SECTION B APPLICATIONS OF FRICTION IN MACHINES

In Section B we investigate the action of friction in various machine applications. Because the conditions in these applications are normally either limiting static or kinetic friction, we will use the variable μ (rather than μ_s or μ_k) in general. Depending on whether motion is impending or actually occurring, μ can be interpreted as either the static or kinetic coefficient of friction.

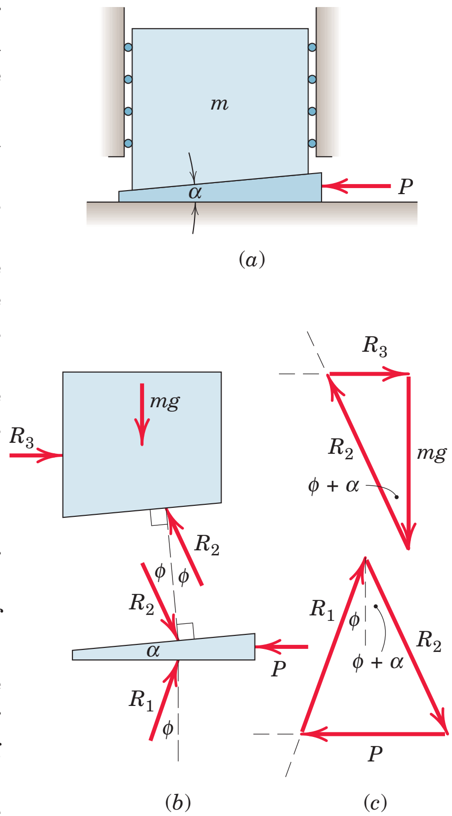
6/4 Wedges

A wedge is one of the simplest and most useful machines. A wedge is used to produce small adjustments in the position of a body or to apply large forces. Wedges largely depend on friction to function. When sliding of a wedge is impending, the resultant force on each sliding surface of the wedge will be inclined from the normal to the surface by an amount equal to the friction angle. The component of the resultant along the surface is the friction force, which is always in the direction to oppose the motion of the wedge relative to the mating surfaces.

Figure 6/3*a* shows a wedge used to position or lift a large mass m, where the vertical loading is mg. The coefficient of friction for each pair of surfaces is $\mu = \tan \phi$. The force P required to start the wedge is found from the equilibrium triangles of the forces on the load and on the wedge. The free-body diagrams are shown in Fig. 6/3*b*, where the reactions are inclined at an angle ϕ from their respective normals and are in the direction to oppose the motion. We neglect the mass of the wedge. From the free-body diagrams we write the force equilibrium conditions by equating to zero the sum of the force vectors acting on each body. The solutions of these equations are shown in part *c* of the figure, where R_2 is found first in the upper diagram using the known value of mg. The force P is then found from the lower triangle once the value of R_2 has been established.

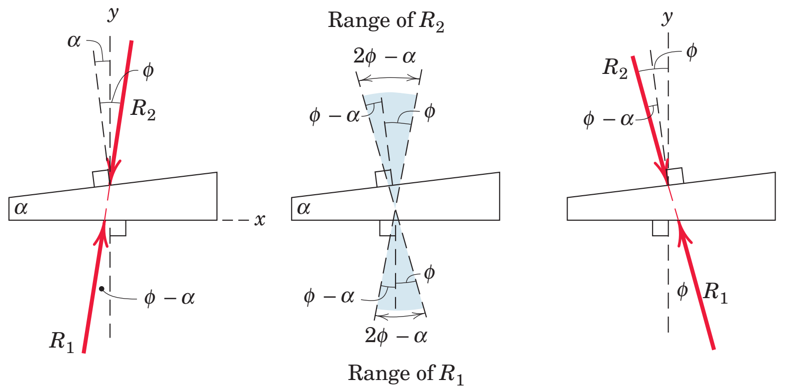
If P is removed and the wedge remains in place, equilibrium of the wedge requires that the equal reactions R_1 and R_2 be collinear as shown in Fig. 6/4, where the wedge angle α is taken to be less than ϕ. Part *a* of the figure represents impending slippage at the upper surface, and part *c* of the figure represents impending slippage at the lower surface. In order for the wedge to slide out of its space, slippage must occur at *both* surfaces simultaneously; otherwise, the wedge is *self-locking*, and there is a finite range of possible intermediate angular positions of R_1 and R_2 for which the wedge will remain in place. Figure 6/4*b* illustrates this range and shows that simultaneous slippage is not possible if $\alpha < 2\phi$. You are encouraged to construct additional diagrams for the case where $\alpha > \phi$ and verify that the wedge is self-locking as long as $\alpha < 2\phi$.

