} = \vec{F}_{av}\Delta t$$

SI unit: $N \cdot s = kg \cdot m/s$

- Impulse is a vector, in the same direction as the average force.
- We can rewrite

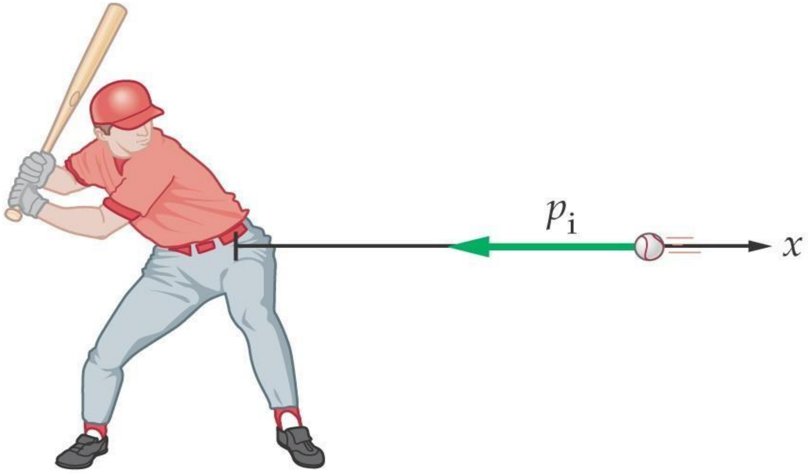
$$\vec{F}_{av}\Delta t = \Delta\vec{p}$$

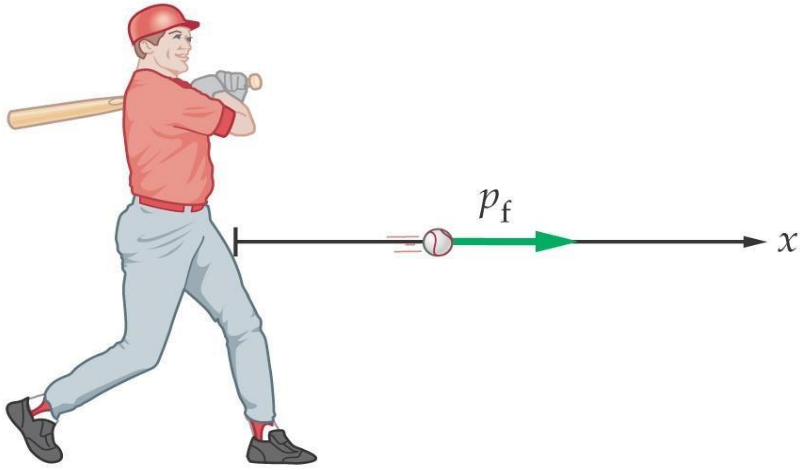
as So we see that

$$\vec{I} = \vec{F}_{av}\Delta t = \Delta\vec{p}$$

- The impulse is equal to the change in momentum.

- Therefore, the same change in momentum may be produced by a large force acting for a short time, or by a smaller force acting for a longer time.





Example

- An object of mass 3.0 kg is allowed to fall from rest under the force of gravity for 3.4 seconds. What is the change in momentum? Ignore air resistance.

Want $\Delta \mathbf{p} = m\Delta \mathbf{v}$.

$$\Delta \mathbf{v} = \mathbf{a}\Delta t$$

$$\Delta \mathbf{v} = -g\Delta t = -33.3 \text{ m/sec}$$

$$\Delta \mathbf{p} = m\Delta \mathbf{v} = -100 \text{ kg m/s (downward)}$$

Example

- What average force is necessary to bring a 50.0-kg sled from rest to 3.0 m/s in a period of 20.0 seconds? Assume frictionless ice.

$$\Delta \mathbf{p} = \mathbf{F}_{av}\Delta t$$

$$\mathbf{F}_{av} = \frac{\Delta \mathbf{p}}{\Delta t} = \frac{m\Delta \mathbf{v}}{\Delta t}$$

$$F_{av} = \frac{(50.0\ \text{kg})(3.0\ \text{m/s})}{20.0\ \text{s}} = 7.5\ \text{N}$$

The force will be in the direction of motion.

