Lecture 11 first stage

Work Done by a Constant Force

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Work Done by a Constant Force

Definition: Work

The work done by a constant force $\mathbf{F}$ on an object is equal to the component of the force in the direction of the displacement times the magnitude of the displacement:

Note that the component of the force in the direction of the

$$\mathbf{W}=\mathbf{F}\cdot\mathbf{r} = |\mathbf{F}||\mathbf{r}| \cos\theta = |\mathbf{F}| \cos\theta|\mathbf{r}|$$

displacement can be positive, zero, or negative so the work may be positive, zero, or negative

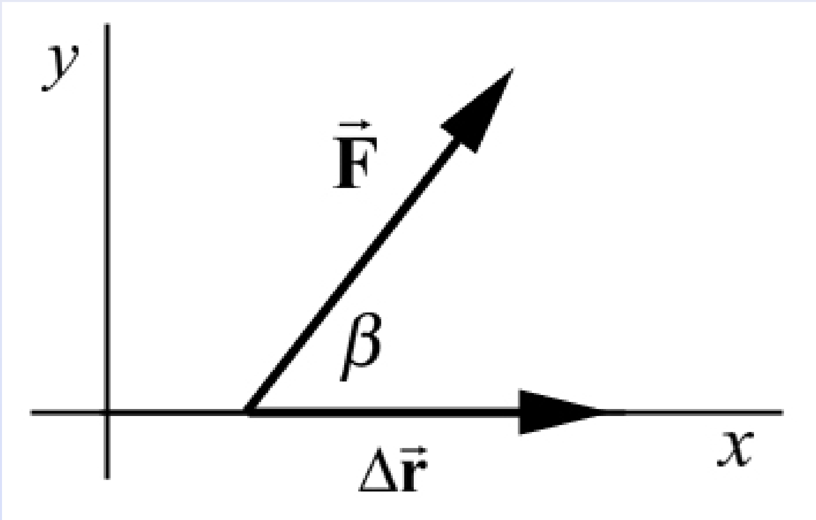
Worked Example: Work Done by a Constant Force in Two Dimensions

Force exerted on the object:

$$\vec{\mathbf{F}} = F_x\,\hat{\mathbf{i}} + F_y\,\hat{\mathbf{j}}$$

Components:

$$F_x = F\cos\beta \qquad F_y = F\sin\beta$$



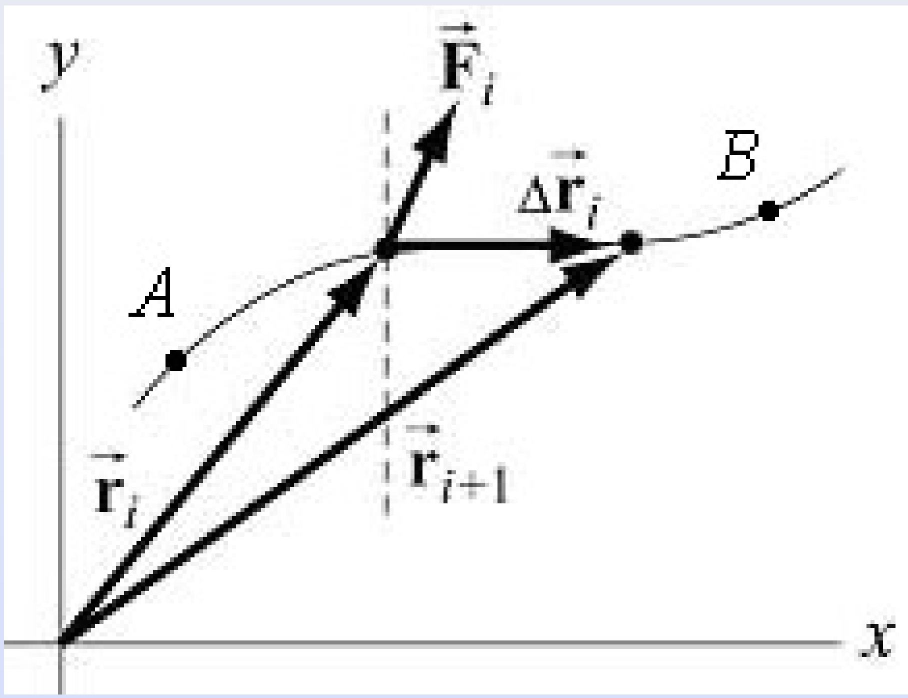
Consider an object undergoing displacement:

