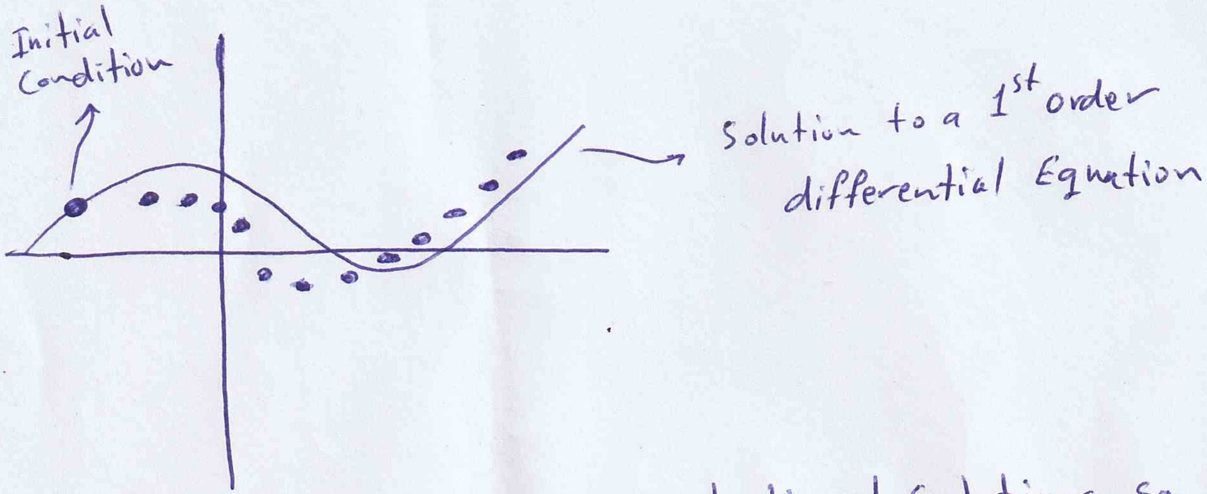
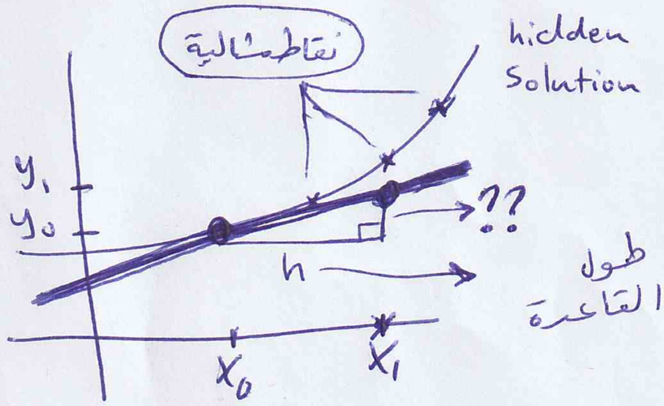
Numerical Methods

Euler's Method



Euler's Method depends on computational Solutions, so, let's assume that the above graphical solution is not exist and solve to find close points to the solution curve with some error envolved



How does it work ??

Start

$x_0 = x_0 \quad , y_0 = y_0$

لا يمكن إيجاد النقاط بشكل مثالي التي تقع
على الحل الصحيح ولهنا يجب علينا الاعتماد
على الحل التقريبي بالاعتماد على خط التماس

$x_1 = x_0 + h$

$\frac{dy}{dx} = f(x,y) \rightarrow \boxed{y(x_0) = y_0} \rightarrow$ initial condition

the slope if we substitut $f(x_0, y_0) = \frac{??}{h} = \frac{y_1 - y_0}{x_1 - x_0}$

$y_1 = y_0 + h \cdot f(x_0, y_0)$

$$\boxed{\begin{aligned} x_2 &= x_1 + h \\ y_2 &= y_1 + h \cdot f(x_1, y_1) \end{aligned}} \Rightarrow \boxed{\begin{aligned} x_n &= x_{n-1} + h \\ y_n &= y_{n-1} + h \cdot f(x_{n-1}, y_{n-1}) \end{aligned}}$$