If the wedge is self-locking and is to be withdrawn, a pull P on the wedge will be required. To oppose the new impending motion, the reactions R_1 and R_2 must act on the opposite sides of their normals from those when the wedge was inserted. The solution can be obtained as



Forces to raise load

Figure 6/3



(a) Slipping impending at upper surface

(b) Range of $R_1 = R_2$ for no slip

(c) Slipping impending at lower surface

Figure 6/4

with the case of raising the load. The free-body diagrams and vector polygons for this condition are shown in Fig. 6/5.

Wedge problems lend themselves to graphical solutions as indicated in the three figures. The accuracy of a graphical solution is easily held within tolerances consistent with the uncertainty of friction coefficients. Algebraic solutions may also be obtained from the trigonometry of the equilibrium polygons.

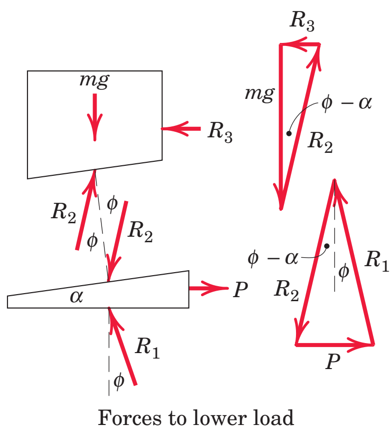


Figure 6/5

6/5 Screws

Screws are used for fastening and for transmitting power or motion. In each case the friction developed in the threads largely determines the action of the screw. For transmitting power or motion the square thread is more efficient than the V-thread, and the analysis here is confined to the square thread.

Force Analysis

Consider the square-threaded jack, Fig. 6/6, under the action of the axial load W and a moment M applied about the axis of the screw. The screw has a lead L (advancement per revolution) and a mean radius r. The force R exerted by the thread of the jack frame on a small representative portion of the screw thread is shown on the free-body diagram of the screw. Similar reactions exist on all segments of the screw thread where contact occurs with the thread of the base.

If M is just sufficient to turn the screw, the thread of the screw will slide around and up on the fixed thread of the frame. The angle ϕ made by R with the normal to the thread is the angle of friction, so that $\tan \phi = \mu$. The moment of R about the vertical axis of the screw is $Rr \sin(\alpha + \phi)$, and the total moment due to all reactions on the threads is $\Sigma Rr \sin(\alpha + \phi)$. Since $r \sin(\alpha + \phi)$ appears in each term, we may factor it out. The moment equilibrium equation for the screw becomes

$$M = [r \sin(\alpha + \phi)]\, \Sigma R$$

Equilibrium of forces in the axial direction further requires that

$$W = \Sigma R \cos(\alpha + \phi) = [\cos(\alpha + \phi)]\, \Sigma R$$

Combining the expressions for M and W gives

$$M = Wr \tan(\alpha + \phi) \tag{6/3}$$

To determine the helix angle α, unwrap the thread of the screw for one complete turn and note that $\alpha = \tan^{-1}(L/2\pi r)$.

We may use the unwrapped thread of the screw as an alternative model to simulate the action of the entire screw, as shown in Fig. 6/7a. The equivalent force required to push the movable thread up the fixed incline is $P = M/r$, and the triangle of force vectors gives Eq. 6/3 immediately.

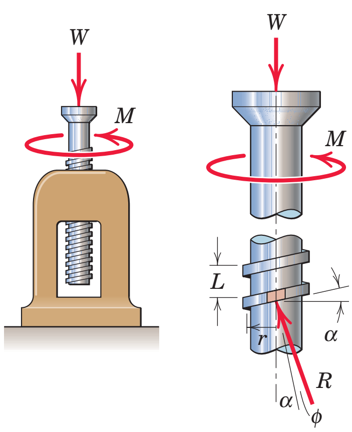
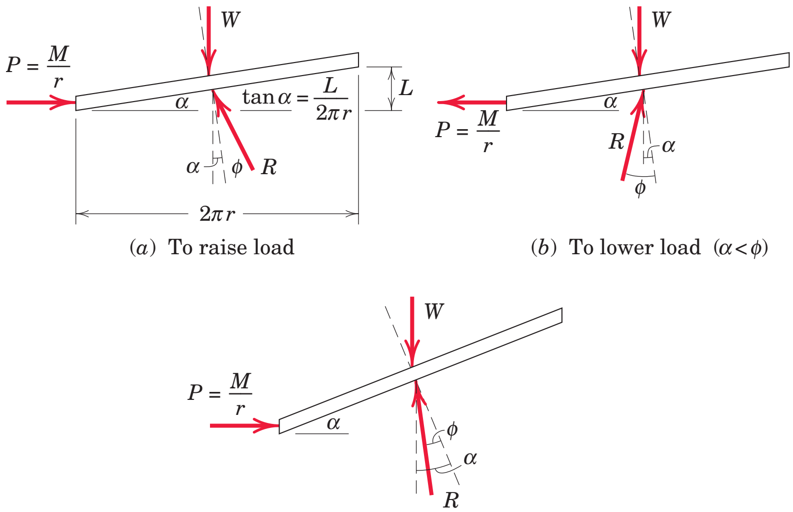


