COMPLEX NUMBER

A Complex Number is a combination of a Real number and an Imaginary Number.

As an example about this the following:

7+3i in this case the number 7 is the real part and 3 is the imaginary part. The only when the imaginary part squared gives negative results on the contrary of the real number.

The "unit" imaginary number (like 1 for Real Numbers) is i, which is the square root of −1

$$i = \sqrt{-1}$$

Because when we square i we get −1

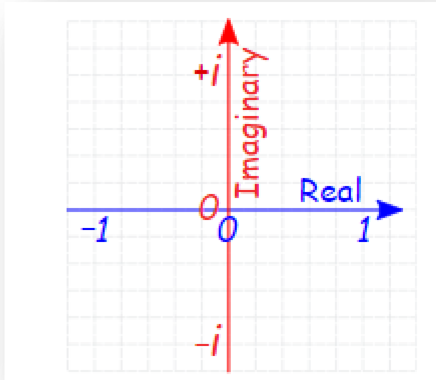
$$i^2 = -1$$

Examples of Imaginary Numbers:

3i	1.04i	−2.8i	3i/4	(√2)i	1998i

And we keep that little "i" there to remind us we need to multiply by √−1

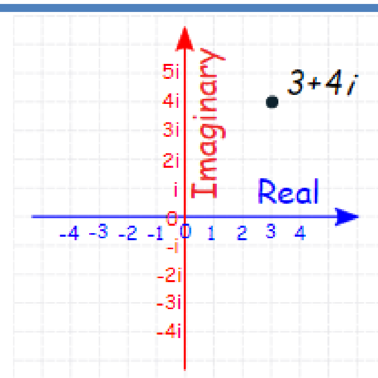
The complex plane is as follows:



So, we can represent the complex number as follows:

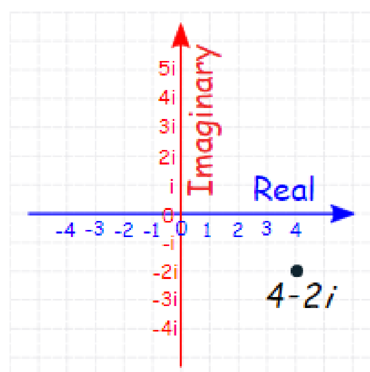
We can then plot a complex number like **3 + 4i** :

- 3 units along (the real axis),
- and 4 units up (the imaginary axis).



And here is **4 - 2i** :

- 4 units along (the real axis),
- and 2 units down (the imaginary axis).



ADDING THE COMPLEX NUMBER:

To add two complex numbers we add each part separately:

$$(a+bi) + (c+di) = (a+c) + (b+d)i$$

EXAMPLE NO.1:

add the complex numbers 3 + 2i and 1 + 7i

add the real numbers, and

add the imaginary numbers:

(3 + 2i) + (1 + 7i)

= 3 + 1 + (2 + 7)i

= 4 + 9i

EXAMPLE NO.2:

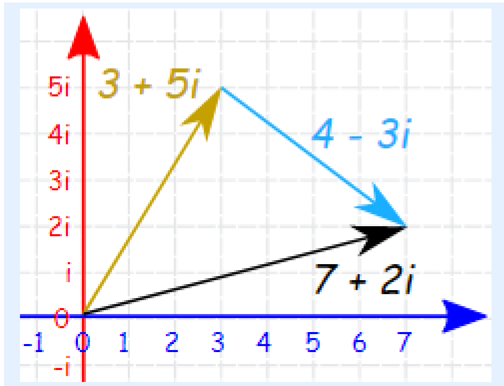
add the complex numbers 3 + 5i and 4 − 3i

(3 + 5i) + (4 − 3i)

= 3 + 4 + (5 − 3)i

= 7 + 2i

On the complex plane it is:



MULTIPLICATION OF COMPLEX NUMBER:

IT IS JUST LIKE THE FOLLOWING EXAMPLE:

Example: $(3 + 2i)(1 + 7i)$

$$
\begin{aligned}
(3 + 2i)(1 + 7i) &= 3\times1 + 3\times7i + 2i\times1 + 2i\times7i \\
&= 3 + 21i + 2i + 14i^2 \\
&= 3 + 21i + 2i - 14 \qquad \text{(because } i^2 = -1\text{)} \\
&= -11 + 23i
\end{aligned}
$$

