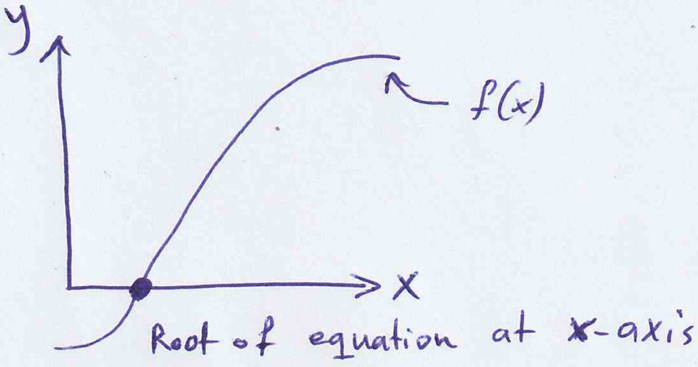
Numerical Methods

Newton Raphson Method

* This method is used to find roots of equation.



خطوات الحل /Procedure

1. اجعل الدالة تساوي صفر

$$f(x) = 0$$

If the initial value is not given, then use intermediate value theorem.
Such that $f(a) < 0$, $f(b) > 0$, initial $\boxed{x_0 = a \mid b}$
-ve +ve

2. إيجاد $f(x_0)$ و $f'(x_0)$

3. جد بشكل تقريبي الـ (Root) بواسطة طريقة نيوتن - رافسون

$$\boxed{x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}}$$

Find $f(x_1)$ and $f'(x_1)$

4. جد التقريب الثاني لـ (Root)
(2nd approximation)

$$\boxed{x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}}$$

General formula $\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$

Ex1/ Using the Newton - Raphson Method Find a real root of equation

$$x^3 - 2x - 5 = 0$$

Sol/

$$f(x) = x^3 - 2x - 5 = 0$$

$$f'(x) = 3x^2 - 2$$

(لا يوجد نقطة بداية) بما انه لا يوجد (intial value) يجب استخدام طريقة intermediat value theorem

مثلاً ($x_0 = 2$)

$$f(x) = x^3 - 2x - 5$$

$f(0) = -5$ (-ve)

$f(1) = (1)^3 - 2(1) - 5 = -6$ (-ve)

$f(2) = (2)^3 - 2(2) - 5 = -1$ (-ve)

$f(3) = (3)^3 - 2(3) - 5 = 16$ (+ve)

تحولت من سالبة الى موجبة وهنا يعني ان (Root) تقع بالوسط

Root lies between (2,3)

Initialization $\underline{\underline{x_0 = 2}} \longrightarrow$

$$f(x_0) = (2)^3 - 2(2) - 5 = \boxed{-1}$$

$$f'(x_0) = 3(2)^2 - 2 = \boxed{10}$$

1st approximation

$$x_1 = x_0 - \frac{f(x)}{f'(x_0)}$$

$$= 2 - \frac{-1}{10} = \boxed{2.1}$$

2nd approximation

$$X_2 = X_1 - \frac{f(x_1)}{f'(x_1)}$$

$$f(x_1) = (2.1)^3 - 2(2.1) - 5 = 0.061$$

$$f'(x_1) = 3(2.1)^2 - 2 = 11.23$$

$$X_2 = 2.1 - \frac{0.061}{11.23}$$

$$X_2 = \underline{\underline{2.0945}}$$

3rd approximation

$$f(x_2) = (2.0945)^3 - 2(2.0945) - 5 = -0.57459 * 10^{-4}$$

$$f'(x_2) = 3(2.0945)^2 - 2 = 11.1608$$

$$X_3 = X_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 2.0945 - \frac{(-0.57459 * 10^{-4})}{(11.1608)}$$

$$X_3 = \underline{\underline{2.0945}}$$

$\boxed{X_2 = X_3}$ نتوقف عند الحصول على نفس الاجابة

Root of equation = $\boxed{2.0945}$ Ans

Ex2/ $\boxed{f(x) = x^3 + 2x^2 + 10x - 20}$

$$\left.\begin{aligned} f(1) &= (1)^3 + 2(1)^2 + 10(1) - 20 = -7 \\ f(2) &= (2)^3 + 2(2)^2 + 10(2) - 20 = 16 \end{aligned}\right\}$$

$[1, 2]$

$\boxed{f'(x) = 3x^2 + 4x + 10}$

let $x_0 = 1.5$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$f(1.5) = (1.5)^3 + 2(1.5)^2 + 10(1.5) - 20 = 2.875$$

$$f'(1.5) = 3(1.5)^2 + 4(1.5) + 10 = 22.75$$

$$x_1 = 1.5 - \frac{2.875}{22.75} = 1.3736$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$f(x_1) = (1.3736)^3 + 2(1.3736)^2 + 10(1.3736) - 20 = 0.1012$$

$$f'(x_1) = 3(1.3736)^2 + 4(1.3736) + 10 = 21.1547$$

$$x_2 = 1.3736 - \frac{0.1012}{21.1547} = 1.3688$$

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$$X_3 = X_2 - \frac{f(x_2)}{f'(x_2)}$$

$$f(x_2) = (1.3688)^3 + 2(1.3688)^2 + 10(1.3688) - 20 = \boxed{-1.7104 * 10^{-4}}$$

$$f'(x_2) = 3(1.3688)^2 + 4(1.3688) + 10 = \boxed{21.096}$$

$$X_3 = 1.3688 - \frac{(-1.7104 * 10^{-4})}{21.096}$$

$$X_3 = 1.3688$$

$$\therefore X_2 = X_3$$

Root is 1.3688

$$\boxed{X = 1.3688}$$

Ans.