Figure 6/6



(a) To raise load (b) To lower load ($\alpha < \phi$)

(c) To lower load ($\alpha > \phi$)

Figure 6/7

Conditions for Unwinding

If the moment M is removed, the friction force changes direction so that ϕ is measured to the other side of the normal to the thread. The screw will remain in place and be self-locking provided that $\alpha < \phi$, and will be on the verge of unwinding if $\alpha = \phi$.

To lower the load by unwinding the screw, we must reverse the direction of M as long as $\alpha < \phi$. This condition is illustrated in Fig. 6/7b for our simulated thread on the fixed incline. An equivalent force $P = M/r$ must be applied to the thread to pull it down the incline. From the triangle of vectors we therefore obtain the moment required to lower the screw, which is

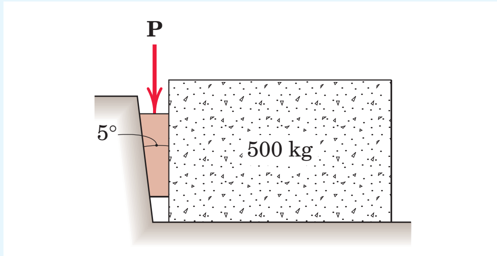
$$M = Wr \tan(\phi - \alpha) \qquad \textbf{(6/3}\boldsymbol{a}\textbf{)}$$

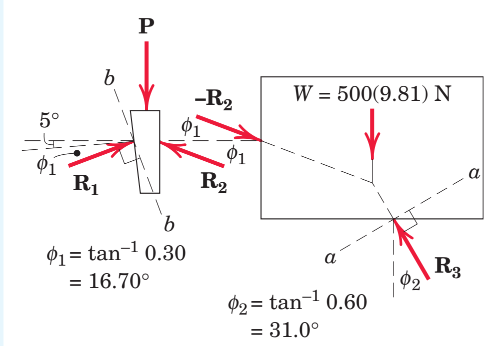
If $\alpha > \phi$, the screw will unwind by itself, and Fig. 6/7c shows that the moment required to prevent unwinding is

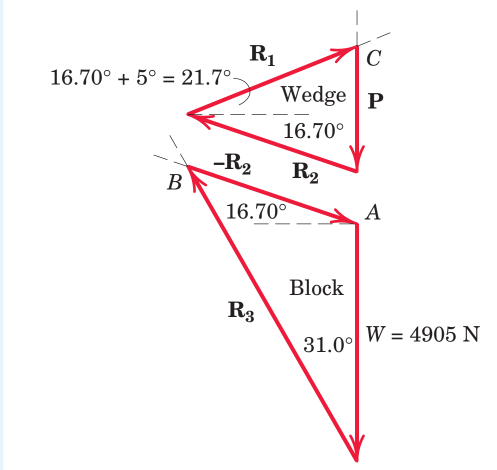
$$M = Wr \tan(\alpha - \phi) \qquad \textbf{(6/3}\boldsymbol{b}\textbf{)}$$

SAMPLE PROBLEM 6/6

The horizontal position of the 500-kg rectangular block of concrete is adjusted by the 5° wedge under the action of the force **P**. If the coefficient of static friction for both wedge surfaces is 0.30 and if the coefficient of static friction between the block and the horizontal surface is 0.60, determine the least force P required to move the block.







Solution. The free-body diagrams of the wedge and the block are drawn with the reactions $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ inclined with respect to their normals by the ①amount of the friction angles for impending motion. The friction angle for limiting static friction is given by $\phi = \tan^{-1} \mu$. Each of the two friction angles is computed and shown on the diagram.