EXAMPLE NO.2:

$(1 + i)^2$

$$
\begin{aligned}
(1 + i)(1 + i) &= 1\times1 + 1\times i + 1\times i + i^2 \\
&= 1 + 2i - 1 \qquad \text{(because } i^2 = -1\text{)} \\
&= 0 + 2i
\end{aligned}
$$

EXAMPLE NO.3:

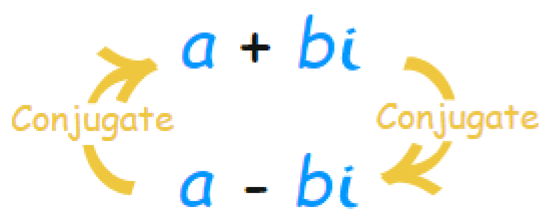
Prove that $i^2 = -1$

We can write i with a real and imaginary part as $0 + i$

$$
\begin{aligned}
i^2 = (0 + i)^2 &= (0 + i)(0 + i) \\
&= (0\times0 - 1\times1) + (0\times1 + 1\times0)\mathbf{i} \\
&= -1 + 0\mathbf{i} \\
&= \mathbf{-1}
\end{aligned}
$$

Conjugates:

A conjugate is where we **change the sign in the middle** like this:



Dividing:

The conjugate is used to help complex division.

The trick is to **multiply both top and bottom** by the **conjugate of the bottom**.

Example: Do this Division:

$$\frac{2 + 3\mathbf{i}}{4 - 5\mathbf{i}}$$

Multiply top and bottom by the conjugate of $4 - 5\mathbf{i}$:

$$\frac{2 + 3\mathbf{i}}{4 - 5\mathbf{i}} \times \frac{4 + 5\mathbf{i}}{4 + 5\mathbf{i}} = \frac{8 + 10\mathbf{i} + 12\mathbf{i} + 15\mathbf{i}^2}{16 + 20\mathbf{i} - 20\mathbf{i} - 25\mathbf{i}^2}$$

Now remember that $i^2 = -1$, so:

$$= \frac{8 + 10\mathbf{i} + 12\mathbf{i} - 15}{16 + 20\mathbf{i} - 20\mathbf{i} + 25}$$

Add Like Terms (and notice how on the bottom $20\mathbf{i} - 20\mathbf{i}$ cancels out!):

$$= \frac{-7 + 22\mathbf{i}}{41}$$

Lastly we should put the answer back into $a + b\mathbf{i}$ form:

$$= \frac{-7}{41} + \frac{22}{41}\mathbf{i}$$

Exercise:1

What is (3 - 5i) + (-4 + 7i)?

A -1 + 2i	B 1 + 2i
C 7 - 12i	D -2 + 3i

2.

What is (-5 + 3i) - (4 + 7i)?

A -1 - 4i	B -1 + 4i
C -9 - 4i	D -9 + 4i

3.

If $z_1 = 2 + 5i$ and $z_2 = 3 - 2i$, what is $z_1 \times z_2$ as a single complex number?

A -4 + 11i	B 16 - 19i
C 6 - 10i	D 16 + 11i

4.

What is (4 - 5i)(-2 + 7i)?

A 27 + 18i	B -43 + 18i
C -43 + 38i	D 27 + 38i

5.

What is (3 + 4i)(3 - 4i)?

A 9 - 16i	B 9 + 16i
C 25	D -7

6.

What is $\frac{2+5i}{3+i}$ as a single complex number?

A $\frac{2}{3}+5i$

B $\frac{3}{10}+\frac{13}{10}i$

C $\frac{11}{10}+\frac{13}{10}i$

D $\frac{11}{8}+\frac{13}{8}i$

7.

What is $\frac{7-3i}{5-2i}$ as a single complex number?

A $\frac{7}{5}+\frac{3}{2}i$

B $1-\frac{1}{29}i$

C $\frac{41}{29}-\frac{1}{29}i$

D $\frac{41}{29}-i$

8.

What are the roots of the quadratic equation $x^2 - 2x + 3 = 0$

A $x = 3$ or -1

B $x = 2 + i\sqrt{2}$ or $2 - i\sqrt{2}$

C $x = 1 + i\sqrt{2}$ or $1 - i\sqrt{2}$

D $x = 1 + 2i$ or $1 - 2i$

9.

