Mathematic I

2.4 Matrix

$$A = \begin{bmatrix} a11 & a12 \\ a21 & a22 \end{bmatrix}$$

Example: -Find the matrix A+B and A-B

$$A = \begin{bmatrix} 3 & 1 \\ 4 & 2 \\ 2 & -1 \end{bmatrix}, \qquad B = \begin{bmatrix} 2 & 5 \\ 1 & 2 \\ 0 & 1 \end{bmatrix}$$

Solution//

$$C = A + B = \begin{bmatrix} 3+2 & 1+5 \\ 4+1 & 2+2 \\ 2+0 & -1+1 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 5 & 4 \\ 2 & 0 \end{bmatrix}$$

$$D = A - B = \begin{bmatrix} 3-2 & 1-5 \\ 4-1 & 2-2 \\ 2-0 & -1-1 \end{bmatrix} = \begin{bmatrix} 1 & -4 \\ 3 & 0 \\ 2 & -2 \end{bmatrix}$$

Example:-Find the matrix A×B

$$A = \begin{bmatrix} 1 & -1 \\ 2 & 0 \\ -1 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & 0 & -1 \\ 1 & 1 & 2 \end{bmatrix}$$

Solution//

$$= \begin{bmatrix} (1 \times 4) + (-1 \times 1) & (1 \times 0) + (-1 \times 1) & (1 \times -1) + (-1 \times 2) \\ (2 \times 4) + (0 \times 1) & (2 \times 0) + (0 \times 1) & (2 \times -1) + (0 \times 2) \\ (-1 \times 4) + (3 \times 1) & (-1 \times 0) + (3 \times 1) & (-1 \times -1) + (3 \times 2) \end{bmatrix}$$

$$A \times B = \begin{bmatrix} 3 & -1 & -3 \\ 8 & 0 & -2 \\ -1 & 3 & 7 \end{bmatrix}$$

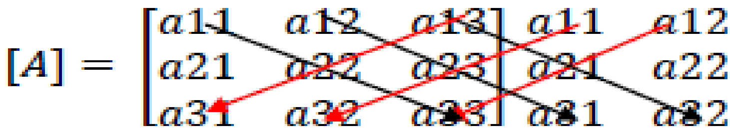
2.5 Determinates

1. Minors and Cofactors:

$$[A] = \begin{bmatrix} a11 & a12 & a13 \\ a21 & a22 & a23 \\ a31 & a32 & a33 \end{bmatrix} \quad \begin{matrix} + & - & + \\ - & + & - \\ + & - & + \end{matrix}$$

$$[A] = a11 \times \begin{bmatrix} a22 & a23 \\ a32 & a33 \end{bmatrix} - a12 \times \begin{bmatrix} a21 & a23 \\ a31 & a33 \end{bmatrix} + a13 \times \begin{bmatrix} a21 & a22 \\ a31 & a32 \end{bmatrix}$$

2. Special method is used only when matrix 3×3



Properties of Determinates

1. If all elements of row or column are equal a zero→

$$determinate\ value = 0$$

$$\begin{bmatrix} 2 & 3 & 1 \\ 0 & 0 & 0 \\ 5 & 3 & -2 \end{bmatrix} = [D] = 0$$

2. If similar elements a row two or column two →

$$\boldsymbol{determinate\ value = 0}$$

$$\begin{bmatrix} -1 & 2 & 3 \\ 3 & 0 & -2 \\ -1 & 2 & 3 \end{bmatrix} = [D] = 0 \ \ or \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 2 \\ 4 & 2 & 4 \end{bmatrix} = [D] = 0$$

Example //Find the Determinate of $[A]$

$$\mathbf{A} = \begin{bmatrix} \mathbf{3} & \mathbf{1} & \mathbf{2} \\ \mathbf{4} & \mathbf{2} & \mathbf{-2} \\ \mathbf{2} & \mathbf{-1} & \mathbf{3} \end{bmatrix}$$

Solution/

$$[A] = 3\begin{bmatrix} 2 & -2 \\ -1 & 3 \end{bmatrix} - 1\begin{bmatrix} 4 & -2 \\ 2 & 3 \end{bmatrix} + 2\begin{bmatrix} 4 & 2 \\ 2 & -1 \end{bmatrix}$$

$$= 3[6-2] - 1[12-(-4)] + 2[-4-4] = 12 - 16 - 16 = -20$$

OR

$$\begin{bmatrix} 3 & 1 & 2 \\ 4 & 2 & -2 \\ 2 & -1 & 3 \end{bmatrix} \begin{matrix} 3 & 1 \\ 4 & 2 \\ 2 & -1 \end{matrix}$$

$$= 3*2*3 + 1*-2*2 + 2*4*(-1) - 2*2*2 - 3*(-2)*(-1) - 1*4*3$$

=-20

Exercise: $\begin{vmatrix} 3 & 1 & 2 \\ 4 & 2 & -K \\ 2 & -1 & 3 \end{vmatrix} = 0.\ \textit{Find value } K$

2.6 Grammer Rule

$$x = \frac{Dx}{D}.\ y = \frac{Dy}{D}.\ \ z = \frac{Dz}{D}$$

Example//Find $x.y\ and\ z$ from these equations by using Grammer Rule.

x+y+z=2 ……. 1

$3x$-y+2z=2 ….. 2

$2x$-3y+3z=1 …. 3

Solution //

$$\boldsymbol{D} = \begin{bmatrix} 1 & 1 & 1 \\ 3 & -1 & 2 \\ 2 & -1 & 3 \end{bmatrix}$$

$$= 1\begin{bmatrix} -1 & 2 \\ -3 & 3 \end{bmatrix} - 1\begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix} + 1\begin{bmatrix} 3 & -1 \\ 2 & -3 \end{bmatrix}$$

$$= 3 - 5 - 7 = -9$$

$$\boldsymbol{Dx} = \begin{bmatrix} 2 & 1 & 1 \\ 2 & -1 & 2 \\ 1 & -1 & 3 \end{bmatrix}$$

$$= 2\begin{bmatrix} -1 & 2 \\ -3 & 3 \end{bmatrix} - 1\begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix} + 1\begin{bmatrix} 2 & -1 \\ 1 & -3 \end{bmatrix}$$

$$= 6 - 4 - 5 = -3$$

$$\boldsymbol{x} = \frac{\boldsymbol{Dx}}{\boldsymbol{D}} = \frac{-3}{-9} = \frac{1}{3}$$

$$\boldsymbol{Dy} = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 2 \\ 2 & 1 & 3 \end{bmatrix} = 1\begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix} - 2\begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix} + 1\begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}$$

$$= 4 - 10 - 1 = -7$$

$$y = \frac{Dy}{D} = \frac{-7}{-9} = \frac{7}{9}$$

$$Dz = \begin{bmatrix} 1 & 1 & 2 \\ 3 & -1 & 2 \\ 2 & -3 & 1 \end{bmatrix}$$

$$= 1\begin{bmatrix} -1 & 2 \\ -3 & 1 \end{bmatrix} - 1\begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix} + 2\begin{bmatrix} 3 & -1 \\ 2 & -3 \end{bmatrix}$$

$$= 5 + 1 - 14 = -8$$

$$z = \frac{Dz}{D} = \frac{-8}{-9} = \frac{8}{9}$$

Exercise: Find x,y and z from these equations by using Grammer Rule.

x+y =1 ……. 1

y+z=1 ……. 2

x+y+z=1…. 3