$$\vec{\mathbf{r}} = x\,\hat{\mathbf{i}}$$

Work done by force on object:

$$W = \vec{\mathbf{F}}\cdot\vec{\mathbf{r}} = Fx\cos\beta = (F_x\,\hat{\mathbf{i}} + F_y\,\hat{\mathbf{j}})\cdot(x\,\hat{\mathbf{i}}) = F_x . x$$

Work Done Along an Arbitrary Path



Work done by force for small displacement

$$W_i = \mathbf{F}_i \cdot \mathbf{r}$$

Work done by force along path from A to B

$$W_{AB} = \lim_{N\to\infty} \sum_{i=1} \mathbf{F}_i \cdot \mathbf{r}_i \equiv \int_A^B \mathbf{F} \cdot d\mathbf{r}$$

Work-Energy Theorem in Three-Dimensions

As you will show in the problem set, the one dimensional work-kinetic energy theorem generalizes to three dimensions

$$W_{AB} = \int_A^B \mathbf{F} \cdot d\mathbf{r} = \int_A^B m\mathbf{a} \cdot d\mathbf{r} = \int_A^B m\frac{d\mathbf{v}}{dt} \cdot d\mathbf{r} = \int_A^B m\,d\mathbf{v} \cdot \frac{d\mathbf{r}}{dt} = \int_A^B m\,d\mathbf{v} \cdot \mathbf{v}$$

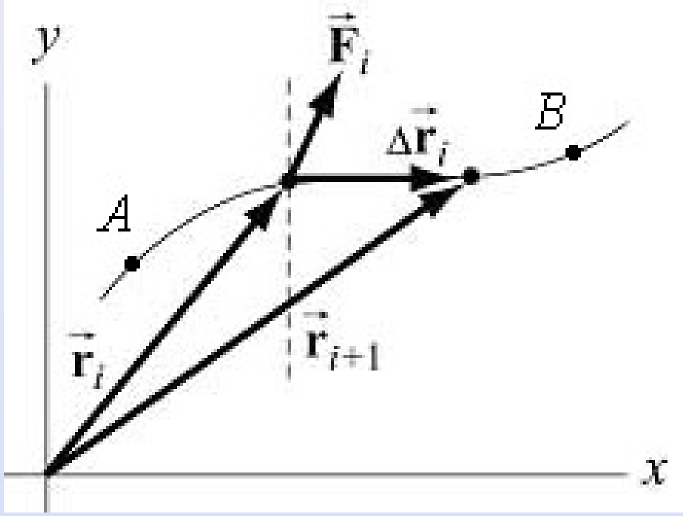
$$= \frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2 = K_B - K_A$$

$$= K$$

Work: Path Dependent Line Integral

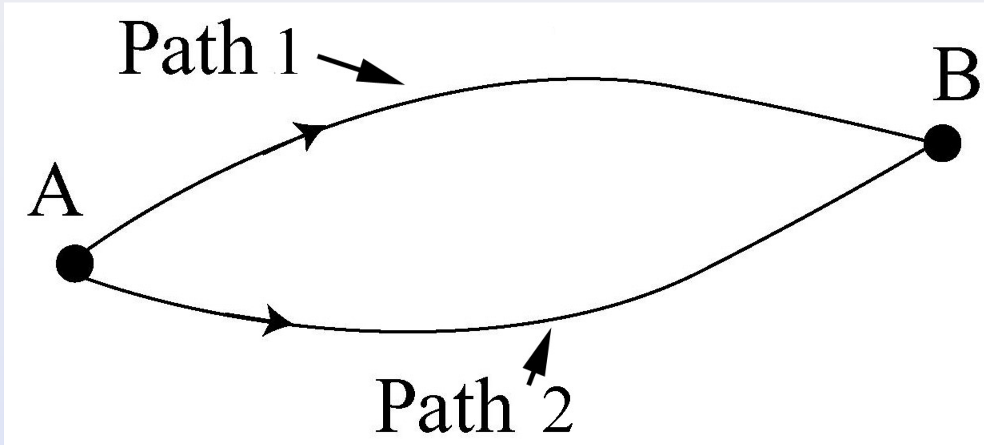
Work done by force along path from A to B