What are the roots of the quadratic equation $x^2 + 4x + 7 = 0$?

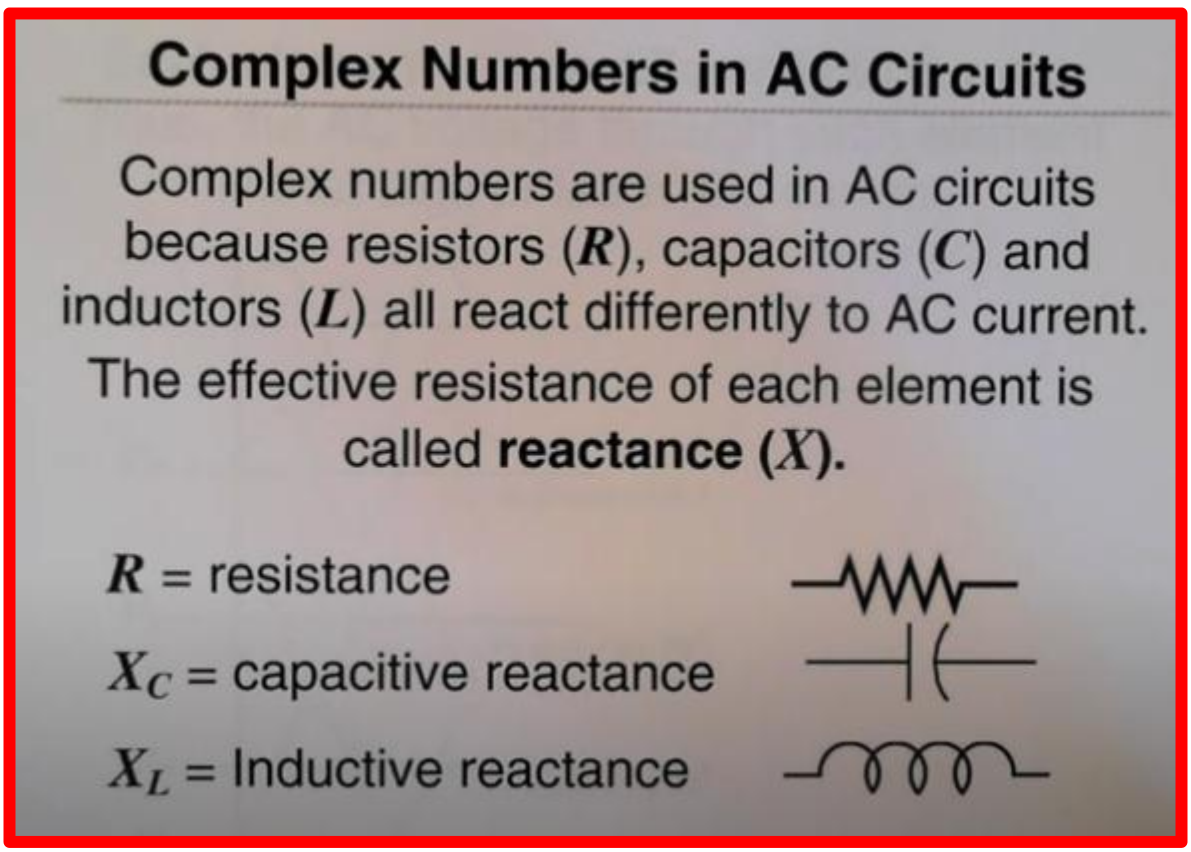
A $x = -2 + i\sqrt{3}$ or $-2 - i\sqrt{3}$	B $x = 2 + i\sqrt{3}$ or $2 - i\sqrt{3}$
C $x = -1 + i\sqrt{3}$ or $-1 - i\sqrt{3}$	D $x = -7$ or 1