4- Conservation of linearmomentum

- The net force acting on an object is the rate of change of its momentum:

$$\sum \vec{F} = \frac{\Delta \vec{p}}{\Delta t}$$

- If the net force is zero, the momentum does not change:

Conservation of Momentum

If the net force acting on an object is zero, its momentum is conserved; that is, $\vec{p}_f = \vec{p}_i$

- **<u>Internal Versus External Forces</u>**:
- **Internal forces** act between objects within the system.
- As with all forces, they occur in action-reaction pairs. As all pairs act between objects in the system, **the internal forces always sum to zero**:

$$\sum \vec{\mathbf{F}}_{\text{int}} = 0$$

- Therefore, the net force acting on a system is the sum of the external forces acting on it.

- Furthermore, internal forces cannot change the momentum of a system.

Conservation of Momentum for a System of Objects

Internal forces have absolutely no effect on the net momentum of a system.

- If the *net external* force acting on a system is zero, its net momentum is conserved. That is,

$$\vec{p}_{1,f} + \vec{p}_{2,f} + \vec{p}_{3,f} + \ldots = \vec{p}_{1,i} + \vec{p}_{2,i} + \vec{p}_{3,i} + \ldots$$

- However, the momenta of components of the system may change.

Example

- A rifle has a mass of 4.5 kg and it fires a bullet of 10.0 grams at a muzzle speed of 820 m/s. What is the recoil speed of the rifle as the bullet leaves the barrel?

As long as the rifle is horizontal, there will be no net external force acting on the rifle-bullet system and momentum will be conserved.

