Application of partial derivative

Small increments

If $z = f(x, y)$ then $\delta z = \frac{\partial z}{\partial x}\delta x + \frac{\partial z}{\partial y}\delta y$

Example 1:

If $I = \frac{V}{R}$, and V=250 volts and $R = 50$ ohms, find the change in I resulting from an increase of 1 volt in V and an increase of 0.5 ohm in R.

I=f(V,R) then $\delta I = \frac{\partial I}{\partial V}\delta V + \frac{\partial I}{\partial R}\delta R$

$\frac{\partial I}{\partial V} = \frac{1}{R}$ and $\frac{\partial I}{\partial R} = -\frac{V}{R^2}$

Then

$$\delta I = \frac{1}{R}\delta V - \frac{V}{R^2}\delta R$$

So when R=50, V=250, $\delta V = 1$, and $\delta R = 0.5$,

$\delta I = -0.03$

i.e. I decreases by 0.03 amperes.

Rates- of-change Problems

$$\frac{dV}{dt} = \frac{\partial V}{\partial r}.\frac{dr}{dt} + \frac{\partial V}{\partial h}.\frac{dh}{dt}$$

Example:

The radius of a cylinder increases at the rate of 0.2 cm/ sec while the height decreases at the rate of 0.5 cm/ sec. Find the rate at which the volume is changing at the instant when r=8 cm and h=12 cm.

$$V = \pi r^2 h$$

$$\delta V = \frac{\partial V}{\partial r}\delta r + \frac{\partial V}{\partial h}\delta h$$

Then

$$\frac{dV}{dt} = \frac{\partial V}{\partial r}.\frac{dr}{dt} + \frac{\partial V}{\partial h}.\frac{dh}{dt}$$

$$\frac{\partial V}{\partial r} = 2\pi r h, \quad \frac{\partial V}{\partial h} = \pi r^2$$

$$\frac{dV}{dt} = 2\pi r h\frac{dr}{dt} + \pi r^2\frac{dh}{dt}$$

Now r=8, h=12, $\frac{dr}{dt} = 0.2, \frac{dh}{dt} = -0.5$

$$\frac{dV}{dt} = 20.1 \ \ cm^3/sec.$$

K A Stroud, engineering mathematics , second edition