We start our vector diagram expressing the equilibrium of the block at a convenient point A and draw the only known vector, the weight **W** of the block. Next we add $\mathbf{R}_3$, whose 31.0° inclination from the vertical is now known. The vector $-\mathbf{R}_2$, whose 16.70° inclination from the horizontal is also known, must close the polygon for equilibrium. Thus, point B on the lower polygon is determined by the intersection of the known directions of $\mathbf{R}_3$ and $-\mathbf{R}_2$, and their magnitudes become known.

For the wedge we draw $\mathbf{R}_2$, which is now known, and add $\mathbf{R}_1$, whose direction is known. The directions of $\mathbf{R}_1$ and **P** intersect at C, thus giving us the solution for the magnitude of **P**.

Algebraic solution. The simplest choice of reference axes for calculation purposes is, for the block, in the direction a-a normal to $\mathbf{R}_3$ and, for the wedge, in ②the direction b-b normal to $\mathbf{R}_1$. The angle between $\mathbf{R}_2$ and the a-direction is $16.70° + 31.0° = 47.7°$. Thus, for the block

$$[\Sigma F_a = 0] \qquad 500(9.81) \sin 31.0° - R_2 \cos 47.7° = 0$$

$$R_2 = 3750 \text{ N}$$

For the wedge the angle between $\mathbf{R}_2$ and the b-direction is $90° - (2\phi_1 + 5°) = 51.6°$, and the angle between **P** and the b-direction is $\phi_1 + 5° = 21.7°$. Thus,

$$[\Sigma F_b = 0] \qquad 3750 \cos 51.6° - P \cos 21.7° = 0$$

$$P = 2500 \text{ N} \qquad \textit{Ans.}$$

Graphical solution. The accuracy of a graphical solution is well within the uncertainty of the friction coefficients and provides a simple and direct result. By laying off the vectors to a reasonable scale following the sequence described, we obtain the magnitudes of **P** and the **R**'s easily by scaling them directly from the diagrams.

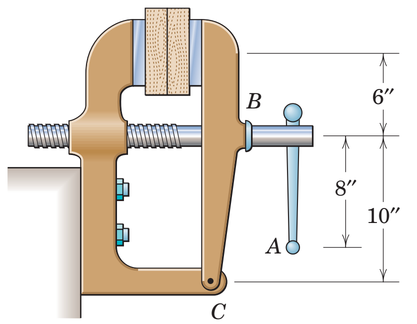
Helpful Hints

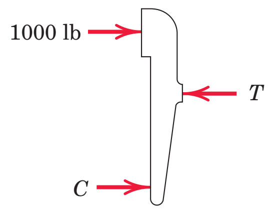
① Be certain to note that the reactions are inclined from their normals in the direction to oppose the motion. Also, we note the equal and opposite reactions $\mathbf{R}_2$ and $-\mathbf{R}_2$.

② It should be evident that we avoid simultaneous equations by eliminating reference to $\mathbf{R}_3$ for the block and $\mathbf{R}_1$ for the wedge.

SAMPLE PROBLEM 6/7

The single-threaded screw of the vise has a mean diameter of 1 in. and has 5 square threads per inch. The coefficient of static friction in the threads is 0.20. A 60-lb pull applied normal to the handle at A produces a clamping force of 1000 lb between the jaws of the vise. (a) Determine the frictional moment M_B, developed at B, due to the thrust of the screw against the body of the jaw. (b) Determine the force Q applied normal to the handle at A required to loosen the vise.





Solution. From the free-body diagram of the jaw we first obtain the tension T in the screw.

$[\Sigma M_C = 0]$ $\qquad 1000(16) - 10T = 0 \qquad T = 1600$ lb

The helix angle α and the friction angle ϕ for the thread are given by

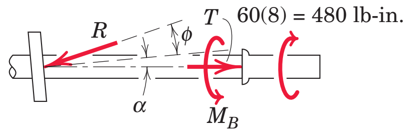
1

$$\alpha = \tan^{-1}\frac{L}{2\pi r} = \tan^{-1}\frac{1/5}{2\pi(0.5)} = 3.64°$$

$$\phi = \tan^{-1}\mu = \tan^{-1} 0.20 = 11.31°$$

where the mean radius of the thread is $r = 0.5$ in.

