## 2.16 Integration application/area of surface

Surface area: the area of surface swept out by revolving the curve about the axis.

1. Rotation with x-axis $[S=2\pi\int_a^b y\sqrt{1+(\frac{dy}{dx})^2}\,dx]$

2. Rotation with y-axis $[S=2\pi\int_c^d x\sqrt{1+(\frac{dx}{dy})^2}\,dy]$

**3. If x=f (t) , y =f (t)** $[S=2\pi\int_{t0}^{t1}\rho\sqrt{(\frac{dx}{dt})^2+(\frac{dy}{dt})^2}\,dt\,]$

**Where:-**

$\rho$ *is the distance from the axis of revolving to the element of arc length and expressses as function of t*

**Example: Find the area of surface obtained by revolving curve $y=\sqrt{x}$ *with* $x-$ *axis and* $0\le x\le 2$**

**Solution//**

$$S=2\pi\int_0^2 y\sqrt{1+(\frac{dy}{dx})^2}\,dx$$

$$\frac{dy}{dx}=\frac{1}{2}(x^{\frac{-1}{2}})\rightarrow(\frac{dy}{dx})^2=(\frac{1}{2}\left(x^{\frac{-1}{2}}\right))^2=\frac{1}{4x}$$

$$S=2\pi\int_0^2\sqrt{x}\sqrt{1+\frac{1}{4x}}\,dx=2\pi\int_0^2\sqrt{x}\sqrt{\frac{4x+1}{4x}}\,dx$$

$$=\frac{\pi}{6}(4x+1)^{\frac{3}{2}}\Big|_0^2=\frac{\pi}{6}[27-1]=\frac{13\pi}{3}$$

**Example: Find the area of surface obtained by revolving curve $x = a\cos^3 t$ & $y = a\sin^3 t$ with $x - axis$ and $0 \le x \le \frac{\pi}{2}$**

Solution//

$$\frac{dx}{dt} = -3a\cos^2 t \,.\, \sin t \rightarrow (\frac{dx}{dt})^2 = 9a^2\cos^4 t \,.\, \sin^2 t$$

$$\frac{dy}{dt} = 3a\sin^2 t \,.\, \cos t \rightarrow (\frac{dy}{dt})^2 = 9a\sin^4 t \,.\, \cos^2 t$$

$$\rho = \quad y = a\sin^3 t$$

$$S=2\pi\int_{t0}^{t1} \rho\sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2}\, dt =$$

$$2\pi\int_0^{\frac{\pi}{2}} a\sin^3 t\sqrt{9a^2\cos^4 t \,.\, \sin^2 t + 9a\sin^4 t \,.\, \cos^2 t}\;\, dt =$$

$$2\pi\int_0^{\frac{\pi}{2}} a^2\sin^4 t\cos t\, dt = \frac{6\pi}{5}a^2\, unit^2$$