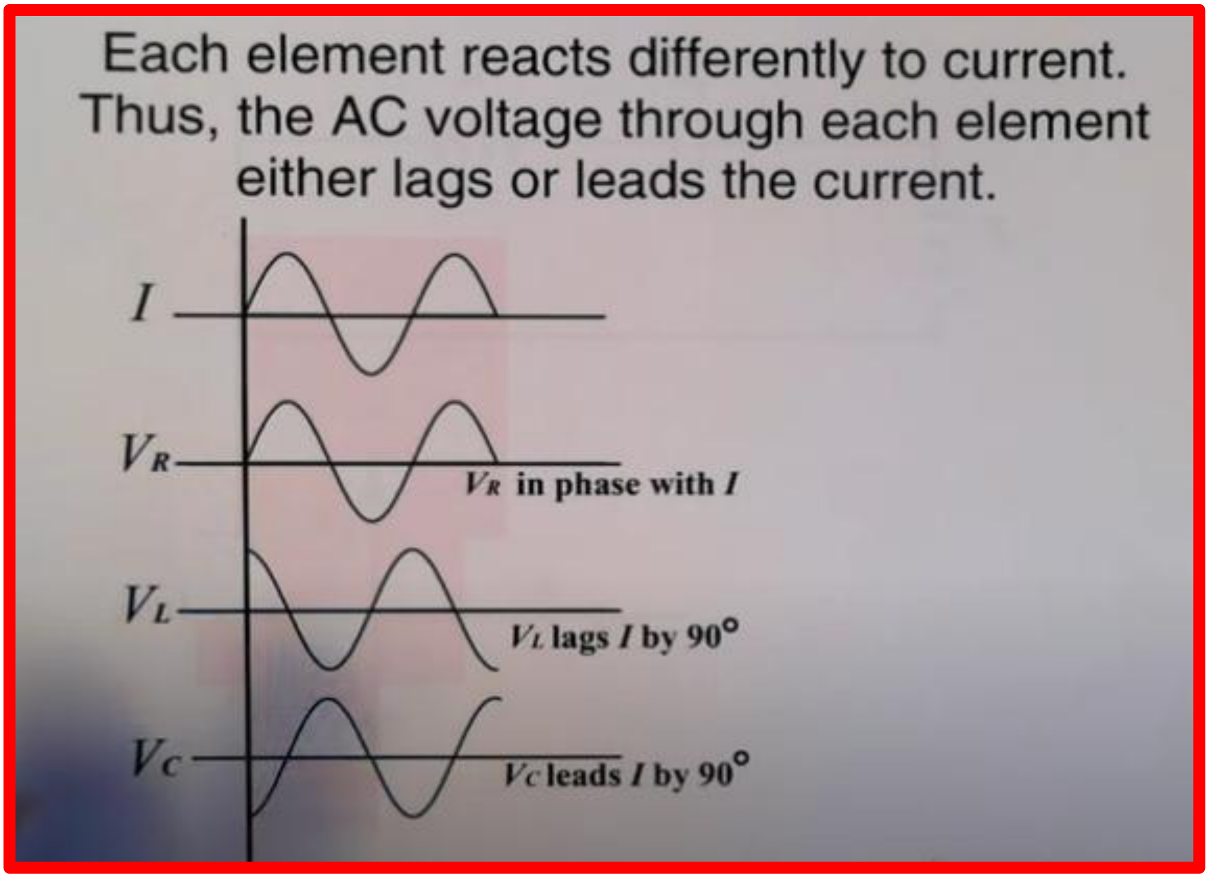
10.