(a) To tighten. The isolated screw is simulated by the free-body diagram shown where all of the forces acting on the threads of the screw are represented by a single force R inclined at the friction angle ϕ from the normal to the thread. The moment applied about the screw axis is $60(8) = 480$ lb-in. in the clockwise direction as seen from the front of the vise. The frictional moment M_B due to the friction forces acting on the collar at B is in the counterclockwise direction to oppose the impending motion. From Eq. 6/3 with T substituted for W the net moment acting on the screw is

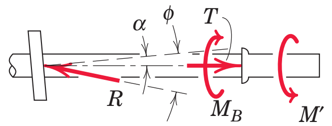


(a) To tighten

$$M = Tr \tan(\alpha + \phi)$$

$$480 - M_B = 1600(0.5) \tan(3.64° + 11.31°)$$

$$M_B = 266 \text{ lb-in.} \qquad \textit{Ans.}$$



(b) To loosen

(b) To loosen. The free-body diagram of the screw on the verge of being loosened is shown with R acting at the friction angle from the normal in the direc-
2 tion to counteract the impending motion. Also shown is the frictional moment $M_B = 266$ lb-in. acting in the clockwise direction to oppose the motion. The angle between R and the screw axis is now $\phi - \alpha$, and we use Eq. 6/3a with the net moment equal to the applied moment M' minus M_B. Thus

$$M = Tr \tan(\phi - \alpha)$$

$$M' - 266 = 1600(0.5) \tan(11.31° - 3.64°)$$

$$M' = 374 \text{ lb-in.}$$

Thus, the force on the handle required to loosen the vise is

$$Q = M'/d = 374/8 = 46.8 \text{ lb} \qquad \textit{Ans.}$$

Helpful Hints

1 Be careful to calculate the helix angle correctly. Its tangent is the lead L (advancement per revolution) divided by the mean circumference $2\pi r$ and not by the diameter $2r$.

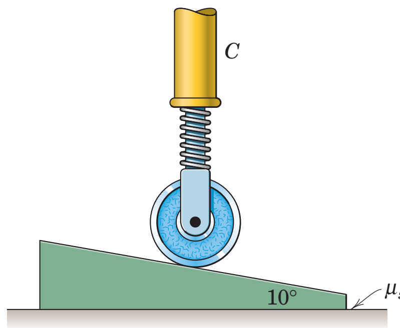
2 Note that R swings to the opposite side of the normal as the impending motion reverses direction.

PROBLEMS

(Unless otherwise instructed, neglect the weights of the wedges and screws in the problems which follow.)

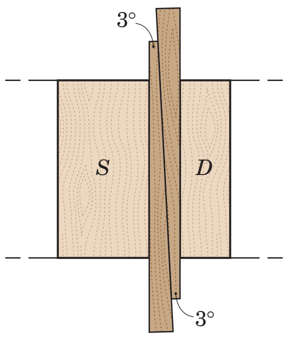
Introductory Problems

6/53 The 10° wedge is driven under the spring-loaded wheel whose supporting strut C is fixed. Determine the minimum coefficient of static friction μ_s for which the wedge will remain in place. Neglect all friction associated with the wheel.



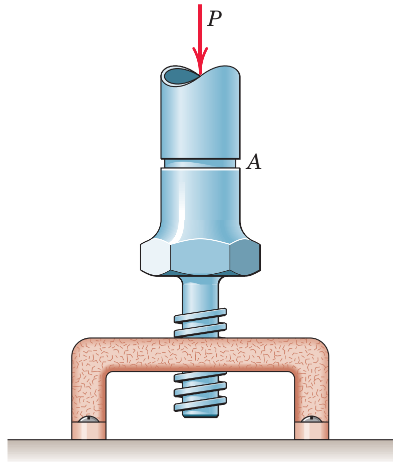
Problem 6/53

6/54 In wood-frame construction, two shims are frequently used to fill the gap between the framing S and the thinner window/door jamb D. The members S and D are shown in cross section in the figure. For the 3° shims shown, determine the minimum necessary coefficient of static friction necessary so that the shims will remain in place.



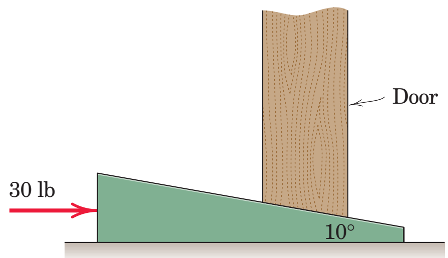
Problem 6/54

6/55 The device shown is used for coarse adjustment of the height of an experimental apparatus without a change in its horizontal position. Because of the slipjoint at A, turning the screw does not rotate the cylindrical leg above A. The mean diameter of the thread is $\frac{3}{8}$ in. and the coefficient of friction is 0.15. For a conservative design which neglects friction at the slipjoint, what should be the minimum number N of threads per inch to ensure that the single-threaded screw does not turn by itself under the weight of the apparatus?



