Numerical Methods

Lagrange's Multipliers Method

$$Pp + Qq = R$$

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

let l, m, n be functions of x, y, z

Each fraction of auxiliary equation will be equal to

$$\frac{l\,dx + m\,dy + n\,dz}{lP + mQ + nR}$$

ملاحظة/ نختار الضارب (Multiplier) بطريقة يكون فيها البسط مشتقة المقام

EX1/ solve $(y+z)p + (z+x)q = x+y$ (2)

Sol/ Lagrange's Auxiliary equation is

$$\frac{dx}{y+z} = \frac{dy}{z+x} = \frac{dz}{x+y}$$

choose 1, -1, 0 as multipliers

$$\frac{dx - dy}{y+\cancel{z} - \cancel{z} - x} = \frac{d(x-y)}{y-x} = \frac{d(x-y)}{-(x-y)} \quad \text{exact differential} \text{ — } ①$$

Again choose 0, 1, -1 as multipliers

$$\frac{dy - dz}{z+\cancel{x} - \cancel{x} - y} = \frac{d(y-z)}{-(y-z)} \text{ — } ②$$

finally choose multiplies 1, 1, 1

$$\frac{dx+dy+dz}{y+z+z+x+x+y} = \frac{dx+dy+dz}{2(x+y+z)} \text{ — } ③$$

Take all ①, ②, ③

$$\frac{d(x-y)}{-(x-y)} = \frac{d(y-z)}{-(y-z)} = \frac{d(x+y+z)}{2(x+y+z)}$$

Take first 2 terms ①, ②

$$\frac{d(x-y)}{-(x-y)} = \frac{d(y-z)}{-(y-z)}$$

$$\frac{d(x-y)}{x-y} = \frac{d(y-z)}{y-z}$$

بالتكامل

$$\log(x-y) = \log(y-z) + \log c_1$$

$$x-y = c_1(y-z)$$

$$\boxed{c_1 = \frac{(x-y)}{(y-z)}}$$

نأخذ الأول والثالث

$$\frac{d(x-y)}{-(x-y)} = \frac{d(x+y+z)}{2(x+y+z)}$$

$$\frac{2\,d(x-y)}{x-y} + \frac{d(x+y+z)}{x+y+z} = 0$$

بالتكامل

$$2\log(x-y) + \log(x+y+z) = 0$$

$$\boxed{(x-y)^2(x+y+z) = c_2}$$

$$\boxed{\phi\left[(x-y)^2(x+y+z),\ \frac{x-y}{y-z}\right] = 0}$$

Ex2/ Solve

$$(2x^2+y^2+z^2-2yz-zx-xy)p$$
$$+(x^2+2y^2+z^2-yz-2zx-xy)q$$
$$=x^2+y^2+2z^2-yz-zx-2xy$$

$$\frac{dx}{2x^2+y^2+z^2-2yz-zx-xy}=\frac{dy}{x^2+2y^2+z^2+yz-2zx-xy}$$

$$=\frac{dz}{x^2+y^2+2z^2-yz-zx-2xy}$$

choose $1,-1,0 \longrightarrow$ multipliers

$$\frac{dx-dy}{x^2-y^2-yz+zx}=\frac{dx-dy}{(x-y)(x+y+z)} \quad -(1)$$

choose $0,1,-1 \longrightarrow$ multipliers

$$\frac{dy-dz}{y^2-z^2-zx+xy}=\frac{dy-dz}{(y-z)(x+y+z)} \quad -(2)$$

choose $-1,0,1 \longrightarrow$ multipliers

$$\frac{dz-dx}{z^2-x^2-xy+yz}=\frac{dz-dx}{(z-x)(x+y+z)} \quad -(3)$$

$$\frac{dx-dy}{(x-y)(x+y+z)} = \frac{dy-dz}{(y-z)(x+y+z)} = \frac{dz-dx}{(z-x)(x+y+z)}$$

نأخذ اول اثنين 2,1

$$\frac{dx-dy}{x-y} - \frac{dy-dz}{(y-z)} = 0$$

بالتكامل

$$\log(x-y) - \log(y-z) = \log c_1$$

$$\boxed{c_1 = \frac{x-y}{y-z}}$$

نأخذ الثاني والثالث

$$\frac{(dy-dz)}{(y-z)} - \frac{(dz-dx)}{(z-x)} = 0$$

بالتكامل

$$\log(y-z) - \log(z-x) = \log c_2$$

$$\boxed{c_2 = \frac{y-z}{z-x}}$$

$$\boxed{\phi\left[\frac{x-y}{y-z}, \frac{y-z}{z-x}\right] = 0}$$

General Solution