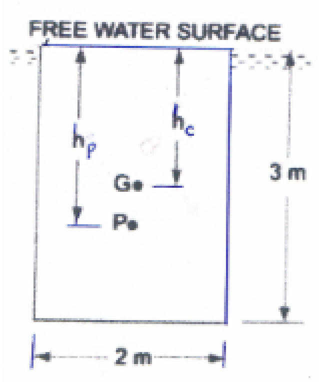
Problem 3.1 /

A rectangular plane surface is 2 m wide and 3 m deep. It lies in vertical plane in water. Determine the total pressure and position of center of pressure on the plane surface when its upper edge is horizontal and (a) coincides with water surface , (b) 2.5 m below the free water surface .

Solution :



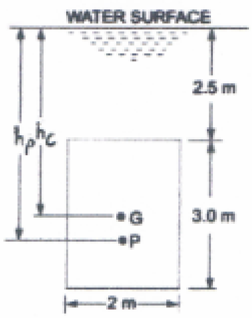
(a) Area (A) = $2 \times 3 = 6\ m^2$, $h_c = \frac{1}{2} \times 3 = 1.5\ m$

$F = \rho\, g\, A\, h_c$, $F = 1000 \times 9.81 \times 6 \times 1.5 = 88290\ N$

$h_p = \frac{I_G}{A\, h_c} + h_c$, $I_G = \frac{b\, d^3}{12} = \frac{2 \times 3^3}{12} = 4.5\ m^4$

$h_p = \frac{4.5}{6 \times 1.5} + 1.5 = 0.5 + 1.5 = 2\ m$

(b)



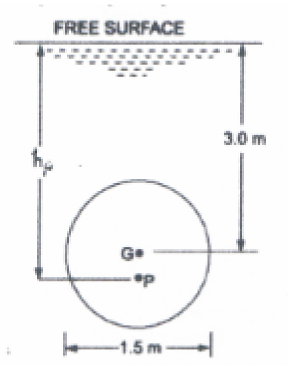
$h_c = 2.5 + \frac{3}{2} = 4\ m$, $F = \rho\, g\, A\, h_c = 1000 \times 9.81 \times 6 \times 4 = 235440\ N$

$$h_p = \frac{I_G}{A h_c} + h_c \quad , \quad h_p = \frac{4.5}{6 \times 4} + 4 = 0.1875 + 4 = 4.1875 \text{ m}$$

Problem 3.2 /

Determine the total pressure force on a circular plate of diameter 1.5 m which is placed vertically in water in such a way that the center of the plate is 3 m below the free surface of water. Find the position of center of pressure also .

Solution :



$$A = \frac{\pi}{4} \times (1.5)^2 = 1.767 \text{ m}^2 \quad , \quad h_c = 3 \text{ m}$$

$$F = \rho\, g\, A\, h_c = 1000 \times 9.81 \times 1.767 \times 3 = 52002.81 \text{ N}$$

$$h_p = \frac{I_G}{A h_c} + h_c \quad , \quad I_G = \frac{\pi\, d^4}{64} = \frac{\pi\,(1.5)^4}{64} = 0.2485 \text{ m}^4$$

$$h_p = \frac{0.2485}{1.767 \times 3} + 3 = 0.0468 + 3 = 3.0468 \text{ m}$$

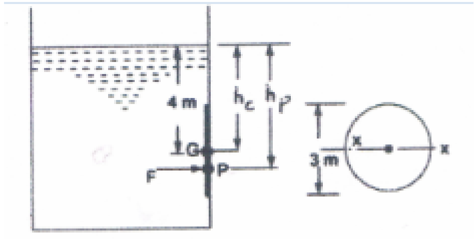
Problem 3.3 /

A circular opening , 3 m diameter , in a vertical side of a tank is closed by a disc of 3 m diameter which can rotate about a horizontal diameter , calculate (1) the force on the disc , (2) the torque required to maintain the disc in equilibrium in the vertical position in the vertical position when the head of water above the horizontal diameter is 4 m .