$$\mathbf{p}_i = \mathbf{p}_f$$

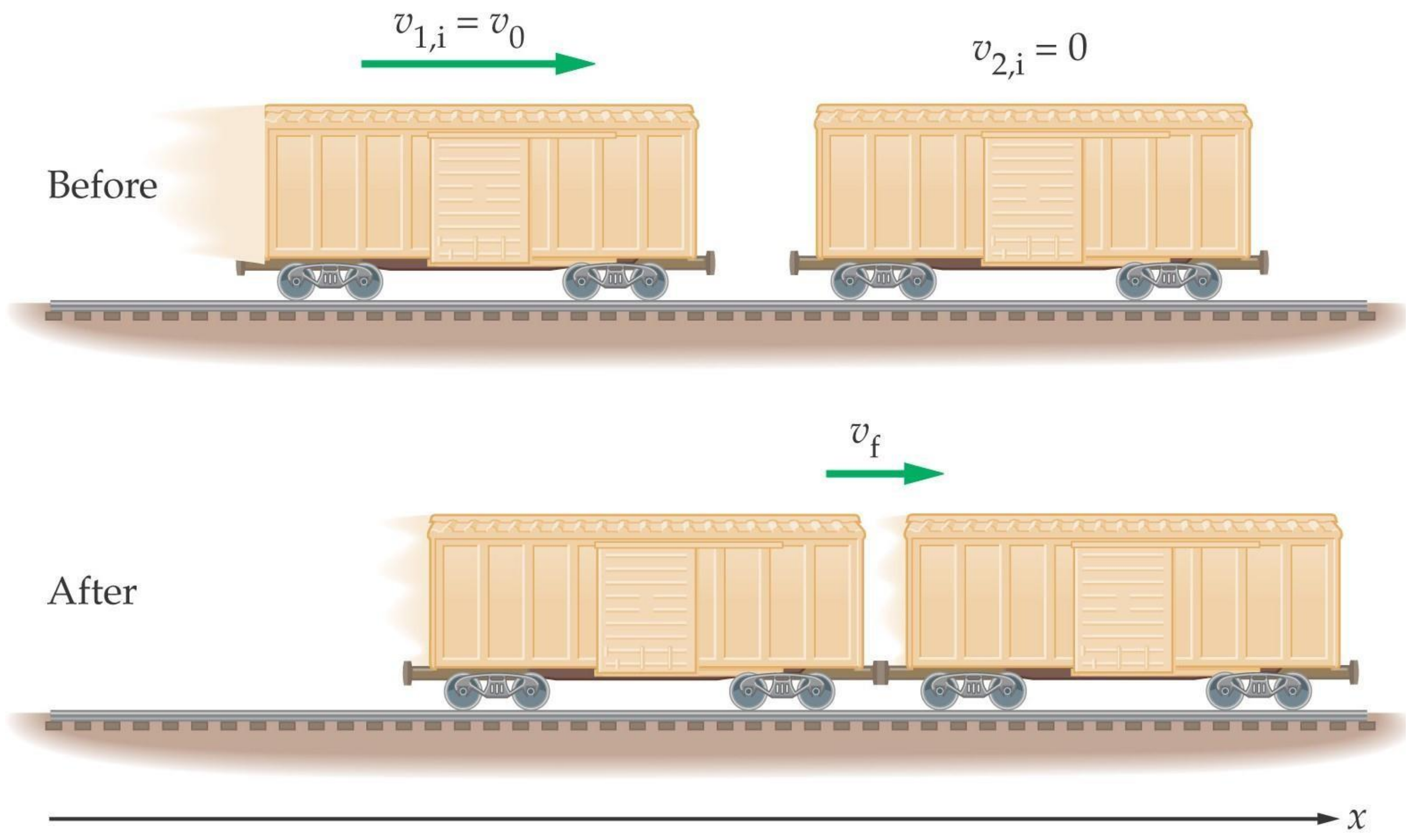
$$0 = m_b v_b + m_r v_r$$

$$v_r = -\frac{m_b}{m_r} v_b = -\left(\frac{0.01\,\text{kg}}{4.5\,\text{kg}}\right) 820\,\text{m/s} = -1.82\,\text{m/s}$$

5-Inelastic collisions

- **Collision**: two objects striking one another
- Time of collision is short enough that external forces may be ignored
- **Inelastic collision**: momentum is conserved but kinetic energy is not
- **Completely inelastic collision**: objects stick together afterwards

- A completely inelastic collision



- Solving for the final momentum in terms of the initial momenta and masses:

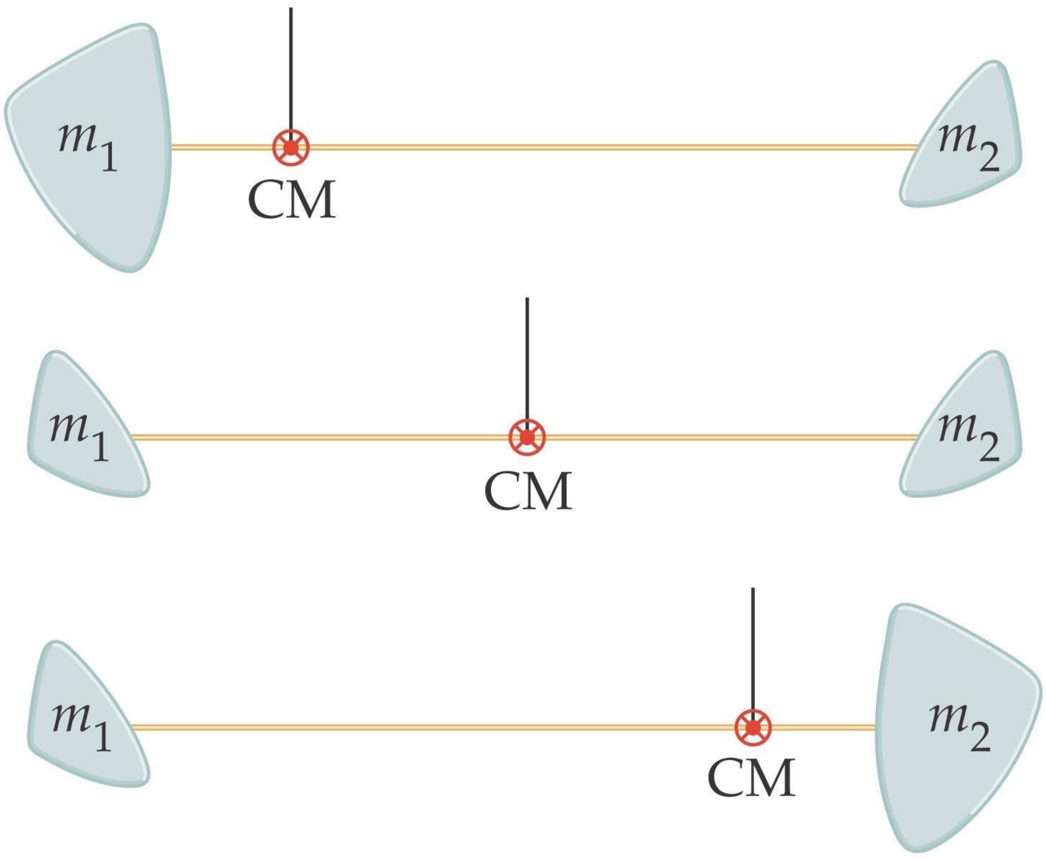
$$p_{\mathrm{i}} = m_1 v_{1,\mathrm{i}} + m_2 v_{2,\mathrm{i}}$$

$$p_{\mathrm{f}} = (m_1 + m_2) v_{\mathrm{f}}$$

$$v_{\mathrm{f}} = \frac{m_1 v_{1,\mathrm{i}} + m_2 v_{2,\mathrm{i}}}{m_1 + m_2}$$

6-Center ofMass

- The center of mass of a system is the point where the system can be balanced in a uniform gravitational field.