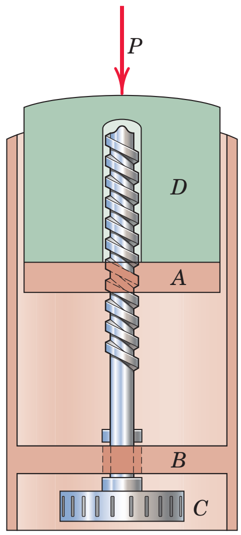
Problem 6/55

6/56 The 10° doorstop is inserted with a rightward horizontal force of 30 lb. If the coefficient of static friction for all surfaces is $\mu_s = 0.20$, determine the values N_U and N_L of the normal forces on the upper and lower faces of the doorstop. With the given information, can you determine the force P required to extract the doorstop?



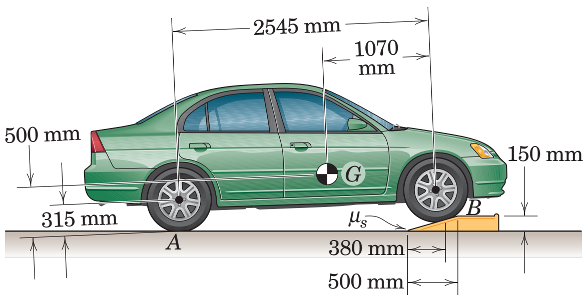
Problem 6/56

6/57 The elements of a deodorant dispenser are shown in the figure. To advance the deodorant D, the knob C is turned. The threaded shaft engages the threads in the movable deodorant base A; the shaft has no threads at the fixed support B. The 6-mm-diameter double square thread has a lead of 5 mm. Determine the coefficient of static friction μ_s for which the deodorant will not retract under the action of the force P. Neglect friction at B.



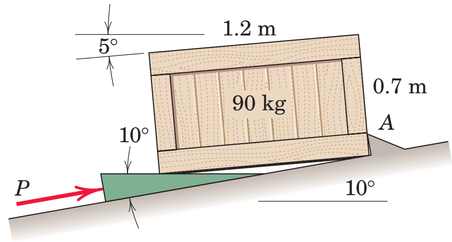
Problem 6/57

6/58 A 1600-kg rear-wheel-drive car is being driven up the ramp at a slow steady speed. Determine the minimum coefficient of static friction μ_s for which the portable ramp will not slip forward. Also determine the required friction force F_A at each rear drive wheel.



Problem 6/58

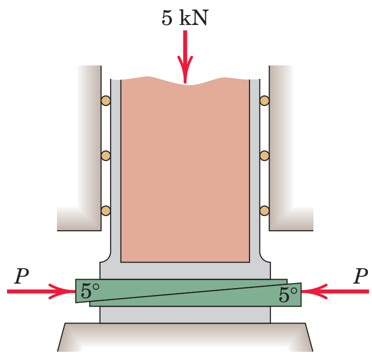
6/59 Determine the force P required to force the 10° wedge under the 90-kg uniform crate which rests against the small stop at A. The coefficient of friction for all surfaces is 0.40.



Problem 6/59

Representative Problems

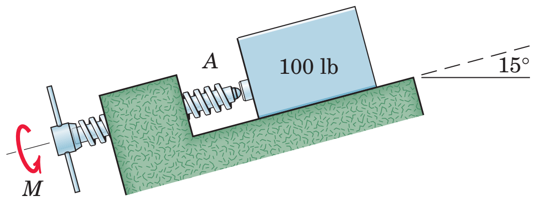
6/60 The two 5° wedges shown are used to adjust the position of the column under a vertical load of 5 kN. Determine the magnitude of the forces P required to raise the column if the coefficient of friction for all surfaces is 0.40.



Problem 6/60

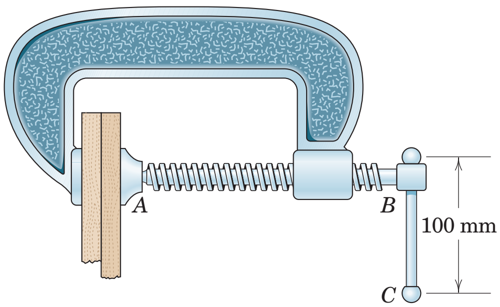
6/61 If the loaded column of Prob. 6/60 is to be lowered, calculate the horizontal forces P' required to withdraw the wedges.