Solution :

(1) Area (A) = $\frac{\pi}{4}\, d^2 = \frac{\pi}{4} \times (3)^2 = 7.068 \text{ m}^2 \quad , \quad h_c = 4 \text{ m}$

$$F = \rho\, g\, A\, h_c = 1000 \times 9.81 \times 7.068 \times 4 = 277368 \text{ N}$$



(2)

$$h_p = \frac{I_G}{A\,h_c} + h_c \quad , \; I_G = \frac{\pi}{64} d^4 = \frac{\pi}{64} \times 3^4 = 3.974 \;\; m^4$$

$$h_p = \frac{3.974}{7.068 \times 4} + 4 = 0.14 + 4 = 4.14 \text{ m}$$

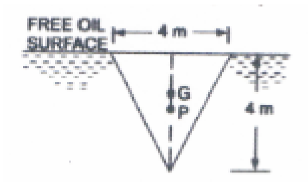
Moment of this force about horizontal diameter X - X ,

$$= F \times (h_p - h_c) = 277368 \; (4.14 - 4) = 38831 \;\; N . m$$

Problem 3.4 /

Determine the total pressure force and center of pressure on an isosceles triangular plate of base 4 m and altitude 4 m , when its immersed vertically in an oil of sp. gr. 0.9. The base of the plate coincides with the free surface of oil .

Solution :



$$\text{Area (A)} = \frac{b \times h}{2} = \frac{4 \times 4}{2} = 8 \;\; m^2 \quad , \; h_c = \frac{1}{3} h = \frac{1}{3} \times 4 = 1.33 \text{ m}$$

$$F = \rho\, g\, A\, h_c = 0.9 \times 1000 \times 9.81 \times 1.33 = 9597.6 \quad N$$

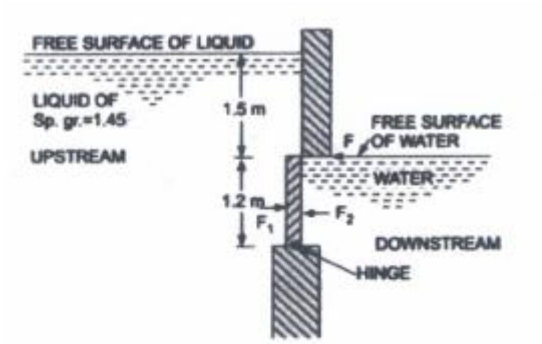
$$h_p = \frac{I_G}{A\,h_c} + h_c \quad , \; I_G = \frac{b\,h^3}{36} = \frac{4 \times 4^3}{36} = 7.11 \; m^4$$

$$h_p = \frac{7.11}{8 \times 1.33} + 1.33 = 0.666 + 1.33 = 1.99 \;\; m$$

<u>Problem 3.5</u> /

A vertical sluice gate is used to cover an opening in a dam. The opening is 2 m wide and 1.2 m high. On the upstream of the gate , the liquid of sp.gr. 1.45 , lies up to a height of 1.5 m above the top of the gate , whereas on the downstream side the water is available up to height touching the top of the gate. (1)Find the resultant force acting on the gate and position of center of pressure.(2) Find also the force acting horizontally at the top of the gate which is capable of opening it. Assume that the gate is hinged at the bottom .

<u>Solution</u> :



(1)

Area (A) = b × d = 2 × 1.2 = 2.4 m^2

For liquid : $h_{cL} = 1.5 + \frac{1.2}{2} = 2.1$ m

$F_1 = \rho_L\, g\, A\, h_{cL} = 1{,}45 \times 1000 \times 9.81 \times 2.4 \times 2.1 = 71691$ N

$h_{pL} = \frac{I_G}{A\, h_{cL}} + h_{cL}$, $I_G = \frac{b\,d^3}{12} = \frac{2 \times 1.2^3}{12} = 0.288\ m^4$

$h_{pL} = \frac{0.288}{2.4 \times 2.1} + 2.1 = 0.0571 + 2.1 = 2.1571$ m

For water : $h_{cw} = \frac{1}{2} \times 1.2 = 0.6$ m

$F_2 = \rho_w\, g\, A\, h_{cw} = 1000 \times 9.81 \times 2.4 \times 0.6 = 14126$ N

$h_{pw} = \frac{I_G}{A\, h_{cw}} + h_{cw}$, $h_{pw} = \frac{0.288}{2.4 \times 0.6} + 0.6 = 0.2 + 0.6 = 0.8$ m

Resultant force on the gate (R) = $F_1 - F_2 = 57565$ N

(2) position of center pressure of resultant force (R) :

Take the moments of forces at the hinge :

$57565 \times h_p = 71691 \times [\ (1.5 + 1.2) - 2.1571\ ] - 14126 \times (1.2 - 0.8)$