$$W_{AB} = \lim_{N\to\infty} \sum_{i=1}^{i=N} \mathbf{F}_i \cdot \mathbf{r}_i \equiv \int_A^B \mathbf{F} \cdot d\mathbf{r}$$



In order to calculate the line integral, in principle, requires a knowledge of the path. However we will consider an important class of forces in which the work line integral is independent of the path and only depends on the starting and end points

Conservative Forces



Definition: Conservative Force If the work done by a force in moving an object from point A to point B is independent of the path (1 or 2),

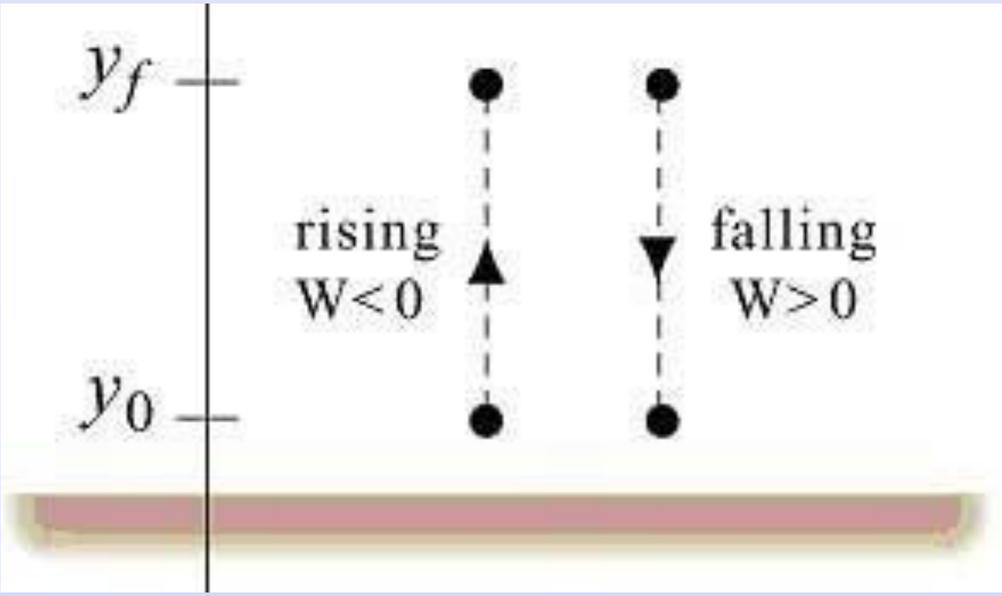
$$W_c \equiv \int_A^B \mathbf{F}_c \cdot d\mathbf{r} \qquad \text{(path independent)}$$

then the force is called a *conservative force* which we denote by $\mathbf{F}_c$. Then the work done only depends on the location of the points A and B.

Example: Gravitational Force

Consider the motion of an object under the influence of a gravitational force near the surface of the earth