Ex1/ Approximate $y(0.4)$ by using E.M.

$$\frac{dy}{dx} = x + 2y \quad , \quad y(0) = 0$$

$f(x,y)$

step size : $h = 0.1$

Sol/

$$x_n = x_{n-1} + h$$

$$y_n = y_{n-1} + h \cdot f(x_{n-1}, y_{n-1})$$

Start

$x_0 = 0 \quad , \quad y_0 = 0$

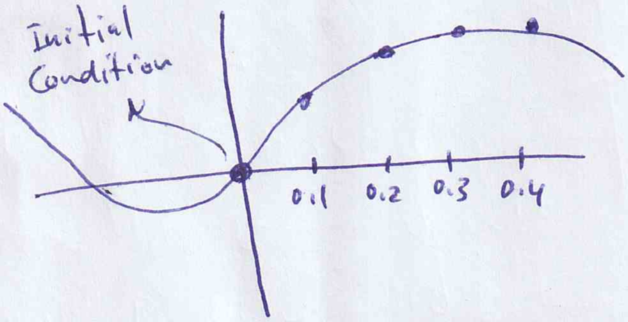
$x_1 = 0.1 \quad , \quad y_1 = 0 + (0.1)(0 + 2(0)) = 0$

$x_2 = 0.2 \quad , \quad y_2 = 0 + (0.1)(0.1 + 2(0)) = 0.01$

$x_3 = 0.3 \quad , \quad y_3 = 0.01 + (0.1)(0.2 + 2(0.01)) = 0.032$

$x_4 = 0.4 \quad , \quad y_4 = 0.032 + (0.1)(0.3 + 2(0.032)) = \boxed{0.0684}$

Ans.



EX2/ Use Euler's Method to find the solution to the differential equation $\frac{dy}{dx} = \cos(x)$ at $x = \frac{\pi}{2}$ with the initial condition $y(0) = 0$ and step size $h = \frac{\pi}{10}$

لاتنسى تحويل
الحاسبة من Deg الى
Rad

Sol/

$$x_{n+1} = x_n + h$$

$$y_{n+1} = y_n + h * f(x_n, y_n)$$

$$f(x,y) = \frac{dy}{dx} = \cos(x) \quad , \quad h = \frac{\pi}{10}$$

starting at the initial point $(x_0, y_0) = (0, 0)$

$$x_1 = x_0 + h = 0 + \frac{\pi}{10} = \boxed{\frac{\pi}{10}}$$

$$y_1 = y_0 + h * f(x_0, y_0) = 0 + \left(\frac{\pi}{10}\right) * \cos(0) = \boxed{\frac{\pi}{10}}$$

$$x_2 = \frac{\pi}{10} + \frac{\pi}{10} = \boxed{\frac{\pi}{5}}$$

$$y_2 = \frac{\pi}{10} + \left(\frac{\pi}{10}\right) \cos\left(\frac{\pi}{10}\right) = \boxed{0.6129}$$

$$x_3 = \frac{\pi}{5} + \frac{\pi}{10} = \boxed{\frac{3\pi}{10}}$$

$$y_3 = 0.6129 + \left(\frac{\pi}{10}\right) \cdot \cos\left(\frac{\pi}{5}\right) = \boxed{0.867}$$

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$$x_4 = \frac{3\pi}{10} + \frac{\pi}{10} = \frac{2\pi}{5}$$

$$y_4 = 0.867 + \left(\frac{\pi}{10}\right)\cos\left(\frac{3\pi}{10}\right) = 1.0516$$

$$x_5 = \frac{2\pi}{5} + \frac{\pi}{10} = \frac{\pi}{2}$$

$$y_5 = 1.0516 + \left(\frac{\pi}{10}\right)\cos\left(\frac{2\pi}{5}\right) = \boxed{1.1486}$$

Ans.