Eigen value

Define: suppose that A square matrix of degree $n \times n$, then nonzero vector $\vec{X}$ called Eigen vector of matrix A if $A\vec{X}$ numerically multiple i.e:

$$A\vec{X} = \lambda\vec{X}$$

λ:Eigen value

$\vec{X}$ Eigen vector

Example:

$$\begin{bmatrix} 2 & 4 \\ 0 & 4 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \end{bmatrix} = 4\begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$\lambda = 4$ is Eigen vector and $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$is Eigen vector of $\begin{bmatrix} 2 & 4 \\ 0 & 1 \end{bmatrix}$

*Eigen value also is called Latent Roots

Find Eigen value of matrix of A

To find Eigen value of matrix A write $A\vec{X} = \lambda\vec{X}$ by $(\lambda I - A)\vec{X} = 0$, where I is unit matrix.For this equation solution single nonzero if and only if

$$\left|(\lambda I - A)\vec{X}\right| = 0$$

Called this equation by Distinctive equation of matrix A, i.e λ is real solution of Distinctive equation

Example: find Latent Roots of this matrix $A = \begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix}$

$$\lambda I - A = \lambda\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \lambda - 3 & -2 \\ 1 & \lambda \end{bmatrix}$$

Determint of matrix

$$\begin{vmatrix} \lambda-3 & -2 \\ 1 & \lambda \end{vmatrix} = \lambda^2 - 3\lambda + 2$$

Distinctive equation is

$$\lambda^2 - 3\lambda + 2 = 0$$

$$(\lambda-1)(\lambda-2) = 0$$

$$\lambda = 1 \quad or \ \lambda = 2$$

Find eigen vector of matrix A:

1. **Find eigen value**
2. **Suppose vector $\vec{X}$ such that number row of $\vec{X}$ equal row matrix**
3. **Solve equation $(\lambda I - A)\vec{X} = 0$ to find eigen value**

Example/ find Eigen vector of matrix A:

$$A = \begin{bmatrix} 3 & -2 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

Sol: find eigen value

$$\lambda I - A = \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 3 & -2 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} \lambda-3 & 2 & 0 \\ 2 & \lambda-3 & 0 \\ 0 & 0 & \lambda-5 \end{bmatrix}$$

$$\begin{vmatrix} \lambda-3 & 2 & 0 \\ 2 & \lambda-3 & 0 \\ 0 & 0 & \lambda-5 \end{vmatrix} = 0$$

$$(\lambda-5)\begin{vmatrix} \lambda-3 & 2 \\ 2 & \lambda-3 \end{vmatrix} = 0$$

$$(\lambda-5)((\lambda-3)^2 - 4) = 0$$

$$(\lambda-5)(\lambda^2 - 6\lambda + 9 - 4) = 0$$

$$(\lambda-5)(\lambda^2 - 6\lambda + 5) = 0$$

$$(\lambda-5)(\lambda-5)(\lambda-1) = 0$$

$$\lambda = 1, 5$$

الان لإيجاده المتجه الذاتي نفرض المتجه $\vec{X} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ ونقوم بحل المعادلة

$$(\lambda I - A)\vec{X} = O$$

$$\begin{bmatrix} \lambda - 3 & 2 & 0 \\ 2 & \lambda - 3 & 0 \\ 0 & 0 & \lambda - 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

عندما $\lambda = 1$ فان

$$\begin{bmatrix} -2 & 2 & 0 \\ 2 & -2 & 0 \\ 0 & 0 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -2x_1 + 2x_2 \\ 2x_1 - 2x_2 \\ -4x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-2x_1 + 2x_2 = 0 \Longrightarrow x_1 = x_2$$

$$-4x_3 = 0 \Longrightarrow x_3 = 0$$

نفرض $x_1 = t$ فان $x_2 = t$. لذلك $\vec{X} = \begin{bmatrix} t \\ t \\ 0 \end{bmatrix} = t \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

معنى ذلك ان المتجه $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ هو المتجه الذاتي عندما $\lambda = 1$.

لايجاد المتجه الذاتي عندما $\lambda = 5$ واجب بيتي

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Good Luke
Sarah