$h_p = 0.578$ m

(3) For calculate the force (F) which acts at the top of gate for opening it :

Take the moments for forces F_1 , F_2 and F at the hinge :

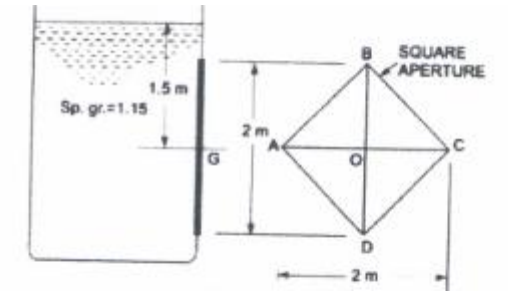
$F \times 1.2 + F_2 \times (1.2 - 0.8) = F_1 \times [(1.5 + 1.2) - 2.1571]$

F = 27725.5 N

Problem 3.6 /

A square aperture in the vertical side of a tank has one diagonal vertical and is completely covered by a plane hinged along one of the upper sides of aperture. The diagonals of the aperture are 2 m long and the tank contains a liquid of specific gravity 1.15 . The center of aperture is 1.5 m below the free surface. Calculate the thrust (force) exerted on the plate by the liquid and position of its center of pressure.

Solution :



Area of square aperture (A) = Area of Δ ACB + Area of Δ ACD

$$= \frac{2 \times 1}{2} + \frac{2 \times 1}{2} = 2 \text{ m}^2$$

$h_c = 1.5$ m

$F = \rho\, g\, A\, h_c = 1.15 \times 1000 \times 9.81 \times 2 \times 1.5 = 33844.5$ N

$$h_p = \frac{I_G}{A\, h_c} + h_c \;\; , \;\; I_G = \frac{b_1\, h_1^3}{12} + \frac{b_2\, h_2^3}{12} = \frac{2 \times 1^3}{12} + \frac{2 \times 1^3}{12}$$

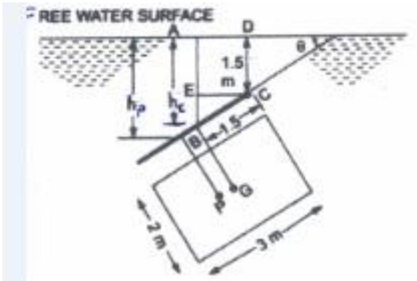
$$= \frac{1}{6} + \frac{1}{6} = \frac{1}{3} \text{ m}^4$$

$$h_p = \frac{\frac{1}{3}}{2 \times 1.5} + 1.5 = 1.611 \text{ m}$$

Problem 3.7 /

A rectangular plane surface 2 m wide and 3 m deep lies in water in such a way that its plane makes an angle of 30^0 with the free surface of water. Determine the total pressure force and position of center of pressure when the upper edge is 1.5 m below the free water surface.

Solution :



$$\text{Area}(A) = 3 \times 2 = 6 \ m^2$$

$$h_c = 1.5 + BC \sin\theta = 1.5 + 1.5 \sin 30^o = 1.5 + 0.75 = 2.25 \ m$$

$$F = \rho_w \, g \, A \, h_c = 1000 \times 9.81 \times 6 \times 2.25 = 132435 \ N$$

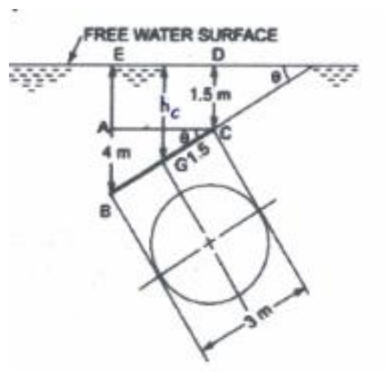
$$h_p = \frac{I_G \sin^2\theta}{A \, h_c} + h_c \quad , \quad I_G = \frac{b\,d^3}{12} = \frac{2 \times 3^3}{12} = 4.5 \ m^4$$

$$h_p = \frac{4.5 \times 0.25}{6 \times 2.25} + 2.25 = 0.0833 + 2.25 = 2.3333 \ m$$

Problem 3.8 /

A circular plate 3 m diameter is immersed in water in such a way that its greatest and least depth below the free surface are 4 m and 3 m respectively . Determine the total pressure force on one face of the plate and position of the center of pressure .