6/62 Determine the torque M which must be applied to the handle of the screw to begin moving the 100-lb block up the 15° incline. The coefficient of static friction between the block and the incline is 0.50, and the single-thread screw has square threads with a mean diameter of 1 in. and advances 0.4 in. for each complete turn. The coefficient of static friction for the threads is also 0.50. Neglect friction at the small ball joint A.



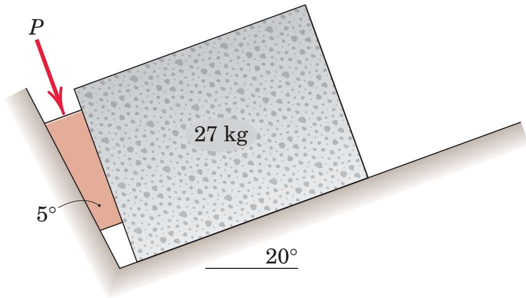
Problem 6/62

6/63 A compressive force of 600 N is to be applied to the two boards in the grip of the C-clamp. The threaded screw has a mean diameter of 10 mm and advances 2.5 mm per turn. The coefficient of static friction is 0.20. Determine the force F which must be applied normal to the handle at C in order to (a) tighten and (b) loosen the clamp. Neglect friction at point A.



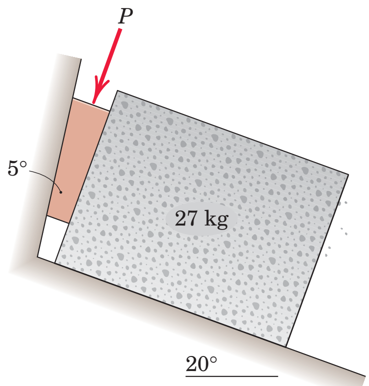
Problem 6/63

6/64 The coefficient of static friction for both wedge surfaces is 0.40 and that between the 27-kg concrete block and the 20° incline is 0.70. Determine the minimum value of the force P required to begin moving the block up the incline. Neglect the weight of the wedge.



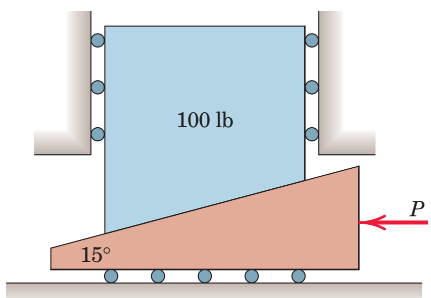
Problem 6/64

6/65 Repeat Prob. 6/64, only now the 27-kg concrete block begins to move down the 20° incline as shown. All other conditions remain as in Prob. 6/64.



Problem 6/65

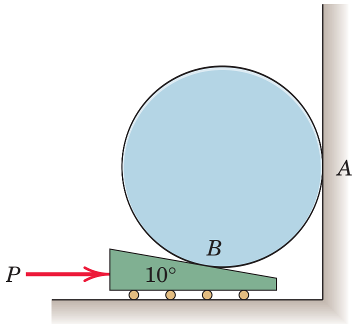
6/66 The coefficient of static friction μ_s between the 100-lb body and the 15° wedge is 0.20. Determine the magnitude of the force P required to begin raising the 100-lb body if (a) rollers of negligible friction are present under the wedge, as illustrated, and (b) the rollers are removed and the coefficient of static friction $\mu_s = 0.20$ applies at this surface as well.



Problem 6/66

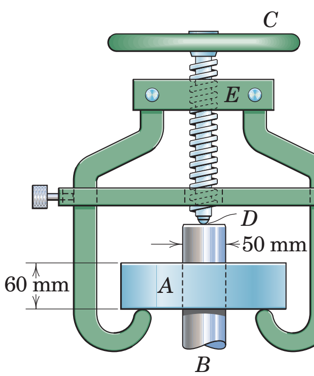
6/67 For both conditions (*a*) and (*b*) as stated in Prob. 6/66, determine the magnitude and direction of the force P' required to begin lowering the 100-lb body.

6/68 Calculate the horizontal force P on the light 10° wedge necessary to initiate movement of the 40-kg cylinder. The coefficient of static friction for both pairs of contacting surfaces is 0.25. Also determine the friction force F_B at point B. (*Caution:* Check carefully your assumption of where slipping occurs.)



