المعادلات التفاضليه الدرجه الاولى

{FIRST ORDER DIFFERENTIAL EQUATION}

The *order* of a differential equation is given by the highest derivative involved in the equation.

$x\frac{dy}{dx} - y^2 = 0$ is an equation of the 1st order

$xy\frac{d^2y}{dx^2} - y^2 \sin x = 0$ is an equation of the 2nd order

$\frac{d^3y}{dx^3} - y\frac{dy}{dx} + e^{4x} = 0$ is an equation of the 3rd order

So that $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 10y = \sin 2x$ is an equation of the ………..order.

طرق حل المعادلات التفاضليه من الدرجه الاولى:

1- THE FIRST METHODE BY DIRECT INTEGRATION.
2- THE SECOND METHODE BY SEPARATING THE VARAIBLES.
3- HOMOGENOUSE EQUATIONS BY SUBISTITUTING y=vx.
4- LINEAR EQUATIONS BY USING INTEGRATING FACTOR.

الان نأخذ الطريقه الاولى:

THE FIRST METHODE BY DIRECT INTEGRATION.

EXAMPLE1:

$$\frac{dy}{dx} = 3x^2 - 6x + 5$$

Then $y = \int (3x^2 - 6x + 5)dx = x^3 - 3x^2 + 5x + C$

i.e. $y = x^3 - 3x^2 + 5x + C$

EXAMPLE2:

Solve $x\frac{dy}{dx} = 5x^3 + 4$

In this case, $\frac{dy}{dx} = 5x^2 + \frac{4}{x}$ So,

$$\boxed{y = \frac{5x^3}{3} + 4\ln x + C}$$

EXAMPLE3:

Find the particular solution of the equation $e^x\frac{dy}{dx} = 4$, given that $y = 3$ when $x = 0$.

First rewrite the equation in the form $\frac{dy}{dx} = \frac{4}{e^x} = 4e^{-x}$.

Then $y = \int 4e^{-x}dx = -4e^{-x} + C$

THEN USE y=3 and x=0

So,

$3= -4e^0 + C$

$C=7$

$y= -4e^x + 7$

THE SECOND METHODE BY SEPARATING THE VARAIBLES.

في هذه الطريقه نجعل المتغيرy في جانب والمتغير x في الجانب الاخر

EXAMPLE1:

Solve $\dfrac{dy}{dx} = \dfrac{2x}{y+1}$

We can rewrite this as $(y+1)\dfrac{dy}{dx} = 2x$

Now integrate both sides with respect to x:

$$\int (y+1)\frac{dy}{dx}\,dx = \int 2x\,dx \quad \text{i.e.}$$

$$\int (y+1)\,dy = \int 2x\,dx$$

and this gives $\dfrac{y^2}{2} + y = x^2 + C$

EXAMPLE2:

Solve $\frac{dy}{dx} = (1+x)(1+y)$

$$\frac{1}{1+y}\frac{dy}{dx} = 1+x$$

Integrate both sides with respect to x:

$$\int \frac{1}{1+y}\frac{dy}{dx}\,dx = \int (1+x)\,dx \qquad \therefore \int \frac{1}{1+y}\,dy = \int (1+x)\,dx$$

$$\ln(1+y) = x + \frac{x^2}{2} + C$$

EXAMPLE3:

Solve $\frac{dy}{dx} = \frac{1+y}{2+x}$

This can be written as $\frac{1}{1+y}\frac{dy}{dx} = \frac{1}{2+x}$

Integrate both sides with respect to x:

$$\int \frac{1}{1+y}\frac{dy}{dx}\,dx = \int \frac{1}{2+x}\,dx$$

$$\therefore \int \frac{1}{1+y}\,dy = \int \frac{1}{2+x}\,dx$$

$$\therefore \ln(1+y) = \ln(2+x) + C$$

EXAMPLE4:

Solve $\frac{dy}{dx} = \frac{y^2 + xy^2}{x^2y - x^2}$

First express the RHS in 'x-factors' and 'y-factors':

$$\frac{dy}{dx} = \frac{y^2(1+x)}{x^2(y-1)}$$

THEN,

$$\boxed{\frac{y-1}{y^2}\,dy = \frac{1+x}{x^2}\,dx}$$

We now add the integral signs:

$$\int \frac{y-1}{y^2}\,dy = \int \frac{1+x}{x^2}\,dx$$

and complete the solution:

$$\int \left\{\frac{1}{y} - y^{-2}\right\} dy = \int \left\{x^{-2} + \frac{1}{x}\right\} dx$$

$$\therefore \ \ln y + y^{-1} = \ln x - x^{-1} + C$$

$$\therefore \ \ln y + \frac{1}{y} = \ln x - \frac{1}{x} + C$$

SOLVED EXAMPLES:

Find the general solutions of the following equations:

1 $\dfrac{dy}{dx} = \dfrac{y}{x}$

2 $\dfrac{dy}{dx} = (y+2)(x+1)$

3 $\cos^2 x \dfrac{dy}{dx} = y + 3$

4 $\dfrac{dy}{dx} = xy - y$

5 $\dfrac{\sin x}{1+y} \cdot \dfrac{dy}{dx} = \cos x$

SOLUTIONS:

1.

1 $\dfrac{dy}{dx} = \dfrac{y}{x} \quad \therefore \quad \int \dfrac{1}{y} dy = \int \dfrac{1}{x} dx$

$$\therefore \ln y = \ln x + C$$

$$= \ln x + \ln A$$

$$\therefore y = Ax$$

2 $\frac{dy}{dx} = (y+2)(x+1)$

$\therefore \int \frac{1}{y+2} dy = \int (x+1) dx$

$\therefore \ln(y+2) = \frac{x^2}{2} + x + C$

3 $\cos^2 x \frac{dy}{dx} = y + 3$

$\therefore \int \frac{1}{y+3} dy = \int \frac{1}{\cos^2 x} dx$

$= \int \sec^2 x \, dx$

$\ln(y+3) = \tan x + C$

4 $\frac{dy}{dx} = xy - y \quad \therefore \frac{dy}{dx} = y(x-1)$

$\therefore \int \frac{1}{y} dy = \int (x-1) dx$

$\therefore \ln y = \frac{x^2}{2} - x + C$

5 $\frac{\sin x}{1+y} \cdot \frac{dy}{dx} = \cos x$

$\int \frac{1}{1+y} dy = \int \frac{\cos x}{\sin x} dx$

$\therefore \ln(1+y) = \ln \sin x + C$

$= \ln \sin x + \ln A$

$1 + y = A \sin x$

$\therefore y = A \sin x - 1$

في المحاضره القادمه سنشرح الطريقتين الاخرى بعونه تعالى

انتهت