Solution :



$$\text{Area (A)} = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times 3^2 = 7.068 \ \ m^2$$

$$\sin\Theta = \frac{AB}{BC} = \frac{4 - 1.5}{3} = 0.8333$$

$$h_c = 1.5 + GC \sin\Theta = 1.5 + 1.5 \times 0.8333 = 2.749 \ \ m$$

$$F = \rho_w \, g \, A \, h_c = 1000 \times 9.81 \times 7.068 \times 2.749 = 190621 \ N$$

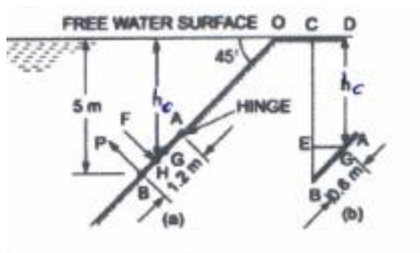
$$h_p = \frac{I_G \sin^2\theta}{A\, h_c} + h_c \quad , \quad I_G = \frac{\pi}{64} d^4 = \frac{\pi}{4} \times 3^2 = 3.976 \ \ m^4$$

$$h_p = \frac{3.976 \times (0.8333)^2}{7.068 \times 2.749} + 2.749 = 0.142 + 2.749 = 2.891 \ m$$

<u>Problem 3.9</u> /

An inclined rectangular sluice gate AB , 1.2 m by 5 m size as shown in Fig. is installed to control the discharge of water. The end A is hinged. Determine the force normal to the gate applied at B to open it .

<u>Solution</u> :



$$\text{Area of gate (A)} = 1.2 \times 5 = 6 \ m^2$$

$$h_c = 5 - BG \sin 45^o = 5 - 0.6 \times \frac{1}{\sqrt{2}} = 4.576 \ m$$

$$F = \rho_w \, g \, A \, h_c = 1000 \times 9.81 \times 6 \times 4.576 = 269\ 343 \ \ N$$

$$h_p = \frac{I_G \ sin^2\theta}{A \ h_c} + h_c \ , \ I_G = \frac{b \ d^3}{12} = \frac{5 \times 1.2^3}{12} = 0.72 \ m^4$$

$$h_p = \frac{0.72 \ (\ sin^2 \ 45)}{6 \times 4.576} + 4.576 = 0.013 + 4.576 = 4.589 \ m$$

$$\sin 45^o = \frac{h_p}{OH} \ , \ OH = \frac{h_p}{sin45} = \frac{4.589}{\frac{1}{\sqrt{2}}} = 6.489 \ m$$

$$\sin 45^o = \frac{5}{BO} \ , \ BO = 5 \times \sqrt{2} = 7.071 \ m$$

$$BH = BO - OH = 7.071 - 6.489 = 0.582 \ m$$

$$AH = AB - BH = 1.2 - 0.582 = 0.618 \ m$$

Taking the moments about the hinge A :

$$P \times AB = F \times AH$$

Where P is the force normal to the gate applied at B ,

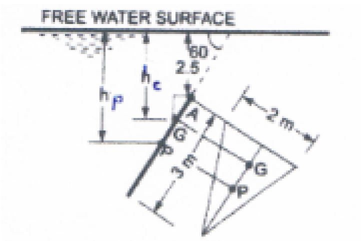
$$P \times 1.2 = 269343 \times 0.618$$

$$P = 138708 \ N$$

Problem 3.10 /

Find the total pressure force and position of center of pressure on a triangular plate of base 2 m and height 3 m which is immersed in water in such a way that the plane of the plate makes an angle of 60° with the free surface of the water. The base of the plate is parallel to water surface and at a depth of 2.5 m from water surface .

Solution :



$$\text{Area (A)} = \frac{b \times h}{2} = \frac{2 \times 3}{2} = 3 \ m^2$$

$$h_c = 2.5 + AG \sin 60^o = 2.5 + \frac{1}{3} \times 3 \times \frac{\sqrt{3}}{2} = 3.366 \ m$$

$$F = \rho_w\, g\, A\, h_c = 1000 \times 9.81 \times 3 \times 3.366 = 99061 \;\; N$$

$$h_p = \frac{I_G \sin^2\theta}{A h_c} + h_c \quad , \quad I_G = \frac{b\, h^3}{36} = \frac{2 \times 3^3}{36} = 1.5 \;\; m^4$$

$$h_p = \frac{1.5 \sin^2 60}{3 \times 3.366} + 3.366 = 0.111 + 3.366 = 3.477 \;\; m$$