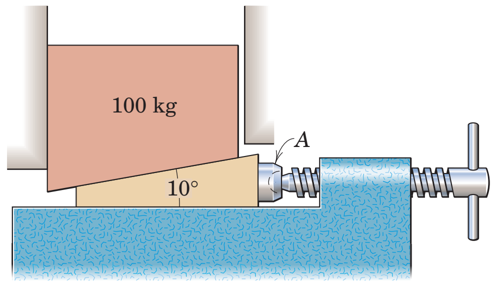
Problem 6/68

6/69 The collar A has a force fit on shaft B and is to be removed from the shaft by the wheel-puller mechanism shown. The screw has a single square thread with a mean diameter of 20 mm and a lead L of 6 mm. If a torque of 24 N·m is required to turn wheel C to slip the collar off the shaft, determine the average pressure p (compressive stress) between the collar and the shaft. The coefficient of friction for the screw at E is 0.25, and that for the shaft and collar is 0.30. Friction at the ball end D of the shaft is negligible.



Problem 6/69

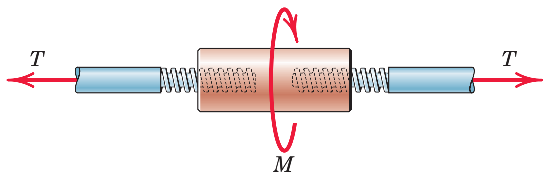
6/70 The vertical position of the 100-kg block is adjusted by the screw-activated wedge. Calculate the moment M which must be applied to the handle of the screw to raise the block. The single-thread screw has square threads with a mean diameter of 30 mm and advances 10 mm for each complete turn. The coefficient of friction for the screw threads is 0.25, and the coefficient of friction for all mating surfaces of the block and wedge is 0.40. Neglect friction at the ball joint A.



Problem 6/70

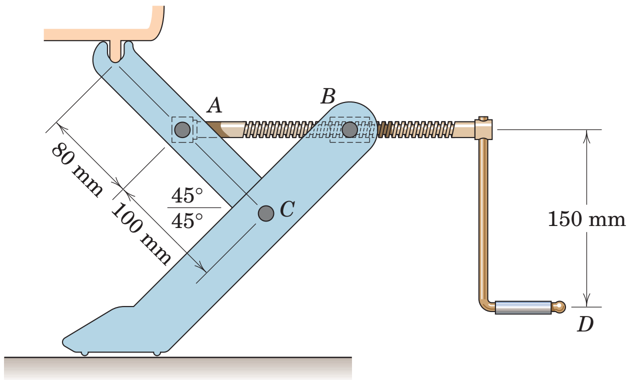
6/71 Calculate the moment M' which must be applied to the handle of the screw of Prob. 6/70 to withdraw the wedge and lower the 100-kg load.

6/72 The threaded collar is used to connect two shafts, both with right-hand threads on their ends. The shafts are under a tension T = 8 kN. If the threads have a mean diameter of 16 mm and a lead of 4 mm, calculate the torque M required to turn the collar in either direction with the shafts prevented from turning. The coefficient of friction is 0.24.



Problem 6/72

6/73 The jack shown is designed to lift small unit-body cars. The screw is threaded into the collar pivoted at B, and the shaft turns in a ball thrust bearing at A. The thread has a mean diameter of 10 mm and a lead (advancement per revolution) of 2 mm. The coefficient of friction for the threads is 0.20. Determine the force P normal to the handle at D required (a) to raise a mass of 500 kg from the position shown and (b) to lower the load from the same position. Neglect friction in the pivot and bearing at A.



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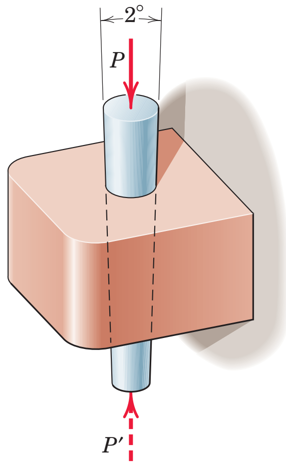
▶6/74 The 2° tapered pin is forced into a mating tapered hole in the fixed block with a force $P = 400$ N. If the force required to remove the pin (with $P = 0$) is $P' = 300$ N, determine the coefficient of friction between the pin and the surface of the hole. (*Hint:* The pressure (stress) normal to the tapered pin surface remains unchanged until the pin actually moves. The distributed forces over the surface of the pin may be replaced by an equivalent single resultant force.)



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