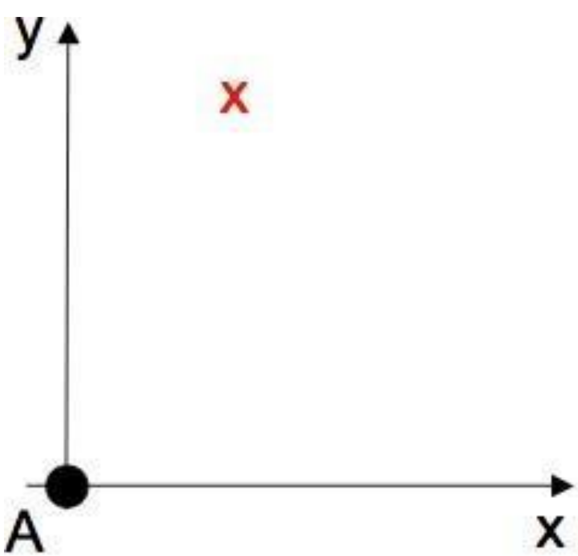
- For two objects:

$$X_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{m_1 x_1 + m_2 x_2}{M}$$

- The center of mass is closer to the more massive object.

Example

- Particle A is at the origin and has a mass of 30.0 grams. Particle Bhas a mass of 10.0 grams. Where must particle B be located so that the center of mass (marked with a red x) is located at the point (2.0 cm,
5.0 cm)?



$$x_{cm} = \frac{m_a x_a + m_b x_b}{m_a + m_b} = \frac{m_b x_b}{m_a + m_b}$$

$$y_{cm} = \frac{m_a y_a + m_b y_b}{m_a + m_b} = \frac{m_b y_b}{m_a + m_b}$$

$$x_{cm} = \frac{(10\text{ g})x_b}{10\text{ g} + 30\text{g}} = 2\text{ cm}$$

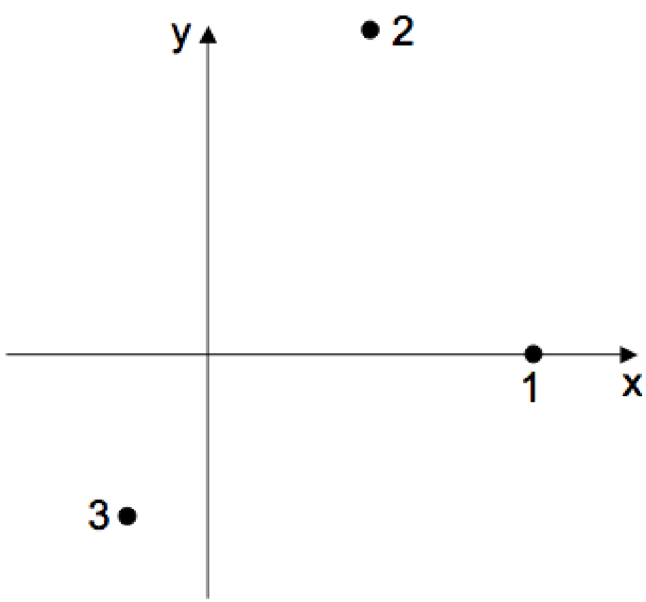
$$x_b = 8\text{ cm}$$

$$y_{cm} = \frac{(10\text{ g})y_b}{30\text{ g} + 10\text{ g}} = 5\text{ cm}$$

$$y_b = 20\text{ cm}$$

Example

- The positions of three particles are (4.0 m, 0.0 m),(2.0 m, 4.0 m), and (−1.0 m, −2.0 m). The masses are 4.0 kg,6.0 kg, and 3.0 kg respectively. What is the location of the center of mass



$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

$$= \frac{(4\,\text{kg})(4\,\text{m}) + (6\,\text{kg})(2\,\text{m}) + (3\,\text{kg})(-1\,\text{m})}{(4+6+3)\text{kg}}$$

$$= 1.92\,\text{m}$$

$$y_{cm} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3}$$

$$= \frac{(4\,\text{kg})(0\,\text{m}) + (6\,\text{kg})(4\,\text{m}) + (3\,\text{kg})(-2\,\text{m})}{(4+6+3)\text{kg}}$$

$$= 1.38\,\text{m}$$