The work done by gravity depends only on the change in the vertical position



$$W_g = F_g y = -mgy$$

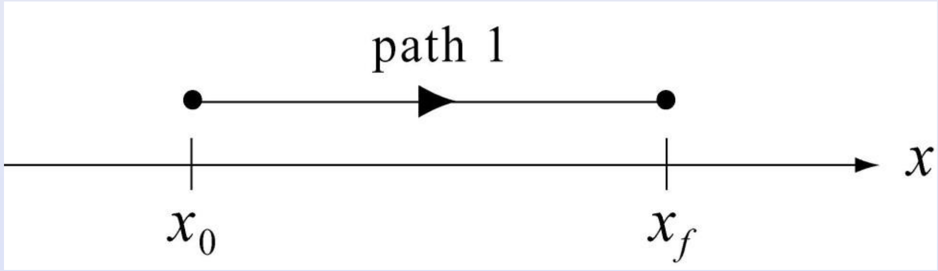
Non-Conservative Forces

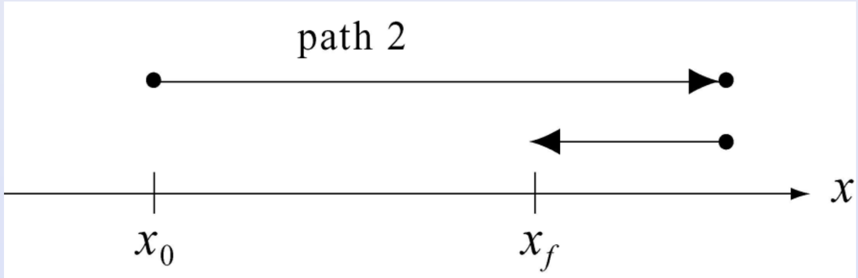
Definition: Non-conservative force Whenever the work done by a force in moving an object from an initial point to a final point depends on the path, then the force is called a *non-conservative force* $\mathbf{F}_{nc}$ and the work done is called *non-conservative work*

$$W_{nc} \equiv \int_A^B \mathbf{F}_{nc} \cdot d\mathbf{r}$$

Non-Conservative Forces

Work done on the object by the force depends on the path taken by the object



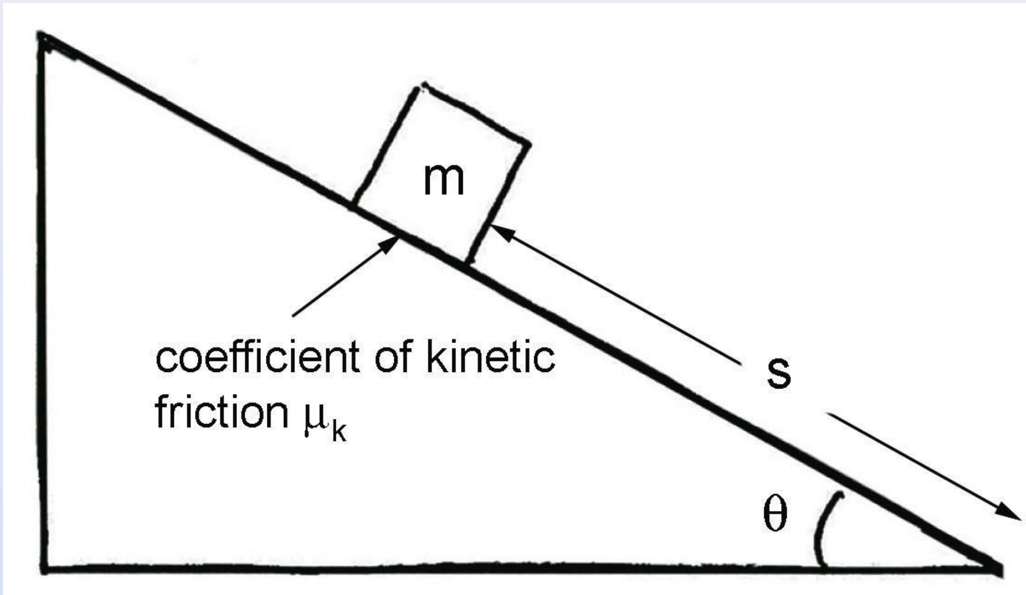


Example: friction on an object moving on a level surface

$$F_{\text{friction}} = \mu_k N$$

$$W_{\text{friction}} = -F_{\text{friction}} x \quad = -\mu_k N x < 0$$

Table Problem: Work Constant Forces and Scalar Product



An object of mass m, starting from rest, slides down an inclined plane of length s. The plane is inclined by an angle of θ to the ground. The coefficient of kinetic friction is μ_k. What is the kinetic energy of the object after it slides down the inclined plane a distance s?