

# Higher Derivatives

Examples: Find ~~the~~ <sup>Fourth</sup> derivatives of the following function:

①  $y = 3x^3 - 4x^2 + 7x + 10$

②  $y = \frac{1}{x} + \sqrt{x^3}$

③  $y = \frac{1}{\sqrt{x}} + 5x^3 - 2\sqrt{x}$

Solution

①  $y = 3x^3 - 4x^2 + 7x + 10$

$$\frac{dy}{dx} = 9x^2 - 8x + 7$$

$$\frac{d^2y}{dx^2} = 18x - 8$$

$$\frac{d^3y}{dx^3} = 18$$

$$\frac{d^4y}{dx^4} = \frac{d^5y}{dx^5} = \dots = \text{Zero}$$

①

$$\textcircled{2} \quad y = \frac{1}{x} + \sqrt{x^3}$$

$$y = \frac{1}{x} + x^{\frac{3}{2}}$$

$$\frac{dy}{dx} = \frac{x \cdot 0 - 1 \cdot 1}{x^2} + \frac{3}{2} x^{\frac{3-2}{2} = \frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{-1}{x^2} + \frac{3}{2} x^{\frac{1}{2}}$$

$$\frac{d^2y}{dx^2} = \frac{x^2 \cdot 0 - (-1) \cdot 2x}{x^4} + \frac{3}{2} \cdot \frac{1}{2} x^{\frac{1-2}{2} = -\frac{1}{2}}$$

$$\frac{d^2y}{dx^2} = \frac{2x}{x^4} + \frac{3}{4} x^{-\frac{1}{2}}$$

$$\frac{d^2y}{dx^2} = \frac{2}{x^3} + \frac{3}{4} x^{-\frac{1}{2}}$$

$$\frac{d^3y}{dx^3} = \frac{x^3 \cdot 0 - 2 \cdot 3x^2}{x^6} + \frac{3}{4} \cdot \frac{-1}{2} x^{-\frac{1}{2}-1} = \frac{-3}{2} x^{-\frac{3}{2}}$$

$$\frac{d^3y}{dx^3} = \frac{-6x^2}{x^6} - \frac{3}{8} x^{-\frac{3}{2}}$$

$$\frac{d^3y}{dx^3} = \frac{-6}{x^4} - \frac{3}{8} x^{-\frac{3}{2}}$$

$$\frac{d^4y}{dx^4} = \frac{x^4 \cdot 0 - (-6) \cdot 4x^3}{x^8} - \frac{3}{8} + \frac{-3}{2} x^{-\frac{3}{2}-1} = \frac{-6}{x^4} - \frac{3}{2} x^{-\frac{5}{2}}$$

$\textcircled{2}$

$$\frac{d^4 y}{dx^4} = \frac{24x^3}{x^8} + \frac{9}{16} x^{-5/2}$$

$$\frac{d^4 y}{dx^4} = \frac{24}{x^5} + \frac{9}{16} x^{-5/2}$$

$$\frac{d^4 y}{dx^4} = \frac{24}{x^5} + \frac{9}{16\sqrt{x^5}}$$

$$\textcircled{3} \quad y = \frac{1}{\sqrt{x}} + 5x^3 - 2\sqrt{x}$$

$$y = x^{-1/2} + 5x^3 - 2x^{1/2}$$

$$\frac{dy}{dx} = -\frac{1}{2} x^{-1/2-1} + 15x^2 - 2 * \frac{1}{2} x^{1/2-1}$$

$$\frac{1-2}{2} = -\frac{1}{2}$$

$$\frac{dy}{dx} = -\frac{1}{2} x^{-3/2} + 15x^2 - x^{-1/2}$$

$$\frac{-3-2}{2} = -\frac{5}{2}$$

$$\frac{-1-2}{2} = -\frac{3}{2}$$

$$\frac{d^2 y}{dx^2} = -\frac{1}{2} * \frac{-3}{2} x^{-3/2-1} + 30x - (-\frac{1}{2}) x^{-1/2-1}$$

$$\frac{d^2 y}{dx^2} = \frac{3}{4} x^{-5/2} + 30x + \frac{1}{2} x^{-3/2}$$

$$\frac{-5-2}{2} = -\frac{7}{2}$$

$$\frac{-3-2}{2} = -\frac{5}{2}$$

$$\frac{d^3 y}{dx^3} = \frac{3}{4} * \frac{-5}{2} x^{-5/2-1} + 30 + \frac{1}{2} * \frac{-3}{2} x^{-3/2-1}$$

③

$$\frac{d^3y}{dx^3} = \frac{-15}{8} X^{-7/2} + 30 - \frac{3}{4} X^{-5/2}$$

$$\frac{d^4y}{dx^4} = \frac{-15}{8} \cdot \frac{-7-2}{2} X^{-7/2-1} + 0 - \frac{3}{4} \cdot \frac{-5-2}{2} X^{-5/2-1}$$

$$\frac{d^4y}{dx^4} = \frac{105}{16} X^{-9/2} + \frac{15}{8} X^{-7/2}$$

$$\frac{d^4y}{dx^4} = \frac{105}{16\sqrt{x^9}} + \frac{15}{8\sqrt{7}}$$