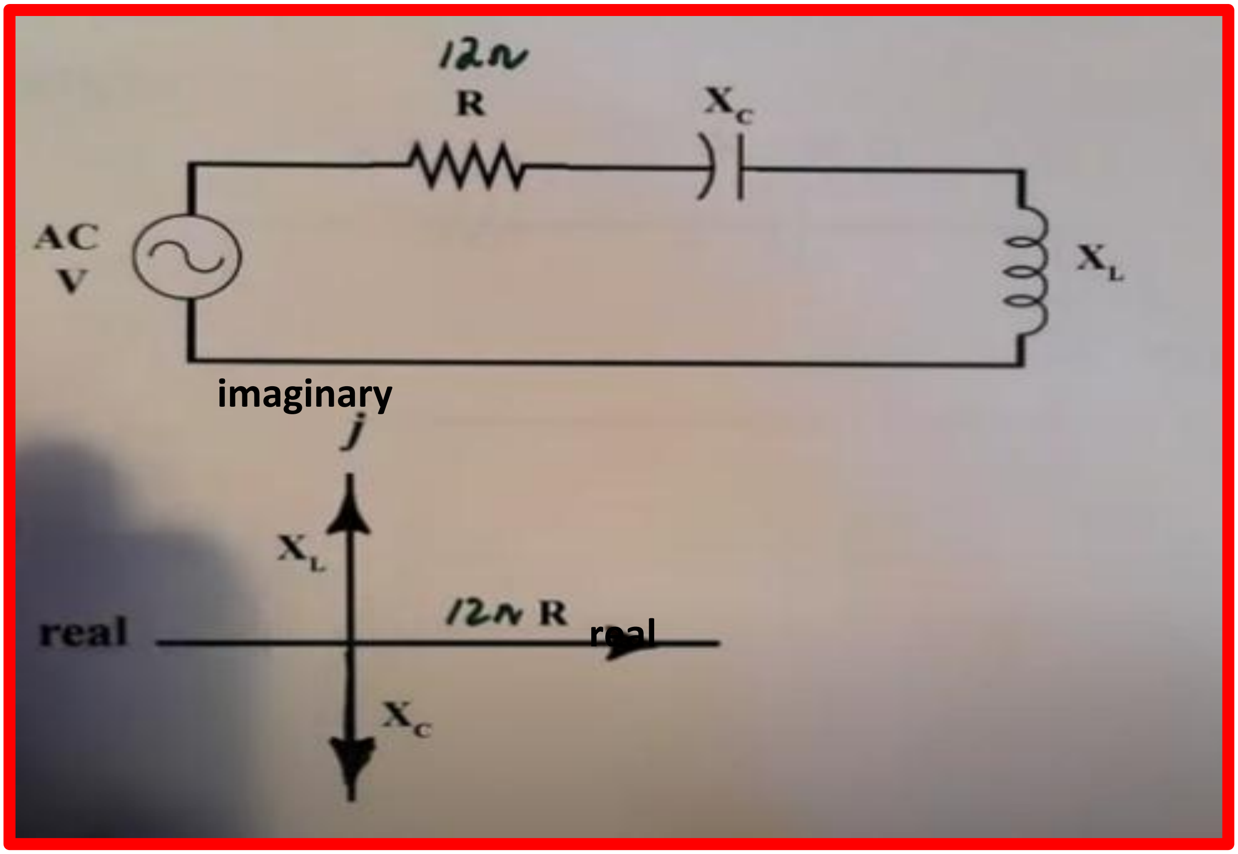
If $z = 1 + i$, what is $z^3 + z^2 + z + 1$ as a single complex number?

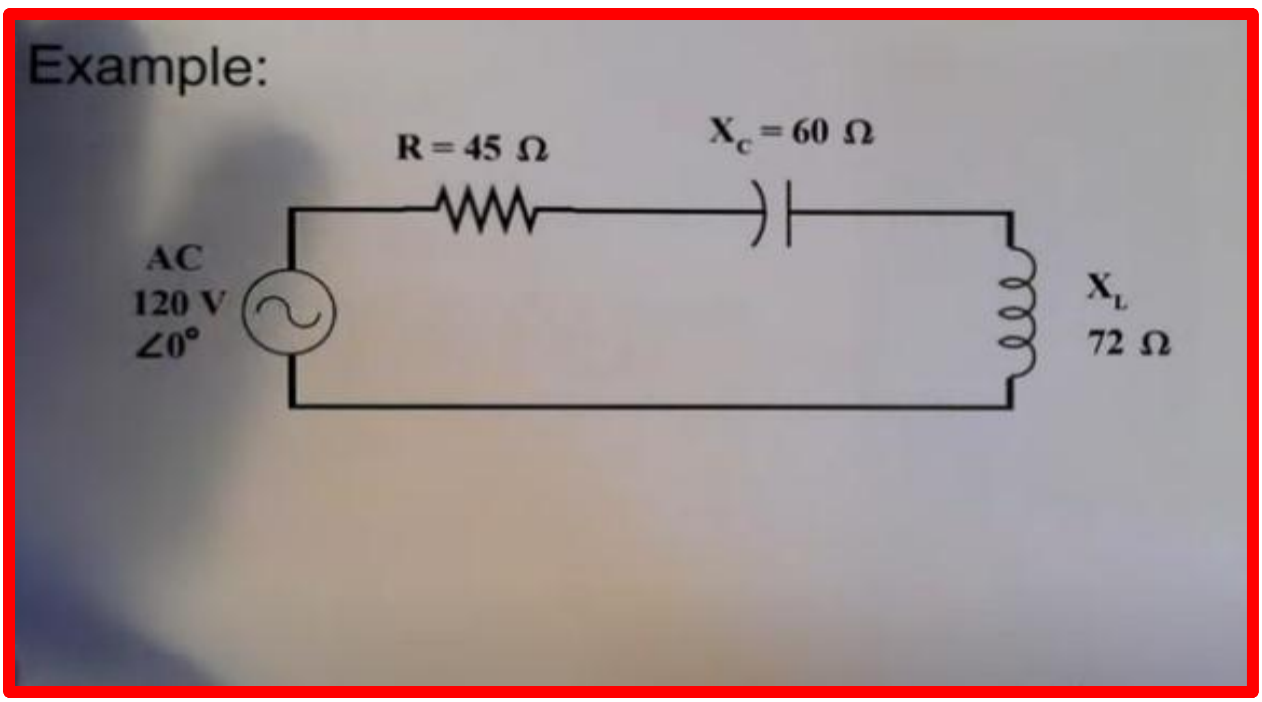
A $5i - 2$	B $5i$
C $5i + 2$	D $3i - 2$

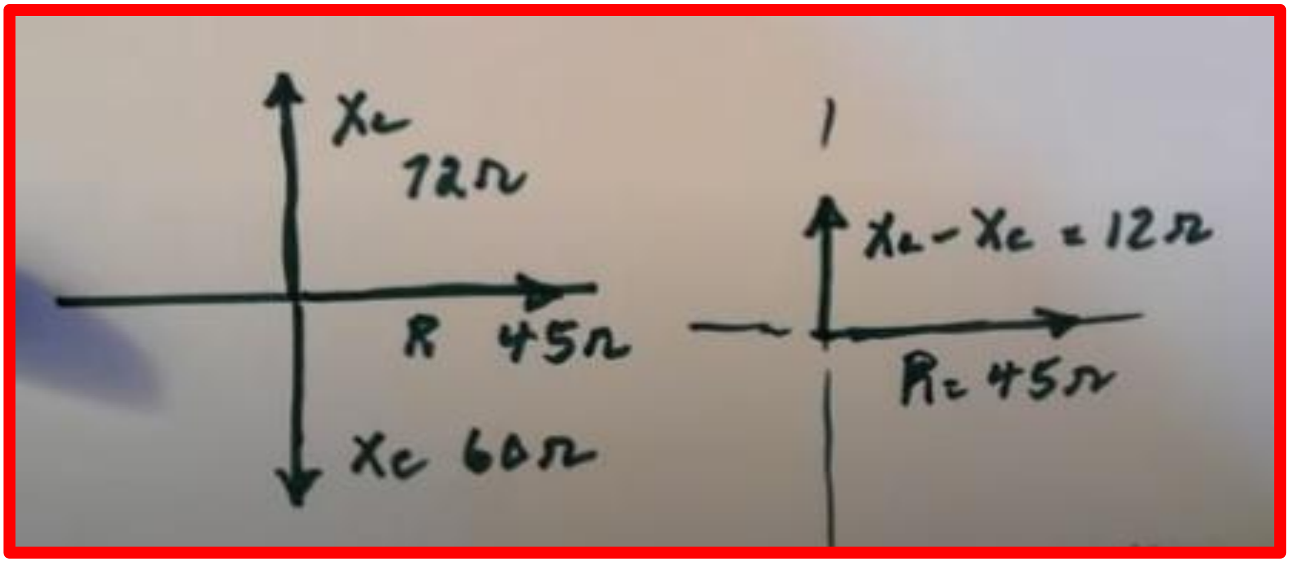
Application of complex number:




Each element reacts differently to current. Thus, the AC voltage through each element either lags or leads the current.
I
V_R
V_R in phase with I
V_L
V_L lags I by 90°
V_C
V_C leads I by 90°


12Ω
R
X_C
AC
V
X_L
imaginary
j
X_L
real
12Ω R
real
X_C


Example:
R = 45 Ω
X_C = 60 Ω
AC
120 V
∠0°
X_L
72 Ω



Impedance=Z=(X_L –X_C)j +45

Z= 12j + 45